Space-Time Block Code Selection for more than two Transmit Antennas

Mehmet E.Çelebi, Selami Şahin, Ümit Aygölü

Istanbul Technical University
Faculty of Electrical and Electronics Engineering
Maslak 34469, Istanbul, Turkey
celebi@ehb.itu.edu.tr, selsah@uekae.tubitak.gov.tr, aygolu@ehb.itu.edu.tr

Abstract

In this paper, we propose a full rate space-time block code selection technique with feedback, which achieves full diversity when more than two transmit antennas are used for transmission of each channel symbol pair. Only one or a few feedback bits are needed for relative state information of the channels. Moreover, all transmitted symbols can be separately decoded at the receiver. It is shown by computer simulations that the new approach provides SNR improvement, especially when feedback errors occur, compared to the antenna selection technique with Alamouti's scheme, for the same number of feedback bits.

I. Introduction

Fading is one of the major problems in wireless communication systems which limits the link performance. One efficient way to reduce the severe attenuations in fading channels is to make use of transmit antenna diversity. Since it does not increase the transmission bandwidth and its cost is paid at the base station by increasing the number of transmit antennas, transmit antenna diversity is extensively studied in recent years. Space-time block coding is the compromising approach when performance vs complexity trade-off is considered. Orthogonal space-time block codes (OSTBC) [1], beside their diversity advantage, provide decoding simplicity since transmitted symbols are separately decoded by means of linear processing. Alamouti's [2] scheme for two transmit antennas is the unique orthogonal space-time block code for complex channel symbols which provides both full diversity and full rate where two symbols are transmitted in each coding step. Several quasi-orthogonal STBCs that provide full rate at the expense of some loss in diversity [3],[4] and OSTBCs that provide full diversity with some loss in code rate [1],[5] have been proposed in the literature. In all of these schemes, it is assumed that only the receiver knows the channel coefficients. On the other hand, in multi-input multi-output (MIMO) systems, it is shown that when the transmitter also has the perfect knowledge of the channels, the performance of the OSTBCs can be improved by transmit antenna selection (TAS)[6],[7] or beamforming [8] techniques.

In this paper, we propose a pure space-time block code selection method where selection is performed among full rate balanced STBCs designed by extending Alamouti's scheme to n number of transmit antennas where $n > 2$. The balanced STBC set covers suitable codes which achieve full diversity for each possible situation of the channels. One or a few feedback bits are needed at the transmitter for the relative state information of the channels. Since the number of transmitted symbols at each coding step is lower than the number of transmit antennas, each transmitted symbol can be separately decoded by linear processing as in the OSTBC case. It is shown by computer simulations that, for the same number of feedback bits, the new approach provides SNR advantage compared to Alamouti's scheme with antenna selection. It is also shown that the benefit of using code selection schemes over pure antenna selection methods becomes more essential in the presence of feedback bit errors.

II. System Model

We consider a wireless system where the base-station is equipped with n antennas and the mobile with single antenna. Data bits are mapped by streams of k bits into M-PSK symbols where $M = 2^k$. At each signaling interval t, n symbols $s_t^1, s_t^2, ..., s_t^n$ are simultaneously transmitted from n antennas as $\frac{1}{\sqrt{n}} s_t^i$ through the quasi-static, flat fading channel whose coefficients h_i, $i = 1, 2, ..., n$ denote the path gains from each transmit antenna to the receive antenna, modeled as independent samples of complex Gaussian random variables with zero-mean and variance 1/2 per dimension. Maintaining constant transmit power constraint through n antennas, the received signal is given as,

$$r_t = \sum_{i=1}^{n} \frac{1}{\sqrt{n}} h_i s_t^i + \eta_t$$

where η_t is the complex zero-mean additive white Gaussian noise samples with variance $N_0/2$ per dimension. For a STBC of two signaling intervals, assuming coherent detection and perfect knowledge of the channel coefficients, the receiver minimizes the maximum likelihood decision metric

$$\sum_{t=1}^{2} |r_t - \frac{1}{\sqrt{n}} \sum_{i=1}^{n} h_i c_t^i|^2$$

over all possible M-PSK symbols c_t^i, with $t = 1, 2$ and $i = 1, 2, ..., n$.

III. Balanced Code Selection

A. Three Transmit Antennas

a) $One\ bit\ feedback$: Consider the full rate STBC pair with transmission matrix

$$C_1 : \begin{pmatrix} s_1 & s_2 & as_2 \\ -s_2^* & s_1^* & as_1^* \end{pmatrix} \quad (1)$$

where $a = \pm 1$. The columns and rows of C_1 denote symbols to be transmitted from three transmit antennas and two signaling intervals, respectively. The received signals at the first and second signaling intervals are

$$r_1 = \frac{1}{\sqrt{3}}[h_1 s_1 + h_2 s_2 + h_3 a s_2] + \eta_1$$

$$r_2 = \frac{1}{\sqrt{3}}[-h_1 s_2^* + h_2 s_1^* + h_3 a s_1^*] + \eta_2$$

respectively. The estimates of s_1 and s_2 are obtained by linear processing as

$$\hat{s}_1 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_2 h_3^*\}]s_1 + \hat{\eta}_1 \quad (2)$$

$$\hat{s}_2 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_2 h_3^*\}]s_2 + \hat{\eta}_2 \quad (3)$$

where $\hat{\eta}_1 = \eta_1 h_1^* + \eta_2^*(h_2 + ah_3)$ and $\hat{\eta}_2 = \eta_1(h_2^* + ah_3^*) - \eta_2^* h_1$. If we choose in (1) the STBC with $a = 1$ and $a = -1$ when $Re\{h_2 h_3^*\}$ is positive and negative, respectively, the contribution of the cross-product term in (2) and (3) will always positive and the gain provided by this scheme will be greater than the sum of the magnitude squares of all path gains which is valid for OSTBCs. We call this type of codes as balanced STBCs.

Let d_{min}^2 be the minimum squared Euclidean distance between two distinct symbols of the M-PSK constellation. Assuming Gray mapping, and by extending the results given in [9] for Alamouti's scheme to our balanced STBC, the bit error probability can be approximated as

$$P_b \simeq \frac{2}{log_2 M} Q\left(\sqrt{\frac{(\sum_{i=1}^{3} |h_i|^2 + 2aRe\{h_2 h_3^*\})}{2N_0} \frac{d_{min}^2}{3}}\right)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$, and the multiplicative factor 2 stands for the number of symbols with distance d_{min} to a given M-PSK symbol.

b) $Two\ bits\ feedback$: Let us consider now, in addition to the STBC pair C_1, the full rate balanced STBC pair

$$C_2 : \begin{pmatrix} s_1 & s_2 & as_1 \\ s_2^* & -s_1^* & as_2^* \end{pmatrix}.$$

The estimates of s_1 and s_2 for this code pair are obtained as

$$\hat{s}_1 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_1 h_3^*\}]s_1 + \hat{\eta}_1$$

$$\hat{s}_2 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_1 h_3^*\}]s_2 + \hat{\eta}_2$$

where $\hat{\eta}_1 = (h_1^* + ah_3^*)\eta_1 - h_2 \eta_2^*$ and $\hat{\eta}_2 = h_2^* \eta_1 + (h_1 + ah_3)\eta_2^*$. If we choose C_1 when $|Re\{h_2 h_3^*\}| > |Re\{h_1 h_3^*\}|$ and C_2, otherwise, and for each selection the STBC with $a = 1$ and $a = -1$ when $Re\{h_j h_k^*\}$, ($j = 2, k = 3$ or $j = 1, k = 3$) is positive and negative, respectively, the contribution of the cross-product terms will be increased with respect to the first case. This selection among 4 codes needs two feedback bits.

c) $Three\ bits\ feedback$: We include now to the balanced STBC set with two code pairs given in (b) a third full rate STBC pair

$$C_3 : \begin{pmatrix} s_1 & as_1 & s_2 \\ s_2^* & as_2^* & -s_1^* \end{pmatrix}.$$

The estimates of s_1 and s_2 for C_3 are obtained as

$$\hat{s}_1 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_1 h_2^*\}]s_1 + \hat{\eta}_1$$

$$\hat{s}_2 = \frac{1}{\sqrt{3}}[(|h_1|^2 + |h_2|^2 + |h_3|^2) + 2aRe\{h_1 h_2^*\}]s_2 + \hat{\eta}_2$$

where $\hat{\eta}_1 = (h_1^* + ah_2^*)\eta_1 - h_3\eta_2^*$ and $\hat{\eta}_2 = h_3^*\eta_1 + (h_1 + ah_2)\eta_2^*$. If we choose the STBC pair C_i, $i = 1, 2, 3$ which provides the maximum product term $|Re\{h_j h_k^*\}|$, and then the STBC with $a = 1$ and $a = -1$ when $Re\{h_j h_k^*\}$ is positive and negative, respectively, the gain provided from the cross-product terms will be further increased with respect to the first and second cases. This total selection among 6 codes needs three feedback bits.

Note that one of the three balanced STBCs C_1, C_2, C_3 and one of the two-tuples $\{C_1, C_2\}$, $\{C_1, C_3\}$, $\{C_2, C_3\}$ can be randomly used, without affecting the error performance, when one and two feedback bits are available, respectively.

B. Four Transmit Antennas

a) *Two bits feedback*: One balanced STBC 4-tuple for four transmit antennas is

$$C_1 : \begin{pmatrix} s_1 & s_2 & as_1 & bs_2 \\ -s_2^* & s_1^* & -as_2^* & bs_1^* \end{pmatrix} \tag{4}$$

where $a = \pm 1$ and $b = \pm 1$, respectively. The estimates of s_1 and s_2 are obtained by linear processing as

$$\hat{s}_1 = \frac{1}{2}[(|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2) + 2Re\{ah_1 h_3^* + bh_2 h_4^*\}]s_1 + \hat{\eta}_1$$

$$\hat{s}_2 = \frac{1}{2}[(|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2) + 2Re\{ah_1 h_3^* + bh_2 h_4^*\}]s_2 + \hat{\eta}_2$$

where $\hat{\eta}_1 = (h_1^* + ah_3^*)\eta_1 + (h_2 + bh_4)\eta_2^*$ and $\hat{\eta}_2 = (h_2^* + bh_4^*)\eta_1 - (h_1 + ah_3)\eta_2^*$. The contribution of the cross-product term $2Re\{ah_1 h_3^* + bh_2 h_4^*\}$ will always be positive and the contribution of this scheme to the symbol estimates will be greater than the sum of the magnitude squares of all path gains, if the code selection from (4) is performed as follows,

$$\begin{cases} Choose \quad a = 1, \quad b = 1 \quad if \quad Re\{h_1 h_3^*\} > 0 \quad and \quad Re\{h_2 h_4^*\} > 0 \\ Choose \quad a = -1, \quad b = -1 \quad if \quad Re\{h_1 h_3^*\} < 0 \quad and \quad Re\{h_2 h_4^*\} < 0 \\ Choose \quad a = 1, \quad b = -1 \quad if \quad Re\{h_1 h_3^*\} > 0 \quad and \quad Re\{h_2 h_4^*\} < 0 \\ Choose \quad a = -1, \quad b = 1 \quad if \quad Re\{h_1 h_3^*\} < 0 \quad and \quad Re\{h_2 h_4^*\} > 0. \end{cases}$$

This selection among four balanced STBCs needs two feedback bits. The two other balanced STBC 4-tuples for four antennas are as follows,

$$C_2 : \begin{pmatrix} s_1 & s_2 & as_2 & bs_1 \\ -s_2^* & s_1^* & as_1^* & -bs_2^* \end{pmatrix} \quad , \quad C_3 : \begin{pmatrix} s_1 & as_1 & s_2 & bs_2 \\ -s_2^* & -as_2^* & s_1^* & bs_1^* \end{pmatrix}.$$

The cross-product terms in symbol estimates for C_2 and C_3 are $2Re\{ah_2 h_3^* + bh_1 h_4^*\}$ and $2Re\{ah_1 h_2^* + bh_3 h_4^*\}$, respectively. One of the C_i s, $i = 1, 2, 3$ given in (4) and (5) can be randomly used when only two feedback bits are available at the transmitter.

b) *Three bits feedback*: When a third feedback bit is permitted, a selection between two preselected balanced STBC 4-tuples from C_is, $i = 1, 2, 3$ is performed first by choosing the code which will produce greater cross-product term. As an example, a selection rule between 4-tuples C_1 and C_2 is given as,

$$\begin{cases} Choose \quad C_1 \quad if \quad |Re\{h_1 h_3^*\}| + |Re\{h_2 h_4^*\}| > |Re\{h_1 h_4^*\}| + |Re\{h_2 h_3^*\}| \\ Choose \quad C_2 \quad if \quad |Re\{h_1 h_3^*\}| + |Re\{h_2 h_4^*\}| < |Re\{h_1 h_4^*\}| + |Re\{h_2 h_3^*\}| \end{cases}$$

Once a code 4-tuple is chosen, the transmitted STBC is determined as in the two bits feedback case (a).

c) *Four bits feedback*: A total selection for four transmit antennas is performed when four feedback bits are available. The balanced STBC 4-tuple from C_is, $i = 1, 2, 3$ which has the maximum cross-product term for the present channel conditions is first selected by two feedback bits, and then the STBC which maximizes the diversity is determined as in (a) by two other feedback bits.

The proposed code selection technique can be easily extended to more than four transmit antennas. The potential gain increase is not only due to diversity enhancement with increasing antennas, but also due to the contribution of the cross-product terms which are obtained via balanced STBC sets with increasing cardinality. The benefit of using code selection schemes over pure antenna selection becomes more essential in the presence of feedback errors as we will see in the next section.

IV. Performance Evaluation

The bit error probabilities of the proposed code selection technique using balanced STBC sets were evaluated for quaternary phase-shift keying (QPSK) modulation by computer simulations. A frame size of 130 symbols were used. The channel considered is a quasi-static Rayleigh fading channel where path gains are zero-mean complex Gaussian random variables with unit variance and statistically independent from one antenna to another. It is assumed that the channel remains constant over one frame but changes independently from one frame to another and that the receiver has perfect CSI. For comparison purposes, the bit error rate (BER) curves of the classical Alamouti's scheme, and the antenna selection with Alamouti's scheme where the best transmit antenna pair is selected by means of feedback bits, are also included to the Figures 1 and 2. For the sake of a fair comparison these schemes were simulated by normalizing each symbol by a factor of $1/\sqrt{2}$.

Figure 1 presents the bit error probabilities of the new 3 transmit antennas balanced STBC selection scheme for 1, 2 and 3 feedback bits. Increasing the number of feedback bits improves the error performance, as expected. Compared to the classical Alamouti's scheme (A2Tx0Fb), the balanced STBC pair C_1 for 3 transmit antennas (CS3Tx1Fb) provides an E_b/N_0 advantage of 5 dB for a BER value of $P_b = 10^{-4}$ with one feedback bit. The BER curves of the Alamouti's scheme with antenna selection using two feedback bits (A3Tx2Fb) to determine the best pair between 3 transmit antennas and CS3Tx1Fb code with one feedback bit are very close. However, the balanced STBC with two feedback bits (CS3Tx2Fb) reaches the same BER value with an E_b/N_0 gain of 0.3 dB at the same number of feedback bits, compared to A3Tx2Fb. With three bits feedback code selection (CS3Tx3Fb) this gain increases approximately to 0.7 dB.

The bit error performances for 4 transmit antennas are shown in Figure 2. Compared to the antenna selection with Alamouti (A4Tx3Fb)which needs 3 feedback bits for this case, the balanced STBC selection with two feedback bits (CS4Tx2Fb) has only 0.2 dB loss of E_b/N_0 while the balanced STBC selection with three (CS4Tx3Fb) and four (CS4Tx4Fb) feedback bits provide E_b/N_0 gains of approximately 0.15 and 0.45 dB, respectively.

The BER performances of the balanced STBC selection schemes were also compared to the Alamouti's scheme with antenna selection in the case of feedback errors. Comparisons were made for both 3 and 4 transmit antennas, on an equal number of feedback bits basis. The feedback channel is assumed to be a binary symmetric one with error probability $\alpha = 0.05$. It is also assumed that the transmitter sends pilot symbols using the STBC decided from the feedback bits and the receiver uses these coded pilot symbols to compute the STBC used by the transmitter [9]. It is seen from Figs.1 and 2 that the E_b/N_0 advantage of the proposed STBC selection schemes become more obvious when feedback errors occur. Specifically for $P_b = 10^{-4}$ and for the same number of feedback bits, the balanced STBCs CS3Tx2Fb and CS4Tx3Fb provide E_b/N_0 gains of 1.4 and 1.7 dB compared to the antenna selection schemes A3Tx2Fb and A4Tx3Fb, respectively.

V. Conclusion

In this paper, a full rate code selection technique has been proposed to improve the error performance of the wireless communication systems with n transmit and one receive antennas where $n > 2$. The approach aims to maximize the diversity by choosing the appropriate STBC for each possible channel situation from a set of balanced STBCs, at the transmitter. Since, instead of the perfect knowledge of the channels, only the relative states of the channel coefficients are needed at the transmitter, only one or a few feedback bits are sufficient. Moreover, due to the number of transmit antennas $n > 2$ for a full rate balanced STBC, the transmitted symbol pairs are separately decoded, as in the OSTBC case. The new scheme provides increasing E_b/N_0 gain by increasing only the number of feedback bits, instead of increasing the number of transmit antennas as in the antenna selection case. It is also less sensitive to feedback errors compared to antenna selection.

References

[1] V.Tarokh, H.Jafarkhani and A.J.Calderbank,“Space-time block codes from orthogonal designs“, IEEE Trans. on Information Theory, vol.45, pp.1456-1467, July 1999.

[2] S.M.Alamouti,“A simple transmit diversity scheme for wireless communications“, IEEE Journal on Selected Areas in Commun., vol.16, pp.1451-1458, October 1998.

[3] H.Jafarkhani,“A quasi-orthogonal space-time block code“, IEEE Trans. on Communications, vol.49, pp.1-4, January 2001.

[4] O.Tirkkonen and A.Hottinen,“Complex space-time block codes for four Tx antennas“, Proceedings GLOBECOM, San Fransisco, U.S.A., pp.1005-1009, 2000.

[5] W.Su and X.G.Xia,“On space-time block codes from complex orthogonal designs“, Wireless Personal Communications, vol.25, pp.1-26, April 2003.

[6] D.A.Gore and A.J.Paulraj,“MIMO antenna subset selection with space-time coding“, IEEE Trans. on Signal Processing, vol.50, pp.2580-2588, October 2002.

[7] R.Heath and A.J.Paulraj,“Antenna selection for spatial multiplexing systems based on minimum error rate“, Proceedings IEEE ICC’2001, Helsinki, Finland, pp.2276-2280, 2001.

[8] G.Ganesan and P.Stoica,“Utilizing space-time diversity for wireless communications“, Wireless Personal Communications, vol.18, pp.149-163, August 2001.

[9] G.Ganesan, P.Stoica and E.G.Larsson,“Orthogonal space-time block codes with feedback“, Wireless Personal Communications, Vol.28, pp.287-312, March 2004.

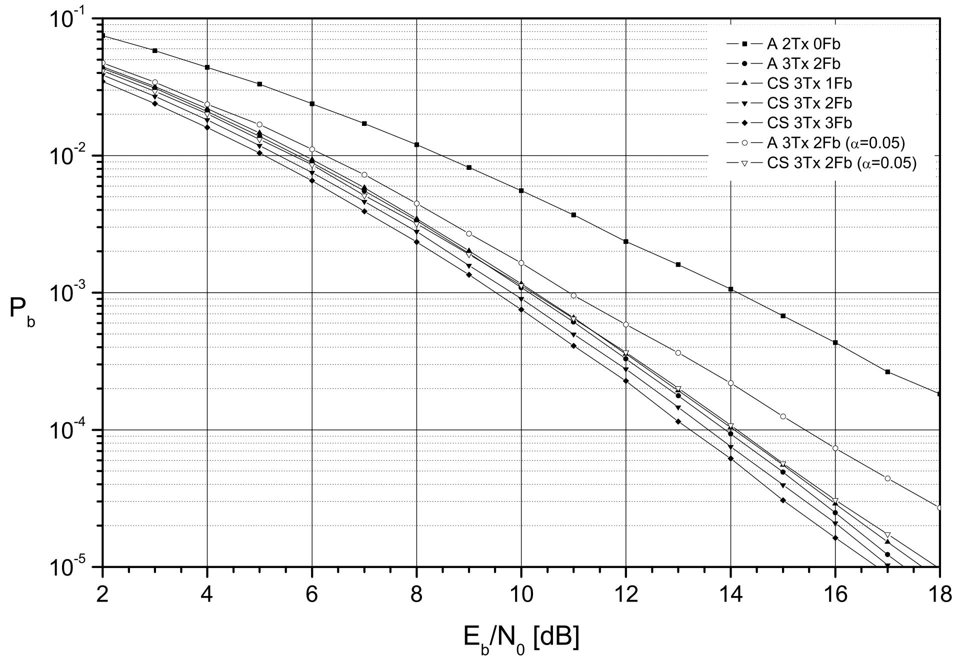

Fig. 1. Bit error probabilities for balanced space-time block code selection with three transmit antennas

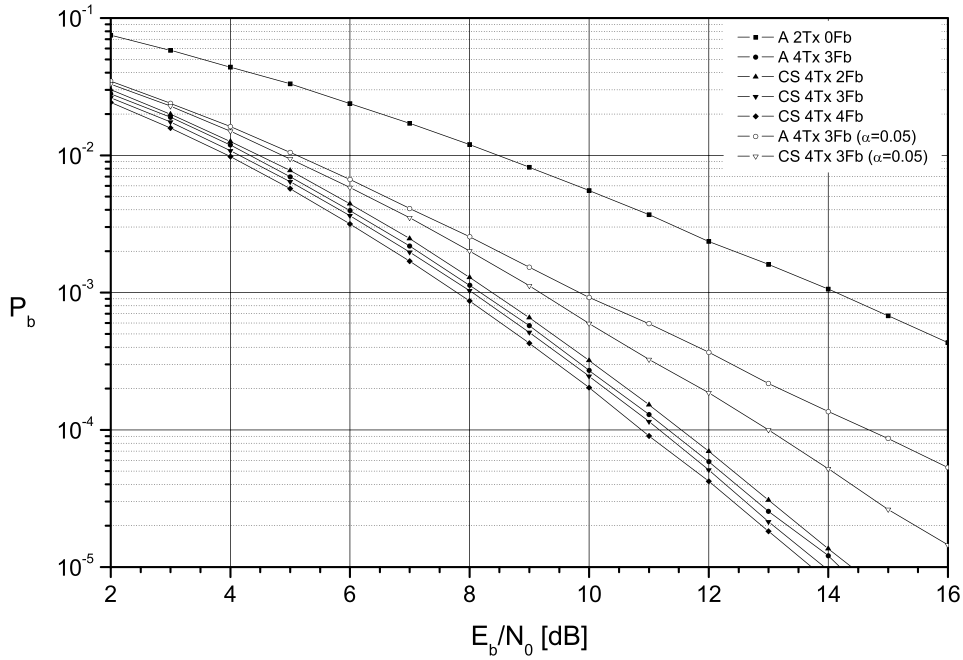

Fig. 2. Bit error probabilities for balanced space-time block code selection with four transmit antennas