On the Multiple-access Capability of a Shared Rayleigh Wireless Channel

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Abstract

In this paper we consider the capacity of a Rayliegh faded wireless channel that is shared by a random number of independent transmitter-receiver pairs. No cooperation is assumed between the different pairs. We examine the possible gain in the sum rate capacity when a binary side information is passed by each receiver to its corresponding transmitter on whether the instantaneous local signal-to-interference-plus-noise-ratio (SINR) is greater or less than a given threshold, and transmitters adapt their transmission power accordingly. It is shown that a substantial improvement in the channel capacity can be achieved by using this "opportunistic" transmission strategy.

I. Introduction

Fading is typically regraded as a serious hindrance in wireless communication. However, recently, it has become clear that fading can be beneficial for multiple-input multiple-output (MIMO) antenna systems and that the channel capacity of MIMO antenna systems increases linearly with the increasing the number of transmit/receive antennas [1]-[3]

On the other hand, a multiuser diversity system can also exploit Rayleigh fading to improve channel capacity [4]. Multiuser diversity arises from the fact that, since the channels of many users in a cell have independent fading statistics, a user whose channel is near the peak may exist at any time.

In contrast to [4], which consider a multiple-access channel model with a common receiver (e.g. the base station in a cellular system), our focus in this paper is on the interference channel model, where several independent transmitter-receiver pairs share randomly a common Rayleigh channel without any coordination between them. This is typical to the situation in the unlicensed frequency band, where several independent users may coexist in the same band. We calculate the total sum capacity under the assumption that a binary side information is made available at each transmitter on whether the local signal to interference plus noise ratio at the intended receiver exceeds or drops below a given threshold. It is worth mentioning that the present model can be considered as a generalization to the CSMA protocol (and its variations) for packet radio networks, where users are allowed to access the channel only if the channel is sensed idle.

The system model is introduced in section II. In section III, we determine the total sum capacity of the channel. In section III, we investigate the error rate performance. Section IV concludes the paper.

II. The Model

Let K independent transmitter-receiver pairs share a common Rayleigh fading channel, Fig. 1. The tap gain from transmitter# i to receiver# j is denoted by $g_{i,j}$. We assume that the different receivers are separated far enough to ensure independently fading channels from each transmitter to every receiver. Therefore, the channel taps are modelled as independent complex Gaussian random variables of equal variance , and $E\left[|g_{i,j}|^2\right] = 1$.

We assume that a binary side information is made available at each transmitter on whether the instantaneous signal-to-interference-plus-noise (SINR) at *its intended receiver* is greater than or less than a given threshold γ. Specifically, the kth transmitter is allowed to access the channel only if $\text{SINR}_k > \gamma$, with SINR_k being the local instantiates SINR at the kth receiver. That is, if s_k is the transmission power of the kth transmitter. Then

$$s_k = \begin{cases} S, & \text{SINR}_k > \gamma \\ 0, & \text{SINR}_k < \gamma. \end{cases} \tag{1}$$

Our aim is to find the optimum threshold γ_{opt} that maximizes the total sum capacity of the shared wireless channel.

III. The Capacity Analysis

The signal to interference plus noise ratio SINR at an arbitrary receiver, say receiver# k, is given by

$$\text{SINR}_k = \frac{|g_{k,k}|^2 S}{\sum_{\substack{i=1 \\ i\neq k}}^{K} s_i |g_{i,k}|^2 + N} \tag{2}$$

where $s_i \in \{0, S\}$ according to SINR_i (The local SINR at the ith receiver) and N is the power of additive white Gaussian noise.

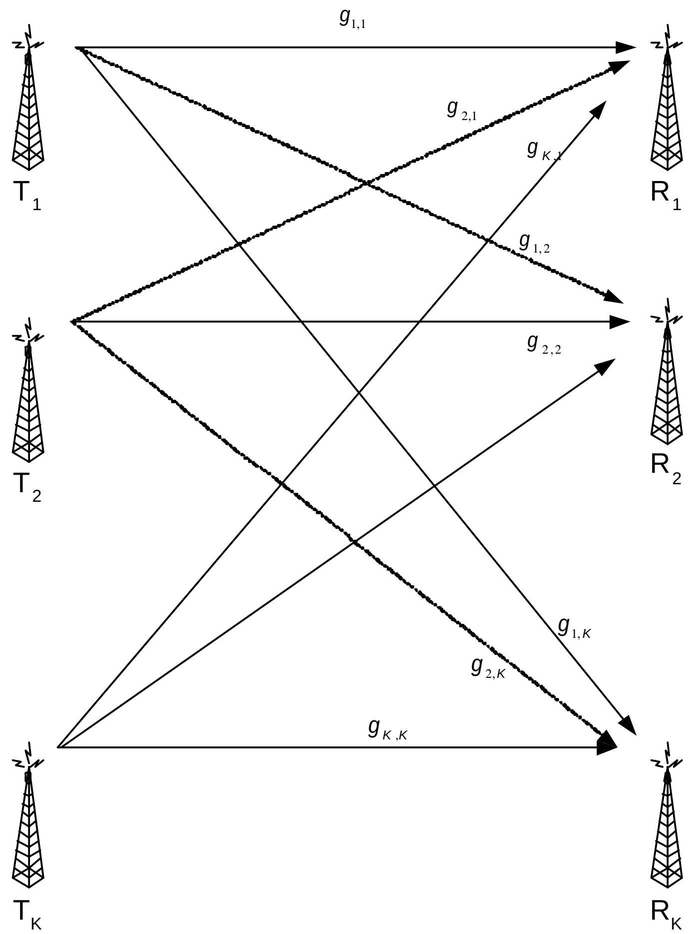

Fig. 1. A number of independent transmitter-receiver pairs share a common Rayleigh fading channel

When the kth receiver knows SINR_k, then the capacity of the kth transmitter-receiver link is given by ([7], [5])

$$C_1\left(\text{SINR}_k\right) = B\log_2\left(1 + \text{SINR}_k\right) \tag{3}$$

where B is the system bandwidth. Without any loss of generality, we let $B = 1$.

Since each receiver is only interested in the information transmitted by its corresponding transmitter, and there is no cooperation between the users (at both decoders and encoders). Then, the shared channel is reduced into K single-user channels, each subjected to multiple-access interference and AWGN (in addition to the Rayleigh fading). Therefore, the capacity of the shared channel can be calculated as the sum of the capacities of each individual single-user channel (see [8] and [9] for similar models). That is

$$C = \max \mathrm{E}\left[\sum_{k=1}^{K}\log_2\left(1 + \text{SINR}_k\right)\right]. \tag{4}$$

Accordingly, the sum capacity is reduced to

$$C = \max_{\gamma} K C_1\left(\gamma\right) \tag{5}$$

where

$$C_1\left(\gamma\right) = \int_{\gamma}^{\infty}\log_2\left(1 + z\right) f_{\text{SINR}_1}\left(z\right) dz \tag{6}$$

is the capacity of a transmitter-receiver pair, and $f_{\text{SINR}}\left(z\right)$ is the probability density function of the SINR experienced by an arbitrary receiver. In the remaining of this section we pay our attention to derive a closed form expression for the cumulative distribution function of the local instantaneous SINR. experienced by an arbitrary receiver.

A. Distribution of the SINR

Notice from (2) that

$$\Pr\left(\text{SINR}_1 > z\right) = \Pr\left\{\frac{\left|g_{1,1}\right|^2 S}{N + \sum_{k=2}^{K} s_k \left|g_{k,1}\right|^2} > z\right\} \tag{7}$$

which, owing to the fact that $\left|g_{1,1}\right|^2$ is an exponential random variable (a consequence to the the fact that $g'_{i,j}s$ are complex Gaussians), becomes

$$\begin{aligned}\Pr\left(\text{SINR}_1 > z\right) &= \mathrm{E}\left[e^{-z\left(\frac{N}{S}+\sum_{k=2}^{K}\frac{s_k}{S}\left|g_{k,1}\right|^2\right)}\right] \\ &= e^{-z\frac{N}{S}}\,\mathrm{E}\left[\prod_{k=2}^{K} e^{-z\frac{s_k}{S}\left|g_{k,1}\right|^2}\right] \\ &= e^{-z\frac{N}{S}}\,\mathrm{E}\left[\prod_{k=2}^{K}\left(\frac{1}{1+\frac{s_k}{S}z}\right)\right].\end{aligned} \tag{8}$$

where the last line of (8) follows because all$\left\{\left|g_{k,j}\right|^2\right\}_{k,j}$ are independent exponential random variables.

In order to evaluate the average in (8), let $K' \in \{0, 1, ..., K-1\}$ denotes the number of transmitters allowed to access the channel. Then

$$\Pr\left(\text{SINR}_1 > z|K'\right) = e^{-z\frac{N}{S}}\left(\frac{1}{1+z}\right)^{K'} \tag{9}$$

Now, it is clear that the SINR at each receiver is not a sequence of independent random variables. However, we introduce the assumption that they are. We note that similar assumptions are quite common in the literature of CSMA packet radio networks (e.g. [10] and [11]), where the validity of the results obtained was claimed by comparing the throughput values against simulation in [10]. Under this assumption, K' in (9) becomes a binomial random variable with

$$\Pr\left(K' = b\right) = \binom{K-1}{b}\alpha^b\left(1-\alpha\right)^{K-1-b} \tag{10}$$

where

$$\alpha = \Pr\left(\text{SINR}_1 > \gamma\right) = ... = \Pr\left(\text{SINR}_K > \gamma\right). \tag{11}$$

We obtain from (9) and (10)

$$\Pr\left(\text{SINR}_1 > z\right) = e^{-z\frac{N}{S}}\left(1 - \frac{\alpha z}{1+z}\right)^{K-1} \tag{12}$$

where α depends on $\frac{S}{N}, \gamma$ and K, and can be obtained by solving the nonlinear equation

$$\alpha = e^{-\gamma\frac{N}{S}}\left(1 - \alpha\frac{\gamma}{1+\gamma}\right)^{K-1}. \tag{13}$$

Though an expression for the probability density function $f_{\text{SINR}}\left(z\right)$, is readily obtained by taking the derivative of (12), however, one can transforms (6) into the more appropriate form (using the integration by parts)

$$\begin{aligned}C_1\left(\gamma\right) &= \log_2\left(1+\gamma\right)\Pr\left(\text{SINR}_1 > \gamma\right) \\ &\quad + \frac{1}{\ln 2}\int_{\gamma}^{\infty}\frac{1}{1+z}\Pr\left(\text{SINR}_1 > z\right)dz.\end{aligned} \tag{14}$$

Substitute (12) into (14) to get the following expression for the average capacity of an arbitrary transmitter-receiver pair

$$C_1\left(\gamma\right) = \alpha\log_2\left(1+\gamma\right) + \frac{1}{\ln 2}\int_{\gamma}^{\infty}\frac{1}{1+z}\left(1 - \frac{\alpha z}{1+z}\right)^{K-1} e^{-z\frac{N}{S}}dz \tag{15}$$

where α depends on γ and is obtained by solving the non-linear equation (13).

B. Random Number of Users

Now, suppose that the number of users (transmitter-receiver) pairs is random. Let the number of transmitter receiver pairs K, be a random variable distributed according to a Poisson process with rate λ. In this case, it can be straitforwardly shown that (15) reduces to

$$C_1(\gamma) = \alpha\lambda \log_2(1+\gamma) + \frac{\lambda}{\ln 2}\int_{\gamma}^{\infty} \frac{1}{1+z} e^{-\lambda \frac{\alpha z}{1+z}} e^{-z\frac{N}{S}} dz \tag{16}$$

where now α is obtained be solving the nonlinear equation

$$\alpha = e^{-\gamma\frac{N}{S}} e^{-\lambda\alpha\frac{\gamma}{1+\gamma}}. \tag{17}$$

which can be solved analytically, to give

$$\alpha = \frac{1+\gamma}{\lambda\gamma} \mathrm{LW}\left(\frac{\lambda\gamma}{1+\gamma} e^{-\gamma\frac{N}{S}}\right)$$

where LW is the Lambert W function.

Notice that, when no side information is made available at the transmitter, then transmissions would take place independently. In this case (15) is reduced to

$$C_1 = \frac{1}{\ln 2}\int_0^{\infty} e^{-z\frac{N}{S}} \left(\frac{1}{1+z}\right)^K dz \tag{18}$$

and the total sum capacity becomes

$$C = \frac{K}{\ln 2}\int_0^{\infty} e^{-z\frac{N}{S}} \left(\frac{1}{1+z}\right)^K dz. \tag{19}$$

When $\frac{S}{N} \to \infty$ and $K \geq 2$, the integration in (19) gives

$$C \to \frac{K}{(K-1)\ln 2} \text{ bits/sec/Hz} \tag{20}$$

which converges to 1.4427 bits/sec/Hz when $K \to \infty$.

In Figures 2-4, we plot the capacity for a given threshold γ, $KC_1(\gamma)$, against γ for different number of users, and various levels of signal to noise ratios $\frac{S}{N} = 5$, 15, and 30 dB. We note that the sum capacity $KC_1(\gamma)$ depends on γ. Furthermore, the optimum threshold γ_{opt} depends on the number of users sharing the channel and the signal to noise ratio S/N. These results reveal an enormous gain in the overall capacity compared with the system without feedback ($\gamma = 0$). For instance, when $S/N = 15$ dB and $K = 25$, the capacity increases from 1.5 [bits/sec/Hz] when $\gamma = 0$ to 11.3 [bits/sec/Hz] at γ_{opt}, which corresponds to more than 500% increase in the sum capacity. This gain is further magnified at higher levels of S/N. For instance, when $S/N = 30$ dB and $K = 100$, the capacity increases from about 1.6 [bits/sec/Hz] when $\gamma = 0$ to about 44 [bits/sec/Hz] at γ_{opt}. That is an increase of more than 2500% in the sum capacity. In Fig. 5, we plot the $\lambda C_1(\gamma)$ against the threshold level γ when $\frac{S}{N} = 20$ dB and for some different user density $\lambda = 1, 2, 5, 10$.

IV. Summary and Conclusions

In this paper, we have investigated the potential benefits of using transmitter side information in a randomly shared Rayleigh wireless channel, where several independent transmitter-receiver pairs randomly share a common Rayleigh fading channel without any coordination between them. We have considered the improvement in the total sum capacity when only a binary side information is made available at each transmitter on whether the signal to interference plus noise ratio at the intended receiver exceeds or drops below a given threshold. It is shown that the total sum capacity is maximized when transmitters that experience bad conditions (the signal to interference plus noise ratio, at their intended receivers, drops below a given threshold) are suspended, and allowing only those which experience better conditions to access the common channel. It is shown that a substantial improvement in the channel capacity can be achieved by using this "opportunistic" transmission strategy. In fact, our numerical results reveal that the total sum capacity increases linearly with the logarithm of the number of users sharing the channels.

References

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311-335, 1998.

[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecomm*., vol. 10, pp. 585-595, Nov. 1999.

[3] W, Rhee, W. Yu, and J. M. Cioffi, "Utilizing multiuser diversity for multiple antenna systems," IEEE WCNC, San Francisco, CA, Sept. 2000, pp. 420-425.

[4] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," *in Proc. IEEE ICC*, pp. 331-334, Nov. 1995.

[5] A. Goldshmit and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inform. Theory*, vol. 43 no. 6, pp. 1986-1992, Nov. 1997.

[6] T. Cover and J. Thomas, *Elements of Information Theory*. New York, Wiley, 1991.

[7] W. C. Y. Lee, "Estimate of channel capacity in Rayleigh fading environment," *IEEE Trans. Veh. Technol.*, vol. 39, pp. 187-189, Aug. 1990.

[8] M. V. Hegde and W. E. Stark, "Capacity of frequency-hop spread-spectrum multiple-access communication systems," *IEEE Trans. on Commun*., vol. 38, pp. 1050-1059, July 1990.

[9] J. G. Goh, and S. V. Maric "The capacities of frequency-hopped code-division multiple-access channels," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 1204 -1211, May 1998.

[10] F. A. Tobagi and L. Kleinrock, "Packet switching in radio channels: Part II- The hidden terminal problem in carrier sense mutliple-access and busy tone solution," *IEEE Trans. on Commun*., vol. 23, no. 12, pp. 1471-1433, Dec. 1975.

[11] H. Takagi and L. Kleinrock, "Optimal transmission ranges for randomly distributed packet radio terminals," *IEEE Trans. on Commun*., vol. 32, no. 3, pp. 246-257, March 1984.

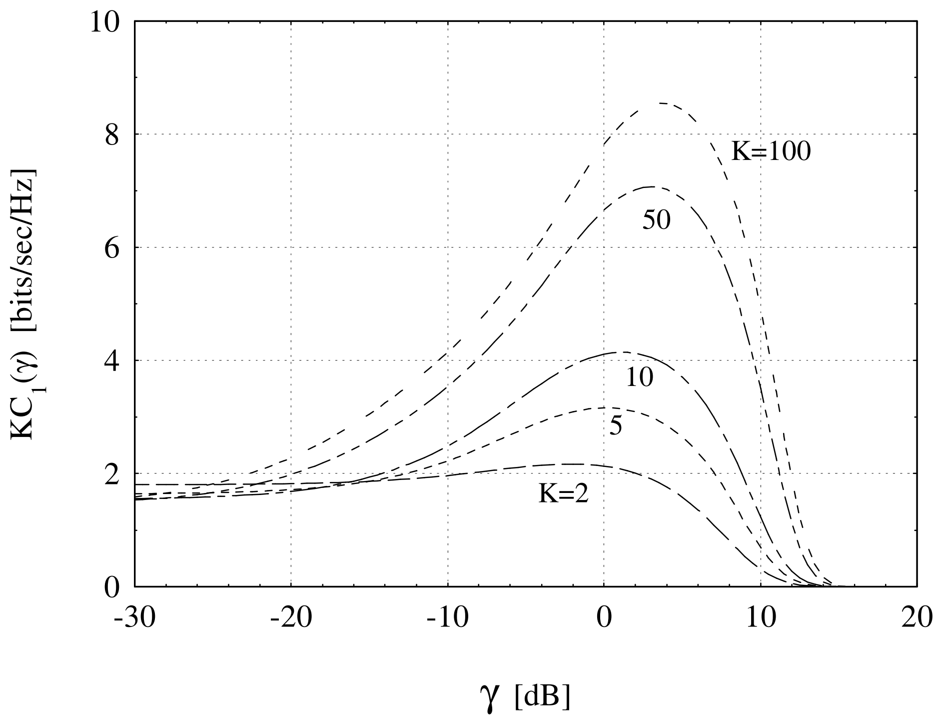

Fig. 2. The sum capacity $KC_1(\gamma)$ against the threshold γ [dB] for different number of transmitter-receiver pairs in the case of $\frac{S}{N} = 5$ dB.

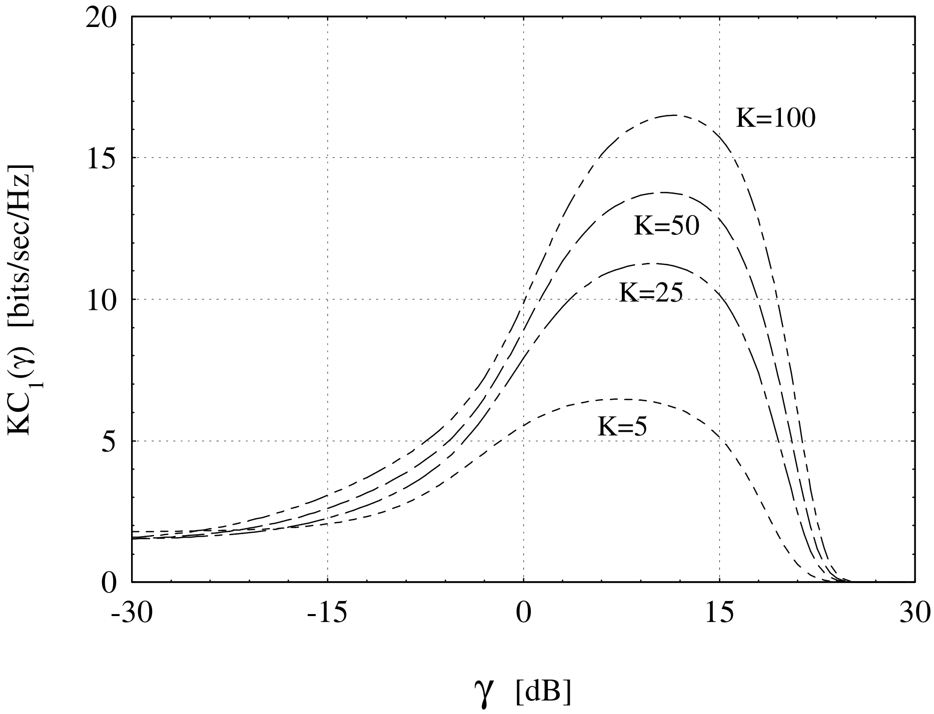

Fig. 3. The sum capacity $KC_1(\gamma)$ against the threshold γ [dB] for different number of transmitter-receiver pairs in the case of $\frac{S}{N} = 15$ dB.

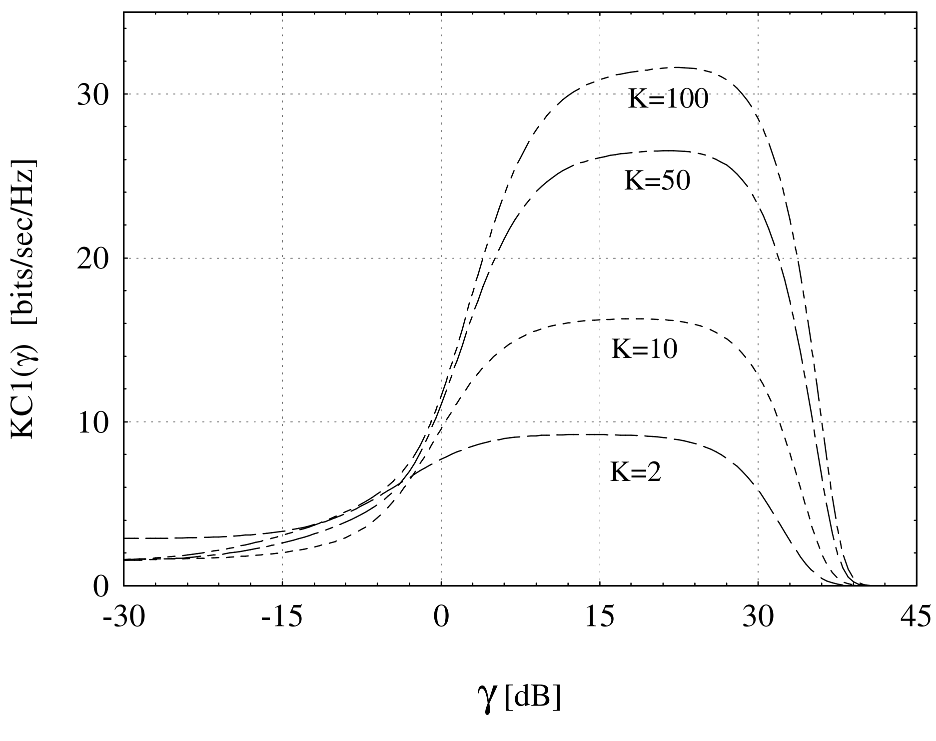

Fig. 4. The sum capacity $KC_1(\gamma)$ against the threshold γ [dB] for different number of transmitter-receiver pairs in the case of $\frac{S}{N} = 30$ dB.

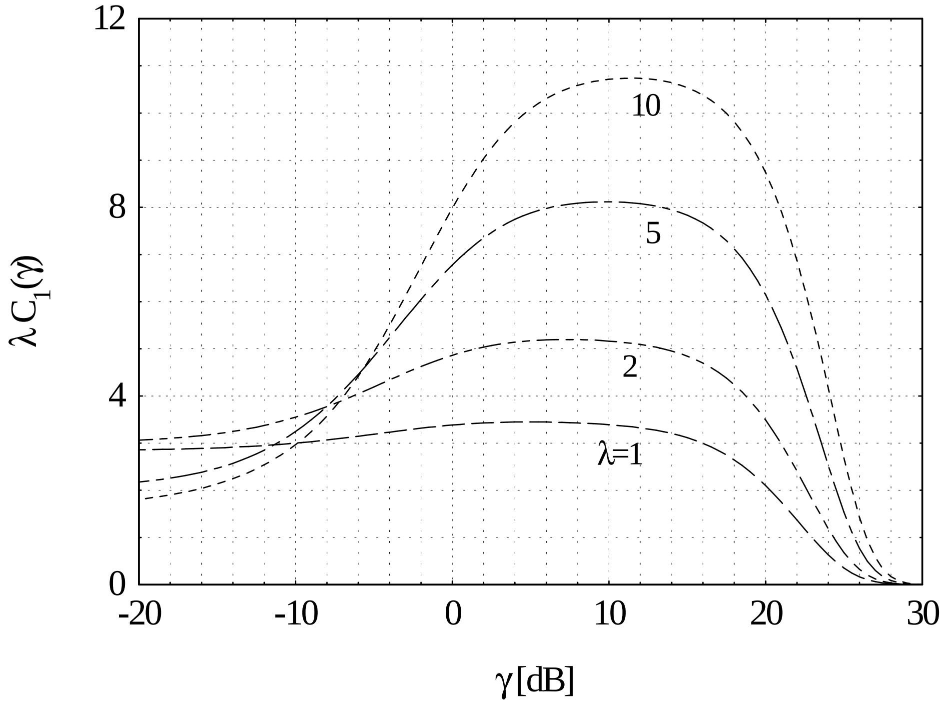

Fig. 5. $\lambda C_1(\gamma)$ against the threshold γ. SNR=20 dB, and $\lambda = 1, 2, 5, 10$.