KL-Expansion Based Channel Estimator for Space-time/frequency Coded OFDM Systems with Transmitter Diversity

Habib Şenol[1], **Hakan A. Çırpan**[2], **Erdal Panayırcı**[3] and **Mesut Çevik**[4]

[1]Kadir Has University, Department of Computer Engineering
Cibali, 34230 İstanbul
e-mail: hsenol@khas.edu.tr

[2,4]Istanbul University, Department of Electrical Engineering
Avcılar, 34320 İstanbul
e-mail: {hcirpan,mcevik}@istanbul.edu.tr

[3]IŞIK University, Department of Electronics Enginering
Maslak, 80670 İstanbul
e-mail: eepanay@isikun.edu.tr

Key words: OFDM Systems, MMSE Channel Estimation, Karhunen-Loeve Expansion, diversity

Abstract

Focusing on transmit diversity orthogonal frequency division multiplexing (OFDM) transmission through frequency selective channels, this paper pursues a channel estimation approach in time-domain for both space-frequency OFDM (SF-OFDM) and space-time OFDM (ST-OFDM) systems. The paper proposes a computationally efficient, pilot-aided linear minimum mean square error (MMSE) time-domain channel estimation algorithm for OFDM systems with transmitter diversity in unknown wireless fading channels. The proposed approach employs a convenient representation of the channel impulse responses based on the Karhunen-Loeve (KL) orthogonal expansion and finds MMSE estimates of the uncorrelated KL series expansion coefficients. Based on such an expansion, no matrix inversion is required in the proposed MMSE estimator. The performance of the proposed approach is studied through analytical and experimental results.

1 Introduction

OFDM has emerged as an attractive and powerful alternative to conventional modulation schemes in the recent past due to its various advantageous in lessening the severe effect of frequency selective fading. The broadband channel undergoes severe multipath fading, the equalizer in a conventional single-carrier modulation becomes prohibitively complex to implement. OFDM is therefore chosen over a single-carrier solution due to lower complexity of equalizers [1]. In OFDM, the entire signal bandwidth is divided into a number of narrow bands or orthogonal subcarriers, and signal is transmitted in the narrow bands in parallel. Therefore, it reduces intersymbol interference (ISI) and obviates the need for complex equalization thus greatly simplifies channel equalization task. Moreover, its structure also allows efficient hardware implementations using fast Fourier transform (FFT) and polyphase filtering. On the other hand, due to dispersive property of the wireless channel, subcarriers on those deep fades may be severely attenuated. To robustify the performance against deep fades, diversity techniques have to be used. Transmit antenna diversity is an effective technique for combatting fading in mobile in multipath wireless channels [2, 3]. Among a number of antenna diversity methods, the Alamouti method is very simple to implement [3]. This is an example for space-time block code (STBC) for two transmit antennas, and the simplicity of the receiver is attributed to the orthogonal nature of the code [4].

The use of OFDM in transmitter diversity systems motivates exploitation of diversity dimensions. Inspired by this, a number of coding schemes have been proposed recently to achieve maximum diversity gain [5, 6, 7]. Among them ST-OFDM have been proposed recently for delay spread channels. On the other hand, transmitter SF-OFDM also offers the possibility of coding in a form of SF-OFDM [5, 6, 7]. Moreover, SF-OFDM and ST-OFDM transmitter diversity systems were compared in [5], under the assumption that the channel responses are known or can be estimated accurately at the receiver. It was shown that the SF-OFDM system has the same performance as a previously reported ST-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments. Also, since, SF-OFDM transmitter diversity scheme performs the decoding within one OFDM block, it only requires half of the memory at the decoder which is needed for the ST-OFDM system of the same block size. Similarly, the decoder latency for SF-OFDM is also half that of the ST-OFDM implementation.

Multipath fading channels have been studied extensively, and several models have been developed to describe their variations. In many cases, the channel taps are modelled as general lowpass stochastic processes (e.g., [8]), the statistics depend on mobility parameters. A different approach explicitly models the multipath channel taps by the Karhunen-Loeve (KL) series representation [9],[10]. KL expansion models have also been used previously in modelling multipath channel within a CDMA scenario. In the case of KL series representation of stochastic process, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated. When these orthogonal bases are employed to expand the channel taps of the multipath channel, uncorrelated coefficients indeed represent the multipath channel. Therefore, KL representation allows one to tackle the estimation of correlated multipath parameters as a parameter estimation problem of the uncorrelated coefficients. Exploiting KL expansion, the main contribution of this paper is to propose a computationally efficient, pilot-aided MMSE channel estimation algorithms for both ST-OFDM and SF-OFDM systems while focusing on transmit diversity OFDM transmissions through unknown frequency selective fading channels. We derive the computationally efficient, MMSE channel estimation algorithms for both transmitter diversity OFDM systems under the assumption that the fading processes are constant over the duration of one code word.

2 Alamouti's Transmit Diversity Scheme for OFDM Systems

We consider the Alamouti transmitter diversity coding scheme, employed in an OFDM system utilizing K subcarrier per antenna transmissions. Note that K is chosen as an even integer. The fading channel between the μth transmit antenna and the receive antenna is assumed to be frequency selective and is described by the discrete-time baseband equivalent impulse response $\boldsymbol{h}_\mu(n) = [h_{\mu,0}(n), \cdots, h_{\mu,L}(n)]^T$, with L standing for the channel order. We *unify* the observation model for SF-OFDM and ST-OFDM as follows:

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 \\ -\mathcal{X}_2^\dagger & \mathcal{X}_1^\dagger \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix} . \tag{1}$$

For convenience, we list the corresponding vectors and matrix for SF-OFDM as

$$\left[\begin{array}{c} \mathbf{Y}_1 \\ \hline \mathbf{Y}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{Y}_e(n) \\ \hline \mathbf{Y}_o(n) \end{array}\right], \quad \left[\begin{array}{c|c} \mathcal{X}_1 & \mathcal{X}_2 \\ \hline -\mathcal{X}_2^\dagger & \mathcal{X}_1^\dagger \end{array}\right] = \left[\begin{array}{c|c} \mathcal{X}_e(n) & \mathcal{X}_o(n) \\ \hline -\mathcal{X}_o^\dagger(n) & \mathcal{X}_e^\dagger(n) \end{array}\right] \tag{2}$$

$$\left[\begin{array}{c} \mathbf{H}_1 \\ \hline \mathbf{H}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{H}_1(n) \\ \hline \mathbf{H}_2(n) \end{array}\right], \quad \left[\begin{array}{c} \mathbf{W}_1 \\ \hline \mathbf{W}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{W}_e(n) \\ \hline \mathbf{W}_o(n) \end{array}\right]$$

and for ST-OFDM as

$$\left[\begin{array}{c} \mathbf{Y}_1 \\ \hline \mathbf{Y}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{Y}(n) \\ \hline \mathbf{Y}(n+1) \end{array}\right], \quad \left[\begin{array}{c|c} \mathcal{X}_1 & \mathcal{X}_2 \\ \hline -\mathcal{X}_2^\dagger & \mathcal{X}_1^\dagger \end{array}\right] = \left[\begin{array}{c|c} \mathcal{X}(n) & \mathcal{X}(n+1) \\ \hline -\mathcal{X}^\dagger(n+1) & \mathcal{X}^\dagger(n) \end{array}\right] \tag{3}$$

$$\left[\begin{array}{c} \mathbf{H}_1 \\ \hline \mathbf{H}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{H}_1(n) \\ \hline \mathbf{H}_2(n) \end{array}\right] , \quad \left[\begin{array}{c} \mathbf{W}_1 \\ \hline \mathbf{W}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{W}(n) \\ \hline \mathbf{W}(n+1) \end{array}\right] .$$

Here, for SF-OFDM: $\mathbf{Y}_e(n)$ and $\mathbf{Y}_o(n)$ are even and odd blocks of $K/2$ tones where the received signal sequence is parsed in. $\mathcal{X}_e(n)$ and $\mathcal{X}_o(n)$ are $K/2 \times K/2$ diagonal matrices whose elements are $\mathbf{X}_e(n)$ and $\mathbf{X}_o(n)$ respectively and $(.)^\dagger$ denotes conjugate transpose. The complex channel gains between adjacent subcarriers are assumed to be approximately constant, i.e., $\mathbf{H}_{1,e}(n) \approx \mathbf{H}_{1,o}(n)$ and $\mathbf{H}_{2,e}(n) \approx \mathbf{H}_{2,o}(n)$ which are even and odd parts of the channel gains. Finally, $\mathbf{W}_e(n)$ and $\mathbf{W}_o(n)$ are zero-mean, i.i.d. Gaussian vectors with covariance matrix $\sigma^2\mathbf{I}_{K/2}$.

And for ST-OFDM: $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are $K \times K$ diagonal matrices whose elements are $\mathbf{X}(n)$ and $\mathbf{X}(n+1)$ respectively. $\mathbf{W}(n)$ and $\mathbf{W}(n+1)$ are zero-mean, i.i.d. Gaussian vectors with covariance matrix

$\sigma^2\mathbf{I}_K$ per dimension.To simplify the problem, we assume that the complex channel gains remain constant over the duration of one ST-OFDM code word, i.e., $\mathbf{H}_1(n) \approx \mathbf{H}_1(n+1)$ and $\mathbf{H}_2(n) \approx \mathbf{H}_2(n+1)$. As will be seen, such an assumption significantly simplifies the channel estimation algorithm. Similarly, the effect of this assumption allows us to omit dependence of channel attenuations on two different time indexes.

Relying on the unifying model (1), we will develop a channel estimation algorithm according to the MMSE criterion. A different approach is adapted here to explicitly model the channel parameters by the KL series representation since, KL expansion allows one to tackle the estimation of correlated parameters as a parameter estimation problem of the uncorrelated coefficients. Note that KL expansion is well known for its optimal truncation property [9]. That is, the KL expansion requires the minimum number of terms among all possible series expansions in representing a random channel for a given mean-squared error. Thus, the optimal truncation property of the KL expansion results in a smaller computational load on the channel estimation algorithm. We therefore will first summarize the KL representation of the multipath channel in the following section.

3 MMSE Estimation of the Multipath Channels

Pilot symbol assisted techniques can provide information about an undersampled version of the channel that may be easier to identify. In this paper, we therefore address the problem of estimating multipath channel parameters by exploiting the distributed training symbols.

Since both SF and ST block coded OFDM systems have symmetric structure in frequency and time respectively, the pilot symbols should be uniformly placed in pairs. Specifically, we also assume that even number of symbols are placed between pilot pairs for SF-OFDM systems. Based on these pilot structures, equation (1) is modified to represent the signal model corresponding to pilot symbols as follows:

$$\underbrace{\begin{bmatrix} \mathbf{Y}_{1,p} \\ \mathbf{Y}_{2,p} \end{bmatrix}}_{\mathbf{Y}_p} = \underbrace{\begin{bmatrix} \mathcal{X}_{1,p} & \mathcal{X}_{2,p} \\ -\mathcal{X}_{2,p}^{\dagger} & \mathcal{X}_{1,p}^{\dagger} \end{bmatrix}}_{\bar{\mathcal{X}}_p} \underbrace{\begin{bmatrix} \mathbf{H}_{1,p} \\ \mathbf{H}_{2,p} \end{bmatrix}}_{\mathbf{H}_p} + \underbrace{\begin{bmatrix} \mathbf{W}_{1,p} \\ \mathbf{W}_{2,p} \end{bmatrix}}_{\mathbf{W}_p} \tag{4}$$

where $(.)_p$ is introduced to represent the vectors corresponding to pilot locations.

Assuming pilot symbols are taken from QPSK modulation, the observation model can be formed by premultiplying both sides of (4) by $\bar{\mathcal{X}}_p^{\dagger}$

$$\bar{\mathcal{X}}_p^{\dagger}\mathbf{Y}_p = \bar{\mathcal{X}}_p^{\dagger}\bar{\mathcal{X}}_p\mathbf{H}_p + \bar{\mathcal{X}}_p^{\dagger}\mathbf{W}_p \ . \tag{5}$$

where $\bar{\mathcal{X}}_p^{\dagger}\bar{\mathcal{X}}_p = 2\,\mathbf{I}_{2K_p}$ and letting $\widetilde{\mathbf{Y}}_p = \bar{\mathcal{X}}_p^{\dagger}\mathbf{Y}_p$ and $\widetilde{\mathbf{W}}_p = \bar{\mathcal{X}}_p^{\dagger}\mathbf{W}_p$, we get the following equation:

$$\widetilde{\mathbf{Y}}_p = 2\,\mathbf{H}_p + \widetilde{\mathbf{W}}_p \ . \tag{6}$$

namely,

$$\begin{bmatrix} \widetilde{\mathbf{Y}}_{1,p} \\ \widetilde{\mathbf{Y}}_{2,p} \end{bmatrix} = 2\begin{bmatrix} \mathbf{H}_{1,p} \\ \mathbf{H}_{2,p} \end{bmatrix} + \begin{bmatrix} \widetilde{\mathbf{W}}_{1,p} \\ \widetilde{\mathbf{W}}_{2,p} \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} \widetilde{\mathbf{Y}}_{1,p} &= \mathcal{X}_{1,p}^{\dagger}\mathbf{Y}_{1,p} - \mathcal{X}_{2,p}\mathbf{Y}_{2,p} \\ \widetilde{\mathbf{Y}}_{2,p} &= \mathcal{X}_{2,p}^{\dagger}\mathbf{Y}_{1,p} + \mathcal{X}_{1,p}\mathbf{Y}_{2,p} \\ \widetilde{\mathbf{W}}_{1,p} &= \mathcal{X}_{1,p}^{\dagger}\mathbf{W}_{1,p} - \mathcal{X}_{2,p}\mathbf{W}_{2,p} \\ \widetilde{\mathbf{W}}_{2,p} &= \mathcal{X}_{2,p}^{\dagger}\mathbf{W}_{1,p} + \mathcal{X}_{1,p}\mathbf{W}_{2,p} \end{aligned} \tag{8}$$

and note that $\widetilde{\mathbf{W}}_{1,p} \sim \mathcal{N}(\mathbf{0}, 2\sigma^2\mathbf{I}_{K_p})$ and $\widetilde{\mathbf{W}}_{2,p} \sim \mathcal{N}(\mathbf{0}, 2\sigma^2\mathbf{I}_{K_p})$. By writing each row of the (7) separately, we get the following observation equation set to estimate the channels $\mathbf{H}_{1,p}$ and $\mathbf{H}_{2,p}$.

$$\widetilde{\mathbf{Y}}_{\mu,p} = 2\,\mathbf{H}_{\mu,p} + \widetilde{\mathbf{W}}_{\mu,p} \qquad \mu = 1,2$$

and substituting $\mathbf{H}_{\mu,p} = \mathbf{F}\boldsymbol{h}_\mu$ in (9), we get the following observation models for the channel impulse responses $\boldsymbol{h}_\mu$,

$$\widetilde{\mathbf{Y}}_{\mu,p} = 2\,\mathbf{F}\boldsymbol{h}_\mu + \widetilde{\mathbf{W}}_{\mu,p} \qquad \mu = 1,2$$

where $\mathbf{F}$ is an $K_p \times L$ FFT matrix generated based on pilot indices and K_p is the number of pilot symbols per one OFDM block. Equations (9) offers a Bayesian linear model representation. Based on these representation, the minimum variance estimator for the time-domain channel vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ can be obtained using the MMSE estimator. We should clearly make the assumptions that impulse responses $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are i.i.d. zero-mean complex Gaussian vectors with covariance $\mathbf{C}_{\boldsymbol{h}}$, and $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are independent with $\widetilde{\mathbf{W}}_{1,p}$ and $\widetilde{\mathbf{W}}_{2,p}$. Therefore, MMSE estimates of $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are given by [11]:

$$\hat{\boldsymbol{h}}_\mu = \left((2\,\mathbf{F})^\dagger \mathbf{C}^{-1}_{\widetilde{\mathbf{W}}_{\mu,p}} (2\,\mathbf{F}) + \mathbf{C}^{-1}_{\boldsymbol{h}} \right)^{-1} (2\,\mathbf{F})^\dagger \mathbf{C}^{-1}_{\widetilde{\mathbf{W}}_{\mu,p}} \widetilde{\mathbf{Y}}_{\mu,p} \qquad \mu = 1,2 \quad . \tag{9}$$

Due to PSK pilot symbol assumption together with the result $\widetilde{\mathbf{W}}_{1,p} \sim \mathcal{N}(\mathbf{0}, 2\sigma^2\mathbf{I}_{K_p})$ and $\widetilde{\mathbf{W}}_{2,p} \sim \mathcal{N}(\mathbf{0}, 2\sigma^2\mathbf{I}_{K_p})$, we can therefore express (9) by

$$\hat{\boldsymbol{h}}_\mu = \left(2\,\mathbf{F}^\dagger\mathbf{F} + \sigma^2\mathbf{C}^{-1}_{\boldsymbol{h}} \right)^{-1} \mathbf{F}^\dagger\,\widetilde{\mathbf{Y}}_{\mu,p} \qquad \mu = 1,2 \tag{10}$$

Under the assumption that uniformly spaced pilot symbols are inserted with pilot spacing interval Δ and $K = \Delta \times K_p$, correspondingly, $\mathbf{F}^\dagger\mathbf{F}$ reduces to $\mathbf{F}^\dagger\mathbf{F} = K_p\mathbf{I}_L$. Then according to (10), and $\mathbf{F}^\dagger\mathbf{F} = K_p\mathbf{I}_L$, we arrive at the expression

$$\hat{\boldsymbol{h}}_\mu = \left(2\,K_p\,\mathbf{I}_L + \sigma^2\mathbf{C}^{-1}_{\boldsymbol{h}} \right)^{-1} \mathbf{F}^\dagger\,\widetilde{\mathbf{Y}}_{\mu,p} \qquad \mu = 1,2 \tag{11}$$

As it can be seen from (11) since MMSE estimation of $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ for SF-OFDM and ST-OFDM systems still requires the inversion of $\mathbf{C}^{-1}_{\boldsymbol{h}}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with an KL expansion.

4 KL Representation of the Multipath Channels

Channel impulse response $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are i.i.d. zero-mean Gaussian process with covariance matrix $\mathbf{C}_{\boldsymbol{h}}$. The KL transformation is therefore employed here to rotate the vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ so that all their components are uncorrelated. The vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ can be expressed as a linear combination of the orthonormal basis vectors as follows:

$$\boldsymbol{h}_\mu = \sum_{l=0}^{L-1} \boldsymbol{\psi}_l\, g_{\mu,l} \;=\; \boldsymbol{\Psi}\,\boldsymbol{g}_\mu \quad , \quad \mu = 1,2 \tag{12}$$

where μ is the multipath channel index, $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \cdots, \boldsymbol{\psi}_{L-1}]$, $\boldsymbol{\psi}_l$'s are the orthonormal basis vectors, $\boldsymbol{g}_\mu = [g_{\mu,0}, g_{\mu,1}, \cdots\ g_{\mu,L-1}]^T$, and $g_{\mu,l}$'s are the weights of the expansion. Due to the fact that $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ vectors are i.i.d. zero-mean complex Gaussian vector, KL coefficient vectors $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ are also i.i.d. zero-mean complex Gaussian vectors. If we form the covariance matrix $\mathbf{C}_{\boldsymbol{h}}$ as

$$\mathbf{C}_{\boldsymbol{h}} = \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^\dagger \tag{13}$$

where $\boldsymbol{\Lambda} = E\{\boldsymbol{g}_\mu\boldsymbol{g}_\mu^\dagger\}$, the KL expansion is the one in which $\boldsymbol{\Lambda}$ of $\mathbf{C}_{\boldsymbol{h}}$ is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\boldsymbol{\Lambda}$ is diagonal, then the form $\boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^\dagger$ is called an *eigendecomposition* of $\mathbf{C}_{\boldsymbol{h}}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\boldsymbol{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in Gaussian case, the uncorrelateness of the coefficients renders them independent as well, providing additional simplicity.

Thus, the channel estimation problem in this application is equivalent to estimating the i.i.d. complex Gaussian vector $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ which represent KL expansion coefficients for multipath channels $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$.

5 MMSE Estimation of KL Coefficients

In contrast to (9) in which only $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are to be estimated, we now assume the KL coefficients $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ are unknown. substituting (12) in (9) unified signal model for SF-OFDM and ST-OFDM systems can be written as

$$\tilde{\mathbf{Y}}_{\mu,p} = 2\,\mathbf{F}\boldsymbol{\Psi}\boldsymbol{g}_\mu + \tilde{\mathbf{W}}_{\mu,p} \qquad \mu = 1,2 \tag{14}$$

which is also recognized as a Bayesian linear model, and recall that $\boldsymbol{g}_\mu \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Lambda})$. As a result, the MMSE estimator of $\boldsymbol{g}_\mu$ for SF-OFDM and ST-OFDM systems is

$$\begin{aligned} \hat{\boldsymbol{g}}_\mu &= \boldsymbol{\Lambda}(2\,K_p\,\boldsymbol{\Lambda} + \sigma^2\mathbf{I}_L)^{-1}\boldsymbol{\Psi}^\dagger\mathbf{F}^\dagger\tilde{\mathbf{Y}}_{\mu,p} & (15)\\ &= \boldsymbol{\Gamma}\;\boldsymbol{\Psi}^\dagger\mathbf{F}^\dagger\tilde{\mathbf{Y}}_{\mu,p} \qquad , \qquad \mu = 1,2 & (16) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\Gamma} &= \boldsymbol{\Lambda}(2\,K_p\,\boldsymbol{\Lambda} + \sigma^2\mathbf{I}_L)^{-1} & (17)\\ &= \mathrm{diag}\left\{\frac{\lambda_0}{2\,K_p\lambda_0 + \sigma^2}, \frac{\lambda_1}{2\,K_p\lambda_1 + \sigma^2}, \cdots, \frac{\lambda_{L-1}}{2\,K_p\lambda_{L-1} + \sigma^2}\right\} \end{aligned}$$

and $\lambda_0, \lambda_1, \cdots, \lambda_{L-1}$ are the singular values of $\boldsymbol{\Lambda}$.

It is clear that the complexity of the MMSE estimator in (11) is reduced by the application of KL expansion. However, the complexity of the $\hat{\boldsymbol{g}}_\mu$ can be further reduced by exploiting the optimal truncation property of the KL expansion [9].

6 Truncated KL Expansion

A truncated expansion $\boldsymbol{g}_\mu(n)$ can be formed by selecting r orthonormal basis vectors among all basis vectors that satisfy $\mathbf{C}_{\boldsymbol{h}}\boldsymbol{\Psi} = \boldsymbol{\Psi}\boldsymbol{\Lambda}$. The optimal one that yields the smallest average mean-squared truncation error is the one expanded with the orthonormal basis vectors associated with the first largest r eigenvalues. However, typically the pattern of eigenvalues for $\boldsymbol{\Lambda}$ splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious. For the optimal truncated KL (rank-r) estimator of (15), $\boldsymbol{\Gamma}$ in (15) now becomes

$$\boldsymbol{\Gamma} = \mathrm{diag}\left\{\frac{\lambda_0}{2\,K_p\lambda_0 + \sigma^2}, \frac{\lambda_1}{2\,K_p\lambda_1 + \sigma^2}, \cdots, \frac{\lambda_{r-1}}{2\,K_p\lambda_{r-1} + \sigma^2}, 0, \cdots, 0\right\} \tag{18}$$

Since our ultimate goal is to obtain MMSE estimator for the channel frequency response $\mathbf{H}_i$, from the invariance property of the MMSE estimator, it follows that if $\hat{\boldsymbol{g}}_\mu$ is the estimate of $\boldsymbol{g}_\mu$, then the corresponding estimate of $\mathbf{H}_\mu$ can be obtained as

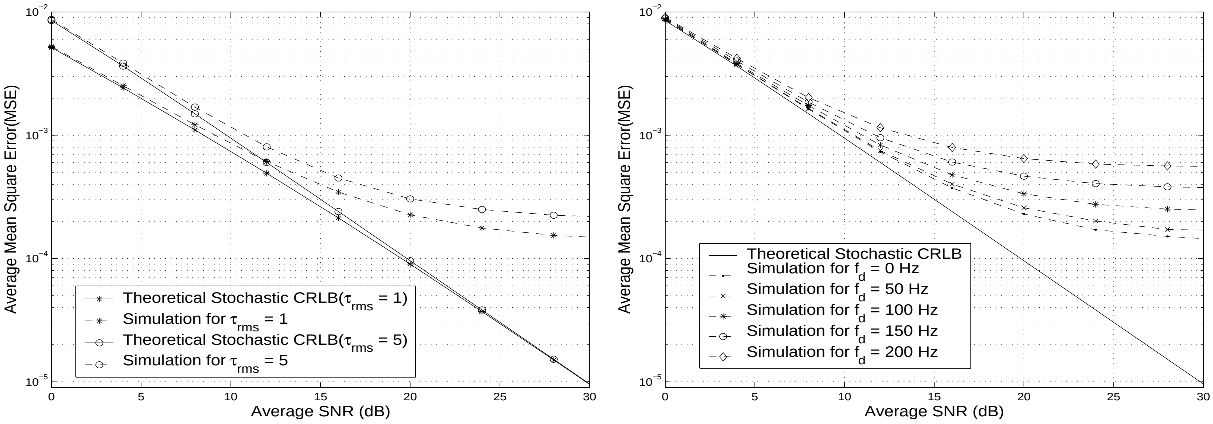
$$\hat{\mathbf{H}}_\mu = \mathcal{F}\boldsymbol{\Psi}\hat{\boldsymbol{g}}_\mu \qquad , \qquad \mu = 1,2 \tag{19}$$

where $\mathcal{F}$ is a $K \times K$ FFT matrix.

7 Simulations

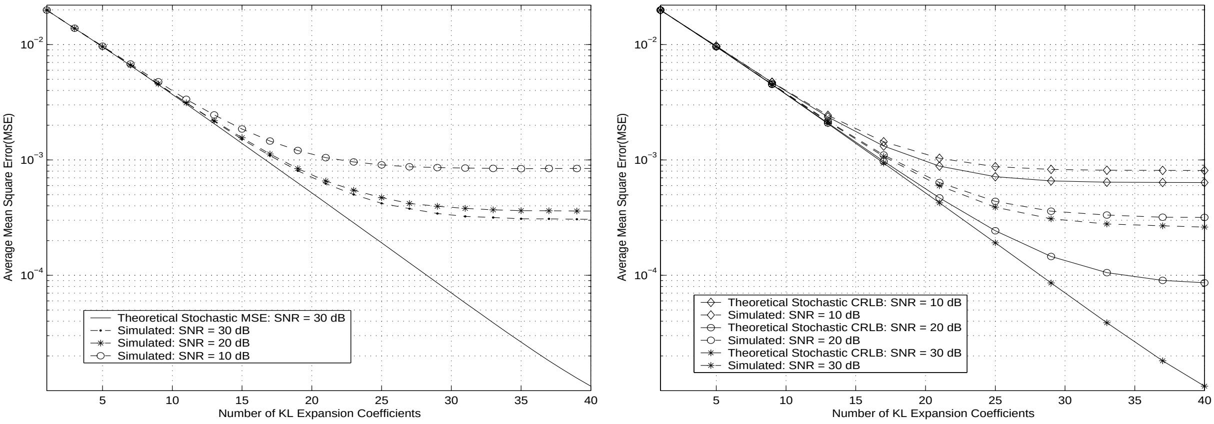
To illustrate the effectiveness of the proposed MMSE channel estimation algorithms for SF-OFDM and ST-OFDM systems, we present simulation results in Figure 1-a and Figure 1-b for the average MSE of multipath channels $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$. Figure 1-a compares the performance of the SF-OFDM for different rms values of multipath delays such as $\tau_{rms} = 1, 5$. Figure 1-b compares the performance of the ST-OFDM for different doppler frequencies such as $f_d = 0, 50, 100, 150, 200\,Hz$ as well.

The truncated estimator performance is also studied as a function of the number of KL coefficients. Figure 2-a is plotted for $L = 40, \tau_{rms} = 5$ samples for SF-OFDM and in addition Figure 2-b is plotted for $L = 40, fd = 100$ Hz for ST-OFDM. Figure 2-a and Figure 2-b present the MSE result of the truncated MMSE estimator for SNR = 10, 20 and 30 dB. If only a few expansion coefficients is employed to reduce the complexity of the proposed estimator, then the MSE between channel parameters becomes large. However, if the number of parameters in the expansion is increased, the irreducible error floor still occurs.



a) **b)**

Figure 1: Performance of the Proposed MMSE for a)SF-OFDM b)ST-OFDM



a) **b)**

Figure 2: MSE as a function of KL Expansion Coefficients for a)SF-OFDM b)ST-OFDM

8 Conclusion

We consider the design of low complexity MMSE channel estimators for SF/ST-OFDM systems in unknown wireless dispersive fading channels. We first derive the MMSE estimator based on the stochastic orthogonal expansion representation of the channel via KL transform. Based on such representation, we show that no matrix inversion is needed in the MMSE algorithm. Therefore, the computational cost for implementing the proposed MMSE estimator is low and computation is numerically stable.

References

[1] R. Van Nee and R. Prasad, "OFDM Wireless Multimedia Communications", in *Artech House Publishers*, 2000.

[2] Y. (G.) Li, L. J. Cimini, N. Seshadri and S. Ariyavistakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels", *IEEE Journal Selected Areas on Commun.*, vol. 17, no.3, pp. 461-471, July 1999.

[3] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Selected Areas Commun.*, vol. 16, pp. 1451-1458, October 1998.

[4] V. Tarokh, Hamid Jafarkhani, and A. R. Calderbank, "Space-time block Codes from orthogonal designs," *IEEE Trans. Inform. Theory* , vol. 45, no.5 pp. 1456-1467, July 1999.

[5] K.F. Lee and D.B. Williams,“A Space-Frequency Transmitter Diversity Technique for OFDM Systems”, *in Proc. IEEE Globecom*, San Francisco, CA, November 2000, pp. 1473-1477.

[6] Z. Liu, Y. Xin, and G.B.Giannakis, "Space-Time-Frequency Coded OFDM for Frequency Selective Fading Channels," *IEEE ,Transactions on Signal Processing,* vol. 50, no 10, pp.2465-2476, October 2002.

[7] H. Bolcskei and A. Paulraj, "Space-Frequency Coded Broadband OFDM Systems," IEEE Wireless Communications and Networking Conference (WCNC) 2000, Vol. 1, pp. 1-6, Sept. 2000.

[8] W.C. Jakes, "Microwave Mobile Communications," *New York Plenum* 1974.

[9] K. Yip and T. Ng, "Karhunen-Loeve Expansion of the WSSUS Channel Output and its Application to Efficient Simulation," *IEEE Journal on Selected Areas in Communications,* vol. 15, no 4, pp.640-646, May 1997.

[10] H. Şenol, H. A. Çırpan and E. Panayırcı, "MMSE Estimation Approach for Karhunen-Loeve Expansion of Multipath Channel in OFDM," *Sixth Baiona Workshop on Signal Procesing in Communications,* pp.85-90, September 8-10, 2003.

[11] S.M. Kay, "Fundamentals of Statistical Signal Processing: Estimation Theory," *Prentice Hall* 1993.