Sub-Optimum MSF-MUD for CDMA Systems

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Abstract *- The microstatistic multi-user receiver (MSF-MUD) belongs to non–linear single–stage multi-user receivers' family (NSS–MUD) [1-3]. The output of the MSF-MUD is taken as the sign of the multi-channel non-linear transformation of the output of a bank of the matched filters (BMF). The non-linear transformation is made by multi-channel conventional microstatistic filter (M-CMF) [2]. The M-CMF consists of the set of M threshold decomposers (TD) and M multi-channel Wiener filters (M-WF). In the case of the optimum MSF-MUD design it is necessary to determine the optimum parameters of the TDs and M-WFs of M-CMF. The design of the TDs (the most complex phase of the optimum MSF-MUD design) consists in a non-linear optimization task solution. In this paper, the scanning method (SC-M), genetic algorithm based method (GA-M) [4] and method of cumulative distribution function (CDF-M) for the optimum and sub-optimum M-CMF TDs design will be described. It will be shown that the most effective approach for sub-optimum MSF-MUD design is provided by the CDF-M. This method is able to reduce the computational complexity of the MSF-MUD design at almost optimum performance properties of the MSF-MUD.*

1. Introduction

In the past decade, a lot of effort in the field of wireless communications was devoted to development of sub-optimum multi-user CDMA receivers [1, 5]. Here, it has been indicated in many references that the group of the NSS-MUD defined in [1] could provide very efficient sub-optimum receivers. This statement results from the fact that the NSS-MUD is able to approximate the non-linear boundary of the decision regions of the CDMA receiver much better then that of linear one and consequently it can outperform the corresponding linear multi-user detectors (MUD).

The MSF-MUD (Fig.1.) is a promising member of the NSS-MUD receivers' family based on the M-CMF application [2-4]. It follows from the Fig. 1, that the M-CMF has the crucial role in the MSF-MUD structure. The M-CMF is a non-linear minimum mean-square estimator a block scheme of which is given in the Fig.2. It can be seen from this figure that the M-CMF consists of TDs and multi-channel M-WFs. It can be shown that the design of the optimum M-CMF and MSF-MUD is the solution of the non-linear optimization task [2-4]. For the solution of the optimization tasks, the SC-M and GA-M have been proposed in [2-4]. The lack of these methods consists in their high complexity because at the optimum MSF-MUD design a number of M-CMF has to be designed.

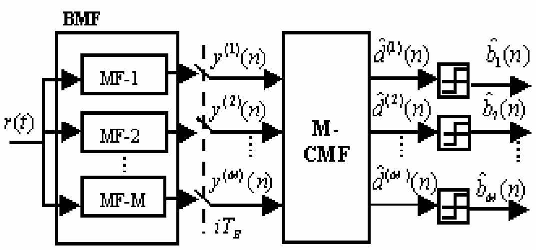

Fig. 1: MSF-MUD receiver

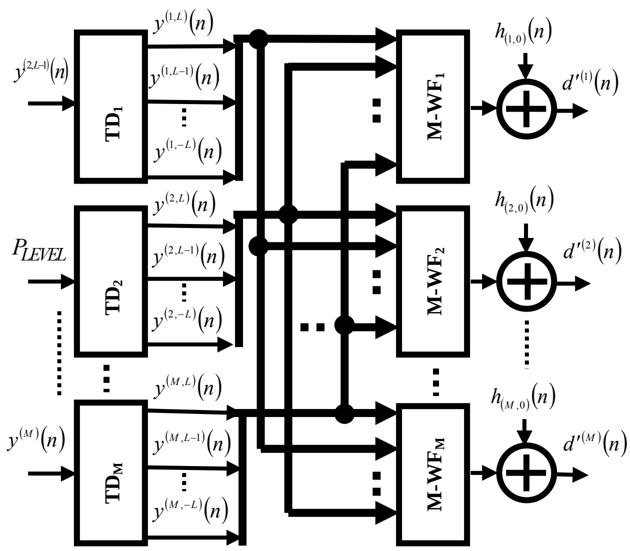

Fig. 2: M-CMF

In this paper, a new method of the MSF-MUD design referred to CDF-M will be introduced. This method can provide the sub-optimum MSF-MUD. It will be shown that the CDF-M is low-complexity method based on estimation of sub-optimum threshold values of the TDs from cumulative distribution functions (CDF) of the outputs of BMFs. In the case of the MSF-MUD designed by the CDF-M, only one M-CMF has to be design. The sub-optimum MFS-MUD designed by that approach can provide MUD with performance properties near to the optimum MSF-MUD. In this paper, some details concerning the M-CMF design procedure for the MFS-MUD will be also presented. Here, the emphasis will be put on the recommendation concerning specification of the TD parameters and the M-WF parameters as well as some additional parameters of the design procedure.

This paper is organized as follows. In the next section, the M-CMF and MSF-MUD will be briefly described. In the Section 3, the basic steps of the M-CMF design procedure will be presented. The methods of the TD parameter estimation such as SC-M, GA-M and CDF-M will be introduced in the Section 4. The results of computer simulations demonstrating the performance properties of the MSF-MUD designed by the methods described in the Section 4 will be given Section 5. In the last section, the conclusion remarks will be given.

2. M-CMF AND MSF-MUD

The block diagram of the M-CMF is given in the Fig. 2. It can be seen from this figure that the M-CMF consists of M TDs and M M-WFs. The performance of the i-th TD (TDi) can be described as the decomposition operation of the signal $y^{(i)}(n)$ into a set of the $O=2L$ signals $y^{(i,j)}(n)$ [6,7]. The output of the TDi $y^{(i,j)}(n)$ is uniquely determined from $y^{(i)}(n)$ by

$$y^{(i,j)}(n)=\begin{cases}0 & for \quad y^{(i)}(n)<l_{j-1}^{(i)}\\ y^{(i)}(n)-l_{j-1}^{(i)} & for \quad l_{j-1}^{(i)}<y^{(i)}(n)\le l_j^{(i)}\\ l_j^{(i)}-l_{j-1}^{(i)} & for \quad l_j^{(i)}<y^{(i)}(n)\end{cases} \quad \text{for } y^{(i,j)}(n)\ge 0,\, l_L^{(i)}=\infty,\, i\in\{1,2,\dots,M\},\, j\in\{1,2,\dots,L\} \quad (1)$$

and

$$y^{(i,j)}(n)=\begin{cases}0 & for \quad y^{(i)}(n)>l_{j-1}^{(i)}\\ y^{(i)}(n)-l_{-j+1}^{(i)} & for \quad l_{-j+1}^{(i)}>y^{(i)}(n)\ge l_{-j}^{(i)}\\ l_{-j}^{(i)}-l_{-j+1}^{(i)} & for \quad l_{-j}^{(i)}>y^{(i)}(n)\end{cases} \quad \text{for } y^{(i,j)}(n)<0,\, l_L^{(i)}=\infty,\, i\in\{1,\dots,M\},\, j\in\{1,\dots,L\} \quad (2)$$

where the parameters $l_j^{(i)}$ and $l_{-j}^{(i)}$ are referred as the threshold levels of the TDi and $-\infty=l_{-L}^{(i)}<\ldots<0<\ldots<l_L^{(i)}=\infty$.

Let us define the threshold value vector $\mathbf{L}$ as

$$\mathbf{L}=\begin{bmatrix}\mathbf{L}^{(1)T} & \mathbf{L}^{(2)T} & \ldots & \mathbf{L}^{(M)T}\end{bmatrix}^T \quad (3)$$

where

$$\mathbf{L}^{(i)}=\begin{bmatrix}l_{-L}^{(i)} & l_{-L+1}^{(i)} & \ldots & l_{L-1}^{(i)} & l_L^{(i)}\end{bmatrix}^T \quad (4)$$

The output signals of all TDs are fed into the k-th M-WF (M-WFk). The k-th output of the M-CMF $\hat{d}^{(k)}(n)$ is then given by the following expression:

$$\hat{d}^{(k)}(n)=h_{(k,0)}(n)+\sum_{i=1}^{M}\sum_{j=-L}^{L}\sum_{l=0}^{N}h_{(k,l)}^{(i,j)}(n)y^{(i,j)}(n-l) \quad (5)$$

The constant term $h_{(k,0)}(n)$ is applied in the M-CMF structure in order to obtain an unbiased M-CMF output. The sequence $h_{(k,l)}^{(i,j)}(n)$ represents the impulse response of the WF fed by signal $y^{(i,j)}(n)$.

Let us assume that the coefficients $h_{(k,0)}(n)$ and $h_{(k,l)}^{(i,j)}(n)$ are arranged into vector $\mathbf{H}_k(n)$ and the outputs of the TDi $y^{(i,j)}(n)$ are arranged by corresponding way into vector $\mathbf{Y}(n)$. Then, the k-th output of the M-CMF

$\hat{d}^{(k)}(n)$ is given by the following expression:

$$\hat{d}^{(k)}(n) = \mathbf{H}_k^T(n)\mathbf{Y}(n) = \mathbf{Y}^T(n)\mathbf{H}_k(n) \tag{6}$$

Then, it can be seen from the figure that the MSF-MUD outputs are given as:

$$\hat{b}_k(n) = sign\left(\hat{d}^{(k)}(n)\right) \tag{7}$$

3. THE M-CMF DESIGN PROCEDURE

Let us assume that the input signals of the M-CMF $y^{(i)}(n)$ and the desired signals $d^{(k)}(n)$ are stationary random processes. Then, the parameters of the optimum M-CMF given by $\mathbf{L}$, $\mathbf{H}_k(n)$ are obtained as the solution that minimizes the cost functions

$$MSE(\mathbf{H}_k(n),\mathbf{L}) = E\left[e_k^2(n)\right] = E\left[\left(d^{(k)}(n) - \hat{d}^{(k)}(n)\right)^2\right] \tag{8}$$

In this expression, $E[.]$ denotes the expectation operator. It follows from (8), that the cost function for the M-CMF design is the well-known mean-square errors of $d^{(k)}(n)$ estimation.

Before $MSE(\mathbf{H}_k(n),\mathbf{L})$ minimization, it is necessary to determine the dimensions of $\mathbf{L}$ and $\mathbf{H}_k(n)$ vectors. Generally, the vector dimensions will be obtained as the results of trade-off between expected performance properties of the MSF-MUD and its computational complexity. The MSF-MUD computational complexity consists of its design complexity and the computation complexity of its response. It is expected also that the performance properties of the MSF-MUD expressed by bit error rate (*BER*) vs. E_b / N_0 should be better than that provided by corresponding linear MUDs. These results should be obtained at the cost of the acceptable increasing MSF-MUD computational complexity.

With regard to these requirements it is recommended to define the vectors $\mathbf{L}^{(i)}$ as follows

$$\mathbf{L}^{(i)} = \begin{bmatrix} l_{-2}^{(i)} & l_{-1}^{(i)} & l_1^{(i)} & l_2^{(i)} \end{bmatrix}^T \; for \; i = 1,2,\ldots,M \tag{9}$$

where

$$l = l_1 = l_1^{(i)} \; and \; -l = -l_1 = l_{-1}^{(i)} \; for \; i = 1,2,\ldots,M \quad l_{-2} = l_{-2}^{(i)} = -\infty \; and \; l_2 = l_2^{(i)} = \infty \; for \; i = 1,2,\ldots,M \tag{10}$$

If the vectors $\mathbf{L}^{(i)}$ are selected according to (9) and (10), then each of TDi can be described by the only parameter (l). Under this condition, the expression (8) can be modified in the form

$$MSE(\mathbf{H}_k(n),\mathbf{L}) = MSE(\mathbf{H}_k(n),l) = E\left[e_k^2(n)\right] \tag{11}$$

The proposed approach for TD complexity specification can decrease the MSF-MUD complexity in a meaningful way.

The dimension of $\mathbf{H}_k(n)$ depends on the M-WF memory. It is recommended to select the M-WF memory by the same way as in the case of the linear MMSE-MUD. E.g. in the case of AWGN transmission channel, the M-WFs can be selected as memory-less.

Under the condition that the dimensions of $\mathbf{L}$ and $\mathbf{H}_k(n)$ are defined, the design procedure of the M-CMF is based on an iteration process, where one iteration consists of three basic steps [2-3].

In the first step, the threshold value l is estimated (the methods of the l parameter estimation are outlined in the Section 4). Then, based on l estimation, the coefficients of the M-WFs are computed (the second step) as follows

$$\mathbf{H}_k^{opt}(n) = \mathbf{R}^{-1}(n)\mathbf{P}_k(n) \tag{12}$$

where $\mathbf{P}_k(n)$ is the cross–correlation function of the signals at the output of the TDi and $\mathbf{P}_k(n)$ is the cross –

correlation vector of the desired signals and the signals at the output of the TDi [2,3]. The $\mathbf{R}(n)$ and $\mathbf{P}_k(n)$ can be estimated based on the training sequence transmission before each information date sequence transmission. It can be done because it is expected that the original training sequence is also available in the receiver. As the training sequence, the set of date with uniform distribution can be applied. The complexity, overhead costs as well as quality of the $\mathbf{R}(n)$ and $\mathbf{P}_k(n)$ estimation depend strongly on the training sequence length. The training sequence length will depend also on the active user number. In the case of AWGN transmission channel it is recommended to take the training sequence of the length of at least 1000 bits.

As the last step of the iteration, the evaluation of the cost functions of the M-CMF (mean square error given by (11)) for the set values of l and $\mathbf{H}_k(n)$ is made. If the values of the cost functions are the minimum once or if they are acceptable from the application point of view, the iteration process is stopped and the values l and $\mathbf{H}_k(n)$ providing the best values of the cost function are declared as the optimum (or sub-optimum) M-CMF and MSF-MUD parameters. If the obtained values of the cost functions are not acceptable, the next iteration of the design procedure has to be started.

4. THRESHOLD DECOMPOSER LEVEL ESTIMATION

It follows from the microstatistic filter theory ([6, 7]) that the optimum value of the threshold levels of the TD have to satisfy the condition $l \in J = (0, Y_{MAX})$, where $Y_{MAX} = \max\left|y^{(i,j)}(n)\right|$. Then, the J interval can be estimated from the histogram of the absolute values of the BMF outputs. An example of a typical histogram of that kind is given in the Fig. 3. On the other hand, l should be selected in such a way as to minimize $MSE(\mathbf{H}_k(n), l)$ and to allow reach the minimum value of BER. An example of the dependence of BER vs. l for an MSF-MUD is illustrated in the Fig.4.

Generally, $MSE(\mathbf{H}_k(n), l)$ and $BER(\mathbf{H}_k(n), l)$ are non-linear multi-modal functions with respect to l, what is also illustrated by Fig. 4. Therefore, the optimum or sub-optimum value of l can be obtained by non-linear optimization task solution. Consequently, the computation of l parameter is the most complex task in the M-CMF and MSF-MUD design. For the l parameter estimation, SC-M, GA-M and CDF-M have been proposed. These methods will be briefly described in the next parts of this section.

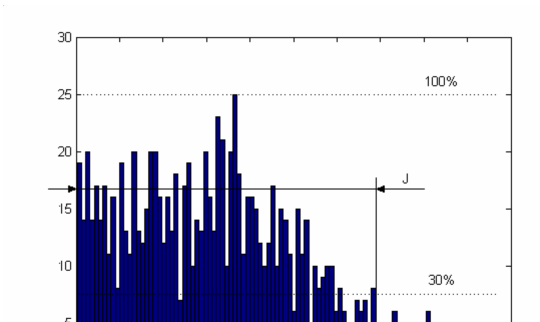

Fig. 3: Histogram of the absolute values of the BMF outputs. J interval estimation.

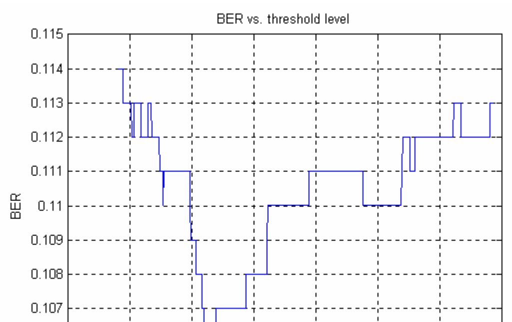

Fig.4: BER vs. threshold level l.

4.1 SCANNING METHOD (SC-M)

In this method, a dense set of possible values of l is taken from the J interval by uniform sampling of the J interval with the step Δl. By that approach, approximately $round(J/\Delta l)$ possible values of l can be found. For each value from the set of possible values, the set of M-CMFs and MSF-MUDs is designed using (12). By using training sequences, the cost function (mean-square error for M-CMF and BER for MSF-MUD) is also evaluated. Then, the parameter of l providing the smallest value of the cost functions corresponds the optimum parameters of the M-CMF and MSF-MUD.

The basic principle of the described SC-M is very simple. The method is able to provide the optimum M-CMF and MSF-MUD. The lack of the SC-M consists in its high computational complexity since for the optimum M-CMF and MSF-MUD design a huge number of M-CMF (approximately $round(J/\Delta l)$) has to be designed.

4.2. GENETIC ALGORITHM BASED METHOD (GA-M)

The computational complexity of the SC-M could be reduced using genetic algorithms (GA). GA is the multi–dimensional and stochastic search method, which can be applied with success to solution of non-linear optimization task [8]. The GA application for the optimum M-CMF and MSF-MUD design referred as the GA-M can be understood as the sophisticated scanning of J interval. The GA-M was originally proposed in [4]. As the GA cost functions, mean-square error for M-CMF and BER for MSF-MUD was selected. It has been shown in [4] that GA-M application can provide the optimum M-CMF and MSF-MUD whereas the optimum MSF-MUD can be designed by GA-M approximately 10-times faster than by the SC-M.

4.3. METHOD OF CUMULATIVE DISTRIBUTION FUNCTION (CDF-M)

The design of the MSF-MUD by the CDF-M is based on the application of the CDF of the absolute values of outputs of BMF ($y^{(i)}(n)$). The CDF of the output of BMF expresses the probability that the output of BMF is less than constant parameter l_{LIM} ($\Pr\left(\left|y^{(i)}(n)\right| < l_{LIM}\right)$). Then, the method of the l estimation using CDF-M is based on the solution of the equation

$$\Pr\left(\left|y^{(i)}(n)\right| < l\right) = P_{LEVEL} \qquad (13)$$

The CDF is estimated from the histogram of the absolute values of the BMF outputs. The value of the probability P_{LEVEL} has to be set from $(0,1)$. Resulting from microstatistic filter theory it is recommended to take P_{LEVEL} from interval $\langle 0,1\,;\,0,2\rangle$. The solution of (13) can be obtained very easily by using the histogram of $\left|y^{(i)}(n)\right|$.

The lack of the CDF-M consists in it that this method is able to provide only the sub-optimum M-CMF and sub-optimum MSF-MUD. In spite of that fact, the performance properties of the sub-optimum MSF-MUD designed by the CDF-M are almost the same as that of the optimum MSF-MUD what will be illustrated in the next section. On the other hand, the computational complexity of the CDF-M is very low since at the M-CMF and MSF-MUD design by the CDF-M it is necessary to design only one M-CMF. Therefore, the combination of almost optimum performance properties and relatively low computational complexity makes the CDF-M the most attractive and proper approach for MSF-MUD design.

5. COMPUTER EXPERIMENTS

In this section, some performance properties of the optimum and sub-optimum MSF-MUD will be presented using properly chosen computer experiments. Here, we will illustrate the influence of the design procedure parameters as the training sequence length and P_{LEVEL} on the MSF-MUD properties. A comparison of performance properties of the MSF-MUD, BMF, the linear minimum mean-square MUD (MMSE-MUD) and decorrelating MUD (D-MUD) will be also given.

In all experiments, a synchronous DS-CDMA base-band model of transmission system, consisting of the sum of antipodally modulated signature waveforms embedded in additive white Gaussian noise (AWGN) has been simulated with power spectrum density N_0. As spreading sequences, the Gold codes with the period of 31 chips were applied. As the performance index of the simulated DS-CDMA transmission system, *BER* vs. E_b / N_0 has been used.

Computer experiment 1

In this experiment, the influence of the training sequence length on the MSF-MUD performance properties has been investigated. The training sequence length was set to 10, 100, 200, 300, 500, 1000, 2000, 5000 and 7000 bits. The number of the active users was set to 30. For the MSF-MUD design, the SC-M has been used. The obtained results are given in the Fig. 5. It follows from this figure that the good and reliable results expressed by *BER* vs. E_b / N_0 can be provided by the MSF-MUD if the training sequence length is at least 1000 bits.

Computer experiment 2

In this experiment, the influence of the selection of the design parameter of P_{LEVEL} on the MSF-MUD performance properties has been studied. The training sequence length was 1000 bits. The number of the active users was set to 2. For the MSF-MUD design, the SC-M and CDF-M have been used. In the case of the CDF-M application, P_{LEVEL} was selected from interval $(0,1;0,9)$ with the step $\Delta P_{LEVEL} = 0,1$. The obtained results are given in the Fig. 6. It follows from this figure that *BER* vs. E_b / N_0 provided by the MSF-MUD designed by the CDF-M (sub-optimum MSF-MUD) and the MSF-MUD designed by the SC-M (optimum MSF-MUD) are almost the same if $P_{LEVEL} = 0,1$.

Computer experiment 3

In this experiment, the performance properties of BMF, D-MUD, MMSE-MUD and MSF-MUD expressed by *BER* vs. E_b / N_0 are compared. The number of the active users was 20. For the design of BMF, MMSE-MUD and D-MUD, the design procedures described in [5] have been applied. For the MSF-MUD design, the SC-M has been used. The training sequence length for MSF-MUD design was set to 300 bits (MSF P tren 300) and 5000 bits (MSF opt). The obtained results are given in the Fig. 7. It follows from this figure that the MSF-MUD outperforms clearly the single-user receiver and the other tested MUDs.

6 CONCLUSIONS

In this paper, the design procedure of the optimum and sub-optimum M-CMF and MSF-MUD has been presented. The stress has been put on description of the methods of the evaluation of the TD threshold levels. Here, it has been shown that the CDF-M is the most perspective method for that purpose. It can provide the MSF-MUD with the performance properties near to the optimum MSF-MUD at relatively slow computational complexity.

Because of its non-linear character, the MSF-MUD is able to approximate the decision region of the CDMA receiver much better then that of linear one and consequently it outperforms very clearly the corresponding linear MUD [2-4]. On the other hand, the MSF-MUD structure is relatively simple, because microstatistic filters can be considered to be a special case of piece-wise linear filters [9]. With regard to that fact, the computational complexity of the MSF-MUD response is not very high and it is comparable or lower than the others non-linear MUD. The low-complexity design procedure of the MSF-MUD by using the CDF-M makes therefore the MSF-MUD really attractive and promising MUD for CDMA transmission systems.

The MSF-MUD described in this paper represents only its basic form. Because of availability of the CDF-M, its structure can be easily rearranged in adaptive or blind modifications. Its simple modifications can be also applied for building receivers of more advanced transmission systems like e.g. MC-CDMA transmission systems. The development of the advanced modifications of the MSF-MUD will be the topic of the next research of ours.

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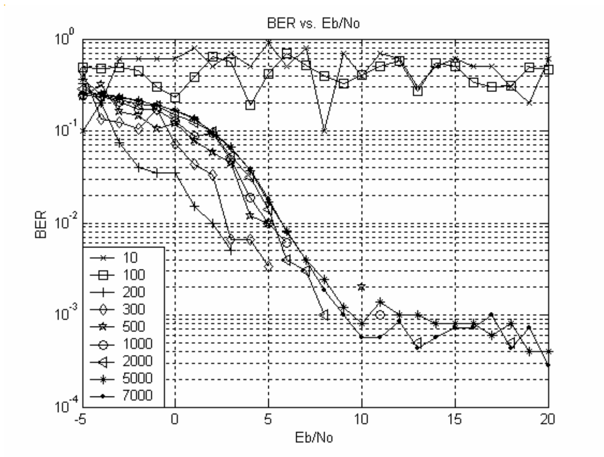

Fig. 5: MSF-MUD.
BER vs. Eb/No vs. different length of training sequences.

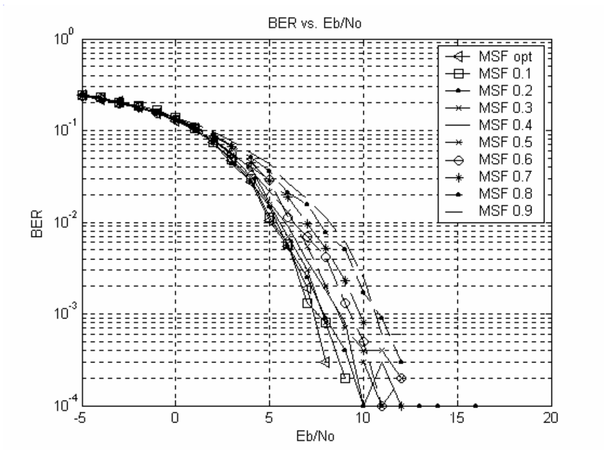

Fig. 6: MSF-MUD.
BER vs. Eb/No vs. different values of P_{LEVEL} .

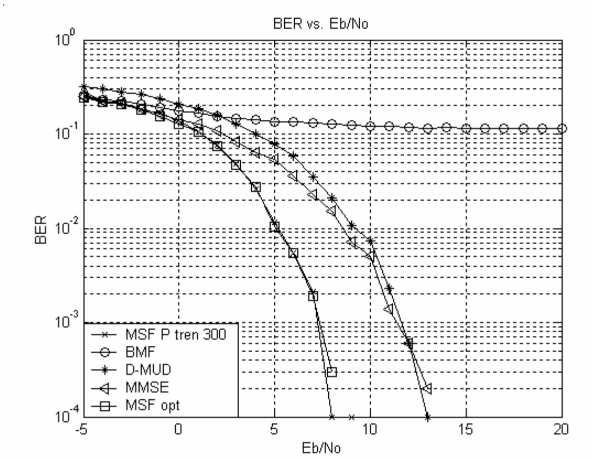

Fig. 7: BER vs. Eb/No for MSF-MUDs, BMF, D-MUD and MMSE-MUD.