Cooperative Distributed Multiuser MMSE Relaying in Wireless Ad-Hoc Networks

Stefan Berger and Armin Wittneben
ETH Zurich, Communication Technology Laboratory, CH-8092 Zurich, Switzerland
Email: {berger, wittneben}@nari.ee.ethz.ch

Abstract

We consider a wireless ad-hoc network with single antenna nodes under a two-hop traffic pattern. Two system architectures are investigated in this paper: Either linear amplify-and-forward relays (LinRel) or a distributed antenna system with linear processing (LDAS) serve as repeater nodes. The gain factors of the repeaters are assigned such that the mean squared error (MSE) of the signal at the destinations is minimized (*multiuser MMSE relaying*). This essentially realizes a distributed spatial multiplexing gain with single antenna nodes as the source-destination pairs can communicate concurrently over the same physical channel. The main contribution of this paper is the derivation of the MMSE gain factors. We evaluate the relaying scheme in comparison to *multiuser zero forcing* (ZF) [1] for the linear relaying network as well as the linear distributed antenna system. The main advantage of multiuser MMSE relaying over ZF relaying is a substantial diversity gain for each source-destination link and a graceful performance degradation for small number of relays.

Keywords – cooperative relaying, ad-hoc networks, distributed spatial multiplexing, minimum mean squared error (MMSE)

I. Introduction

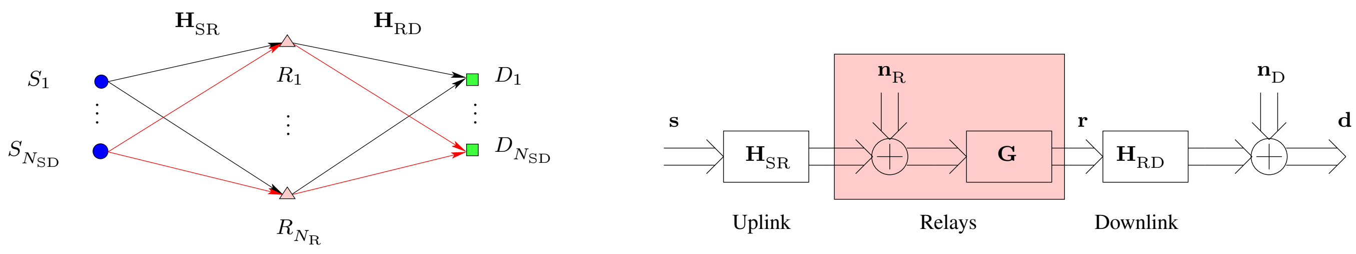
A means of providing the high data rate demanded for future wireless communication systems is spatial multiplexing. In a rich scattering environment Multiple Input/Multiple Output (MIMO) systems already achieve an unprecedented spectral efficiency. In addition to conventional MIMO systems, *distributed antenna systems* (DAS) employ multiple antennas, which are not colocated at one site [2]. A *linear distributed antenna system* (LDAS), which is a distributed antenna system with linear processing, can be used to act as relaying architecture in a multi-hop traffic pattern. Opposed to that, we call the concept of autonomous amplify-and-forward relays which are not connected via a wired backbone a *linear relaying* (LinRel) architecture.

Upper and lower bounds on the capacity of MIMO wireless networks are given in [3], [4]. In [5], [6] cooperative relaying schemes have been proposed to improve wireless communication in multinode networks by achieving diversity. The authors in [7] present amplify-and-forward (AF) as well as decode-and-forward (DF) protocols in relay, cooperative broadcast and cooperative multiple access channels and investigate the achieved diversity-multiplexing tradeoff. A relay selection scheme to find the 'best' end-to-end path between a single source and destination is presented in [8]. In contrast to that, we focus on space-time techniques that are able to achieve a spatial multiplexing gain by enabling more than one source-destination pair to communicate concurrently over the same physical channel.

Distributed antenna systems and linear relaying are proposed to relax the rich scattering requirement of conventional MIMO signalling in [9], [10]. A multiuser relaying space-time coding scheme, which nulls the interference between different source-destination pairs by an appropriate gain allocation at the amplify-and-forward relays is presented in [1]. The authors refer to this scheme as multiuser zero forcing (ZF) relaying. We will see that, compared to ZF relaying, multiuser MMSE relaying offers a substantial diversity gain.

The main focus of the present paper is the derivation and performance analysis of a new gain allocation that minimizes the mean squared error at the destinations.

Notation: The operators $\odot$, $\mathrm{E}_{\{x\}}[\cdot]$, $\mathrm{tr}(\cdot)$ and $(\cdot)^{\mathrm{H}}$ denote the elementwise product, expectation with respect to x, trace operation, and conjugate complex transpose, respectively. $\mathbf{I}_N$ is the identity matrix of size $N \times N$. $\mathrm{diag}(\cdot)$ has two meanings: When the argument is a matrix, it takes the diagonal elements and puts them into a column vector. When the argument is a vector, it puts the elements of the vector into a diagonal matrix. Finally, the conjugate complex transpose of an inverted matrix is denoted by $(\cdot)^{-\mathrm{H}}$.



(a) Two hop system configuration with half-duplex relays

(b) Signal model with AWGN at the relays and destinations

Fig. 1. System configuration and compound signal model for a two hop relay network

II. System Model

We consider a wireless network where N_{SD} source-destination pairs want to communicate concurrently on the same physical channel. N_{R} amplify-and-forward relay nodes assist the communication in a half-duplex scheme. They amplify the signals they receive from the sources by multiplication with complex gain factors before retransmitting them. All nodes in the network employ only one antenna. For the sake of a simple notation, we assume that the number of sources equals the number of destinations. This happens without loss of generality as inactive sources or destinations can be omitted in the consideration. The communication follows a two-hop relay traffic pattern, i.e., each transmission cycle includes two channel uses: one for the *uplink* transmission from the sources to all relays and one for the *downlink* transmission from the relays to the destinations. The direct link is not taken into account in the following considerations. **Figs. 1(a)** and **1(b)** show the system configuration and the compound signal model, respectively. The scalar transmit symbols are stacked in the vector $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{SD}}}$. No channel state information (CSI) is present at the sources. Thus, no power loading is performed. Each source is assumed to have the same transmit power $\sigma_{\mathrm{s}}^2 = \frac{P_{\mathrm{S}}}{N_{\mathrm{SD}}}$, where P_{S} is the total transmit power of all sources. The source signal vector $\mathbf{s}$ is first transmitted over the uplink matrix channel $\mathbf{H}_{\mathrm{SR}} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{SD}}}$ to the relays. The vector $\mathbf{n}_{\mathrm{R}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{R}}}^2 \mathbf{I}_{N_{\mathrm{R}}}\right)$ comprises the additive, white, Gaussian noise (AWGN) contributions at the relay nodes. After multiplication with the gain matrix $\mathbf{G} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{R}}}$, the signal $\mathbf{r}$ is passed through the downlink matrix channel $\mathbf{H}_{\mathrm{RD}} \in \mathbb{C}^{N_{\mathrm{SD}} \times N_{\mathrm{R}}}$ to the N_{SD} destination nodes. The vector $\mathbf{n}_{\mathrm{D}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{D}}}^2 \mathbf{I}_{N_{\mathrm{SD}}}\right)$ comprises the AWGN contribution at the destinations. For all numerical results we let $\sigma_{n_{\mathrm{R}}}^2 = \sigma_{n_{\mathrm{D}}}^2 := \sigma_n^2$ and use the same sum transmit power at the sources and at the relays. We apply this constraint because one of our goals is to study the influence of the number of relays on the performance of the system. Therefore, to have a fair comparison, additional relays shall not increase the received power at the destinations. Note that all channels are assumed to be frequency flat fading channels.

The signals at the destinations are stacked in the vector

$$\mathbf{d} = \mathbf{H}_{\mathrm{RD}} \mathbf{G} \mathbf{H}_{\mathrm{SR}} \cdot \mathbf{s} + \mathbf{H}_{\mathrm{RD}} \mathbf{G} \cdot \mathbf{n}_{\mathrm{R}} + \mathbf{n}_{\mathrm{D}} := \mathbf{H}_{\mathrm{SD}} \mathbf{s} + \mathbf{n}_{\mathrm{RD}}, \tag{1}$$

where $\mathbf{H}_{\mathrm{SD}}$ is called the *equivalent channel matrix* and $\mathbf{n}_{\mathrm{RD}}$ the *equivalent noise vector*. The components of $\mathbf{n}_{\mathrm{RD}}$ are spatially no longer white.

III. MMSE Relaying Gain Factors

The gain matrix $\mathbf{G}$ is to be designed such that the mean squared error of the signal at the destinations is minimized. In addition we demand the transmit power of all relays P_{R}, averaged over the transmit symbols, to be equal to the total transmit power of all sources P_{S}. The constraint optimization we have to perform in order to find the gain matrix $\mathbf{G}$ is then

$$\mathbf{G}_{\mathrm{MMSE}} = \arg\min_{\mathbf{G}} \mathrm{E}_{\{\mathbf{s}, \mathbf{n}_{\mathrm{R}}, \mathbf{n}_{\mathrm{D}}\}} \left[\left\| \mathbf{s} - \gamma^{-1} \mathbf{d} \right\|_2^2 \right] \quad \text{such that} \quad \mathrm{E}_{\{\mathbf{s}, \mathbf{n}_{\mathrm{R}}\}} \left[\mathbf{r}^{\mathrm{H}} \mathbf{r} \right] = P_{\mathrm{S}}. \tag{2}$$

The factor $\gamma \in \mathbb{R}$ allows for received signals which are a scaled version of the transmitted symbols. It turns out that the destinations do not require any channel knowledge. They only have to know the scaling factor γ in order to decode the symbols. This is an advantage over the ZF relaying scheme, where the destinations need to know the instantaneous equivalent channel matrix $\mathbf{H}_{\mathrm{SD}}$.

The choice of a scalar γ certainly is suboptimal. When the mean path loss between each source-relay-destination pair is different, the performance of the LinRel MMSE relaying scheme is expected to drop down. This is because the signal constellation of the received signals will scale differently. Calculating an individual scaling factor for each destination is necessary to overcome this drawback. However, results for this case are not presented here. The LDAS MMSE relaying scheme is not affected by this handicap as all relays can forward all signals jointly. The scaling of the signal constellations due to the instantaneous channel realizations is the same at all destinations.

A. Linear Relaying

Consider a linear relaying (LinRel) system architecture. We assume perfect *global channel knowledge* at the relays, i.e., all relays know the instantaneous uplink channel matrix $\mathbf{H}_{\mathrm{SR}}$ as well as the instantaneous downlink channel matrix $\mathbf{H}_{\mathrm{RD}}$ perfectly. However, they only have *local signal knowledge*, which means that they only know the signals they receive. They have no knowledge about the signals at the other relays. As a consequence the gain matrix $\mathbf{G}$ is diagonal. It comprises the N_{R} gain factors on its main diagonal: $\mathbf{G}_{\mathrm{MMSE}} = \mathrm{diag}\left(\mathbf{g}_{\mathrm{MMSE}}\right)$. Solving (2) with the help of Lagrangian multipliers, we get

$$\mathbf{g}_{\mathrm{MMSE}} = \gamma \cdot \left(\mathbf{B} \odot \mathbf{A}^{*}\right)^{-1} \mathrm{diag}\left(\mathbf{C}^{\mathrm{H}}\right) := \gamma \cdot \tilde{\mathbf{g}}_{\mathrm{MMSE}}, \tag{3}$$

where

$$\mathbf{A} := \left(\mathbf{H}_{\mathrm{SR}}\mathbf{R}_{\mathrm{s}}\mathbf{H}_{\mathrm{SR}}^{\mathrm{H}} + \mathbf{R}_{\mathrm{n_R}}\right) = \mathbf{A}^{\mathrm{H}}, \tag{4}$$

$$\mathbf{B} := \left(\mathbf{H}_{\mathrm{RD}}^{\mathrm{H}}\mathbf{H}_{\mathrm{RD}} + \frac{\mathrm{tr}\left(\mathbf{R}_{\mathrm{n_D}}\right)}{P_{\mathrm{S}}}\mathbf{I}_{N_{\mathrm{R}}}\right) = \mathbf{B}^{\mathrm{H}}, \tag{5}$$

$$\mathbf{C} := \mathbf{H}_{\mathrm{SR}}\mathbf{R}_{\mathrm{s}}\mathbf{H}_{\mathrm{RD}}, \tag{6}$$

with $\mathbf{R}_{\mathrm{s}} = \mathrm{E}_{\{\mathbf{s}\}}\left[\mathbf{s}\mathbf{s}^{\mathrm{H}}\right]$, $\mathbf{R}_{\mathrm{n_D}} = \mathrm{E}_{\{\mathbf{n}_{\mathrm{D}}\}}\left[\mathbf{n}_{\mathrm{D}}\mathbf{n}_{\mathrm{D}}^{\mathrm{H}}\right]$, $\mathbf{R}_{\mathrm{n_R}} = \mathrm{E}_{\{\mathbf{n}_{\mathrm{R}}\}}\left[\mathbf{n}_{\mathrm{R}}\mathbf{n}_{\mathrm{R}}^{\mathrm{H}}\right]$, and

$$\gamma = \sqrt{\frac{P_{\mathrm{S}}}{\tilde{\mathbf{g}}_{\mathrm{MMSE}}^{\mathrm{H}}\left(\mathbf{A} \odot \mathbf{I}_{N_{\mathrm{R}}}\right)\tilde{\mathbf{g}}_{\mathrm{MMSE}}}}. \tag{7}$$

B. Linear Distributed Antenna System

A linear distributed antenna system is considered in this section. As mentioned above, all relays are connected via a wired backbone and can thus share their received signals. We say that they have *global signal knowledge*. In contrast to the LinRel architecture, the gain matrix $\mathbf{G}$ does not have to be diagonal for this case. We still assume perfect *global channel knowledge* at all relays. The gain matrix $\mathbf{G}_{\mathrm{MMSE}}$ can again be found with (2):

$$\mathbf{G}_{\mathrm{MMSE}} = \gamma\, \mathbf{B}^{-1}\mathbf{C}^{\mathrm{H}}\mathbf{A}^{-\mathrm{H}} := \gamma\, \tilde{\mathbf{G}}_{\mathrm{MMSE}} \tag{8}$$

and

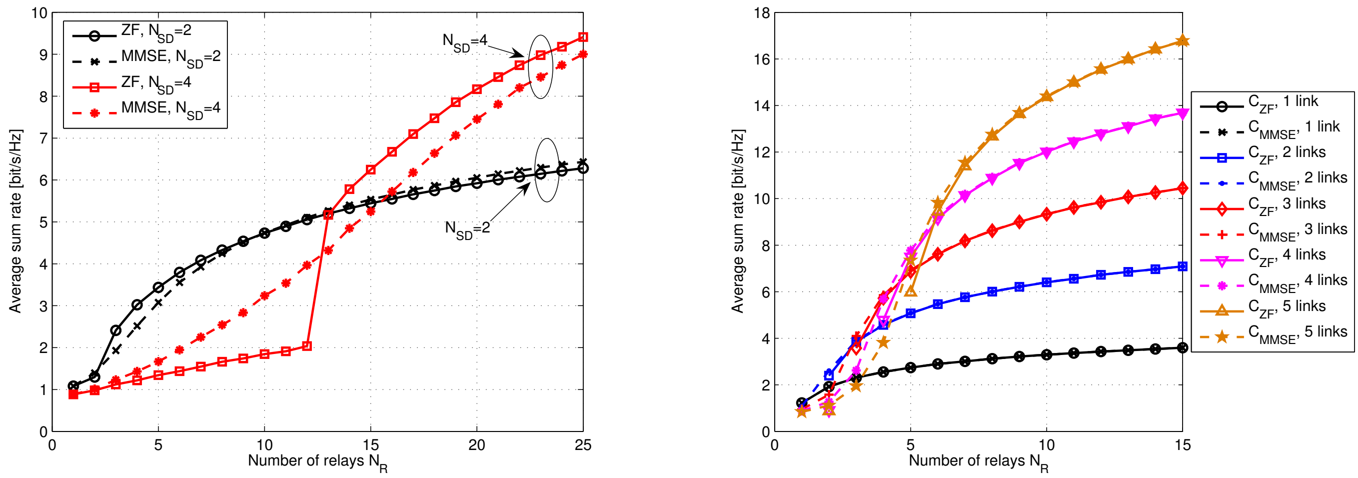
$$\gamma = \sqrt{\frac{P_{\mathrm{S}}}{\mathrm{tr}\left(\tilde{\mathbf{G}}_{\mathrm{MMSE}}\mathbf{A}\tilde{\mathbf{G}}_{\mathrm{MMSE}}^{\mathrm{H}}\right)}}. \tag{9}$$

We again used the substitutions (4), (5), and (6).

IV. Performance Results

In this section, we present the results of Monte Carlo simulations we performed in order to evaluate the MMSE relaying scheme and discuss them. As a reference we look at multiuser ZF relaying [9] and compare the performance of both systems. For all simulations we assumed flat i.i.d. Rayleigh fading $\mathcal{CN}(0,1)$. The channel matrices were constant during each transmission cycle (block fading) and temporally independent.

In **Fig. 2** the average sum rate of a linear relaying architecture and a linear distributed antenna system is plotted versus the number of relays. The parameter of the curves is the number of source-destination pairs N_{SD}. The left figure (**Fig. 2(a)**) shows the linear relaying architecture. We see that for $N_{\mathrm{R}} \leq N_{\mathrm{SD}}^2 - N_{\mathrm{SD}}$, the MMSE relaying with its slowly degrading performance towards small number of relays, achieves a higher average sum rate than ZF relaying. This is because the null space projection with which the zero-forcing gain matrix is found, cannot be performed below that limit [1]. For larger number of relays, the ZF processing scheme will at first achieve a higher average sum rate because it can perform a maximum ratio combining-like weighting of the subchannels when the initial gain vector is chosen accordingly [1]. For the linear distributed antenna system (**Fig. 2(b)**), both signalling schemes perform nearly identical. However, it can be seen that ZF relaying is not possible if the number of relays N_{R} is smaller than the number of source-destination pairs N_{SD}.



(a) LinRel

(b) LDAS

Fig. 2. Average sum rate versus the number of relays for SNR = 10dB

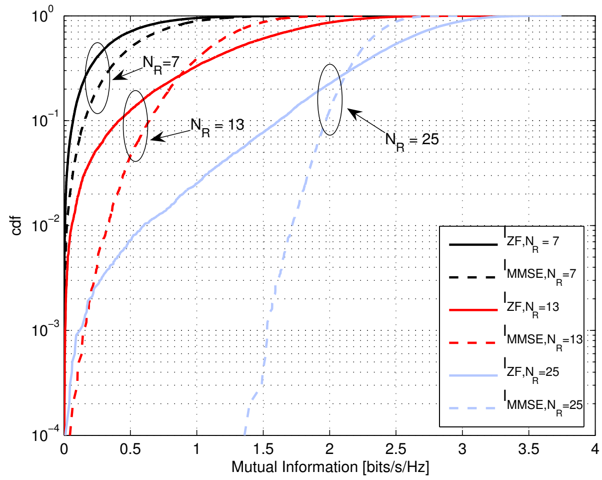
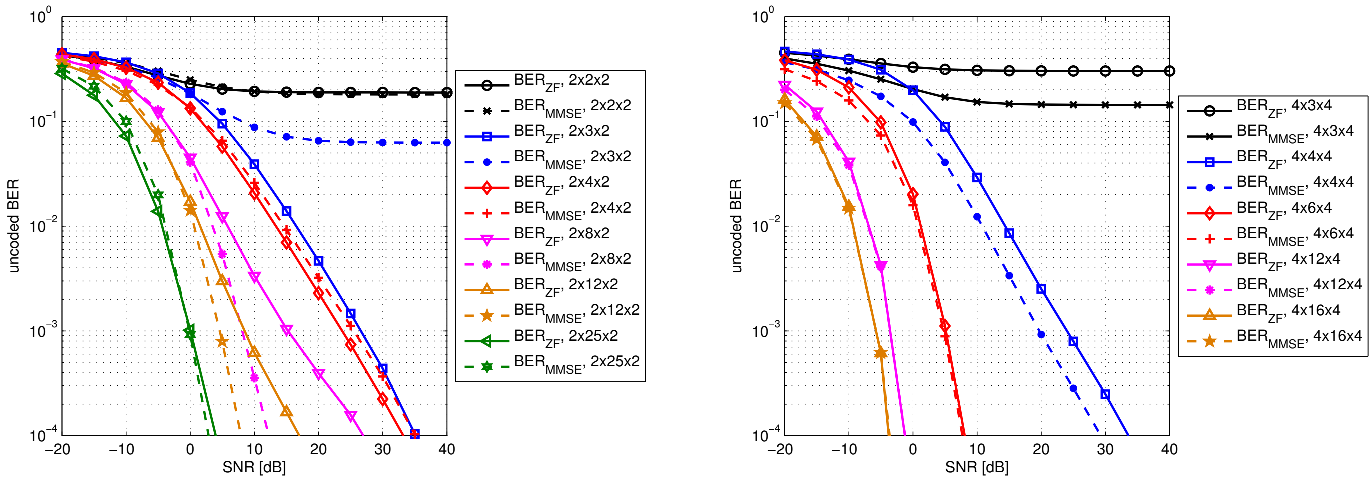


Fig. 3. cdf of the mutual information for one source-destination pair with SNR = 10dB and $N_{\mathrm{SD}} = 4$ in a LinRel architecture

Fig. 3 shows the cumulative distribution function (cdf) of the mutual information for one source-destination pair in a linear relaying architecture with $N_{\mathrm{SD}} = 4$, $N_{\mathrm{R}} = 7, 13, 25$, and $\mathrm{SNR} = 10\,\mathrm{dB}$. We see that MMSE relaying outperforms ZF relaying in terms of outage. The achieved diversity gain can be explained by the fact that the MMSE processing scheme always tries to make the equivalent channel matrix to be $\mathbf{H}_{\mathrm{SD}} = \gamma \mathbf{I}_{N_{\mathrm{SD}}}$. This is a very fair scheme as it provides all destinations with the same signal strength.

Finally, in **Fig. 4** we show uncoded bit error rate (BER) curves for both, the LinRel as well as the LDAS architecture when QPSK modulation is used. We consider configurations with 2 as well as 4 source-destination pairs and N_{R} relays (denoted by $(2 \times N_{\mathrm{R}} \times 2)$ and $(4 \times N_{\mathrm{R}} \times 4)$, respectively). In the lefthand figure (**Fig. 4(a)**) we want to highlight two points: 1) For the case that $N_{\mathrm{R}} = 3$ (which is the minimum relay configuration for ZF relaying [1]) the zero forcing signalling scheme clearly outperforms MMSE relaying. However, when adding one more relay the two schemes nearly perform equally good. 2) For a larger number of relays, the BER performance of ZF- and MMSE relaying drift apart. This can be explained as follows: The choice of the initial gain vector for ZF relaying ([1]) distributes most of the energy to the strong source-destination links and only a few to weak links. As the nullspace projection does nothing to improve bad links it may happen that the signal at one destination is outlyingly weak. This link then produces much more bit errors than the other links. In contrast to that, for the above made assumptions the signal power is equal at all destinations for the MMSE relaying scheme. Averaging over all channel realizations the number of occurring bit errors is the same for all links. The righthand figure (**Fig. 4(b)**) shows the uncoded BER curves for a linear distributed antenna system. For ZF relaying as well as MMSE relaying, there exists an error floor for $N_{\mathrm{R}} < N_{\mathrm{SD}}$. However, it is lower for MMSE relaying. When one relay is added such that $N_{\mathrm{R}} = N_{\mathrm{SD}}$, MMSE



(a) LinRel (b) LDAS

Fig. 4. Sum BER versus SNR for LinRel and LDAS architecture

relaying still outperforms ZF relaying in terms of BER. Adding more relays eventually makes the two BER curves converge.

V. Conclusion

We considered a two hop traffic pattern with amplify-and-forward (AF) relaying nodes organized in a linear distributed antenna system (LDAS) or a linear relaying (LinRel) architecture. All nodes employ only one antenna. The gain factors of the relays were derived such that the mean squared error (MSE) at the destinations is minimized. A scaling factor γ allows for received signals which are a scaled version of the transmitted symbols. However, finding an individual scaling factor for each destination unit will further improve the system. This extension to the present system is currently in preparation.

In Monte Carlo simulations we compared the derived MMSE relaying scheme with multiuser zero forcing (ZF) relaying. For the **LinRel architecture**, MMSE relaying is found to outperform ZF relaying in terms of average sum rate when the number of relays is less than the minimum relay configuration [1]. Adding more relays at first makes ZF relaying outperform MMSE relaying, but eventually the two systems achieve nearly the same average sum rate. We also found that compared to ZF relaying, the MMSE relaying increases the fairness in terms of outage rate as it tries to make the equivalent channel matrix a weighted identity. Consequently, all destinations receive the same signal power.

For **LDAS** both, the ZF relaying as well as the MMSE relaying, perform nearly equal in terms of average sum rate as well as bit error rate when the number of relays N_{R} is larger than the number of source-destination pairs N_{SD}.

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