Effects of Channel on Multi-user CFO Estimation for Interleaved OFDMA Uplink

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***Abstract* -- Synchronization of multi-user Orthogonal Frequency Division Multiplex Access (OFDMA) uplink is a challenging research topic. There are few published algorithms for this purpose. In this paper we analyze the effects of uplink channel on the performance of structure based synchronization algorithm for multi-user OFDMA recently proposed by Cao et. al. It is shown that carrier frequency offset (CFO) estimation error is insensitive to user's channel type because signal power is conserved due to the signal structure. Underestimation of number of CFOs (users) is possible when SNR is low. We examine a modification to original algorithm has that has significantly lower underestimation probability with the same amount of computation. It is also found that both methods are very robust to the near-far effect.**
***Index terms* -- OFDM, OFDMA, Uplink, CFO Estimation, Multi-user, Interleaved subcarriers.**

I. Introduction

Synchronization in OFDM systems is one of the key problems that are still investigated especially in the case of uplink transmission, from the user to the base station [1]. In multi-user system there are several users and each of them has to be synchronized with the base station [2]. Since every user has its own time delay (due to propagation) and frequency offset (due to oscillator mismatch and Doppler shift) base station receiver have to estimate them all so compensation before demodulation could be possible. Time delay can be handled by guard interval but there is no such easy technique for frequency offset. Uncompensated or bad estimation of frequency offset of only one user would introduce multiple access interference to other, synchronized, users.

In multi-user OFDMA system users share available set of subcarriers according to some in advance determined scheme. One simple way is to allocate carriers in blocks like in Frequency Division Multiplex. The other way is to apply interleaved subcarrier assignment scheme, assign one carrier to each user and repeat this regular assignment until all subcarriers are allocated. The most general is "random" assignment of subcarriers according, to lets say, channel state. Interleaved subcarrier allocation scheme is attractive because it allows simple transmitter implementation [3].

There are only several published algorithms for carrier frequency offset (CFO) estimation in the case of multi-user OFDMA uplink. Among them group of blind or semi-blind algorithms is attractive since they do not require pilot signals or carriers, but relies on a priori information of received signal structure.

CFO estimation in the uplink of block subcarrier assignment scheme has been reported in [4-6], where frequency guard band was used so that signals from neighboring users can be separated. Estimation of CFO for arbitrary subcarrier assignment among users is investigated in [7-9]. Method in [7] request that each user send one pilot symbol which is used in the uplink receiver estimator of CFO and time delay. Algorithm proposed in [8] has an estimation bias, while [9] is computationally demanding. The CFO estimation algorithm for interleaved subcarriers was proposed by Cao et al. [10, 11], and performances are analyzed in the case all users have the same type of channel. This assumption is not realistic since some users can have line of sight, while the others at the same time could be in non line of sight channel.

In this paper we examine Cao's of deterministic multi-user CFO estimation for interleaved OFDMA uplink in various channel types: Gaussian (AWGN), non line of sight (NLOS), and line of sight (LOS) which is not done in available literature. We also propose a modification to this algorithm with improved performance and the same amount of computations.

The rest of the paper is organized as follows. In Section II is given mathematical model of generalized OFDMA uplink which is used in a special case of interleaved subcarriers assignment scheme. Original and modified algorithm for CFO estimation are explained in section III. Simulation setup and the results are reported in Section IV, and conclusions are drawn in Section V.

II. Mathematical Model for Multi-user OFDMA Uplink

In this section, we give mathematical model for multi-user OFDMA uplink. It is based on matrix transmission model from [10]. Assume OFDMA system with N subcarriers divided to Q subchannels in interleaved manner, so each subchannel has $P=N/Q$ subcarriers. Subchannel q (q = 0, 1, …, Q-1) contains set of subcarriers $\{q, Q+q, \ldots, (P-1)Q+q\}$. Let there be $K<=Q$ active users in one OFDMA block, than time domain samples of the kth user received at base station after the removal of cyclic prefix (CP) are given by

$$r^k(n) = \sum_{p=0}^{P-1} H_p^{(k)} X_p^{(k)} e^{j2p(pQ+q+x^{(k)})n/N}, \quad (1)$$

where $H_p^{(k)}$ is channel frequency response on *p*th subcarrier of user *k*, $X_p^{(k)}$ is transmitted QAM modulation symbol of *k*th user on *p*th subcarrier in one OFDMA block, *n*= 0, 1, …,*N*-1, $x^{(k)} = \Delta f^{(k)} / \Delta f$ is CFO of *k*th user normalized to subcarrier spacing in OFDMA system.

Expression (1) can be further simplified to

$$r^k(n) = e^{j2pq^{(k)}/P} \sum_{p=0}^{P-1} H_p^{(k)} X_p^{(k)} e^{j2ppn/P}. \quad (2)$$

In (2) $q^{(k)} = (q + x^{(k)})/Q$ is defined as effective CFO. It is shown in [10] that in interleaved scheme received signal of each user has special periodic structure

$$r^k(n + nP) = e^{j2pnq^{(k)}} r^k(n), \quad (3)$$

where *v* is an integer. It means that the received *N* signal samples of *k*th user has periodicity every *P* samples, and let there be *T*=*N*/*P* number of periods within one OFDMA block. We notice common linear phase shift in (3) that depends on effective CFO which has one important property. Different users occupy different subchannels and thus have distinct effective CFO. Range of effective CFO is ((*q*-0.5)/*Q*, (*q*+0.5)/*Q*) with absolute maximum of normalized CFO limited to 0.5. Using such definition the effective CFO of different users do not overlap. One OFDMA block at BS receiver contains superposition of signals from all users.

$$r(n) = \sum_{k=1}^{K} r^k(n), \quad (4)$$

for *n* = 0, 1, …, *N*-1. The next step is to organize samples *r*(*n*) of one OFDMA block at base station in observation matrix

$$Y = VR + N \quad (5)$$

where *V* is a *Q*x*K* Vandermonde matrix that models effective CFO, and its element are$n_{q,k} = e^{j2p(q-1)q^{(k)}}$,

$$R = \begin{bmatrix} r^{(1)}(0) & r^{(1)}(1) & ... & r^{(1)}(P-1) \\ r^{(2)}(0) & r^{(2)}(1) & ... & r^{(2)}(P-1) \\ ... & ... & ... & ... \\ r^{(K)}(0) & r^{(K)}(1) & ... & r^{(K)}(P-1) \end{bmatrix} \quad (6)$$

is signal matrix, and *N* is white Gaussian noise matrix with zero mean and mean power of σ^2.

III. Algorithm for CFO Estimation

In this section, we will give CFO estimation algorithm and explain its modification.

Formulation of the CFO estimation problem corresponds to estimation of a direction of arrival of narrow band signals impinging a linear antenna array from different angles. Thus we can use the same estimation methods. High-resolution subspace technique MUSIC (Multiple SIgnal Clasification) [12] is know as very good and accurate for this purpose, thus the algorithm for CFO estimation proposed in [10] is based on it.

A task of CFO estimation is to find normalized CFOs of all *K* active users. In order to determine the CFO of each user we need to match each of *K* estimates to the corresponding user. It is possible due to the property of effective CFO and assumption that normalized CFOs are in the range (-0.5, 0.5).

From the observation matrix *Y* we calculate, using singular value decomposition, the signal U_s and noise subspace U_n of covariance matrix $F= YY^H/P$, *T*=rank(*F*). Then effective CFOs are estimated as values that correspond to *K*<*T* largest local-maxima of the MUSIC cost function

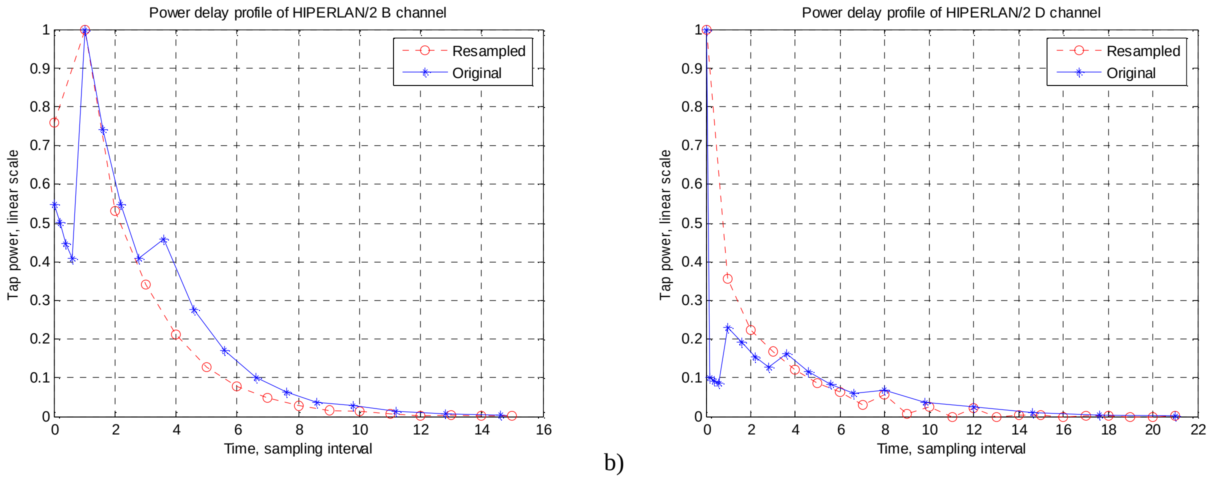
$$P_{MUSIC}(q) = \frac{a^H(q)a(q)}{\left\| a^H(q) U_n U_n^H a(q) \right\|^2}, \quad (7)$$

where $a(q) = [1, e^{j2pq}, ..., e^{j2p(R-1)q}]^T$. Then from estimated effective CFO's of all active users one can calculate normalized CFO for each of them using $x^{(k)} = Qq^{(k)} - q$.

MUSIC algorithm is known to be asymptotically unbiased, but we are limited to its finite sample properties. It is shown in [13] that noise subspace estimation can be improved when using also conjugate values of observation matrix with the following covariance matrix definition $\Phi = (YY^{\mathrm{H}} + J\overline{Y}\overline{Y}^{\mathrm{H}}J)/P$, where *J* is exchange matrix filled with zeros, except on minor diagonal where it has ones. This technique also slightly improves estimation accuracy. This algorithm will be further referred as Modified MUSIC (MMUSIC) as in [13], while in available literature it is also called forward-backward approach.

IV. Simulation Results

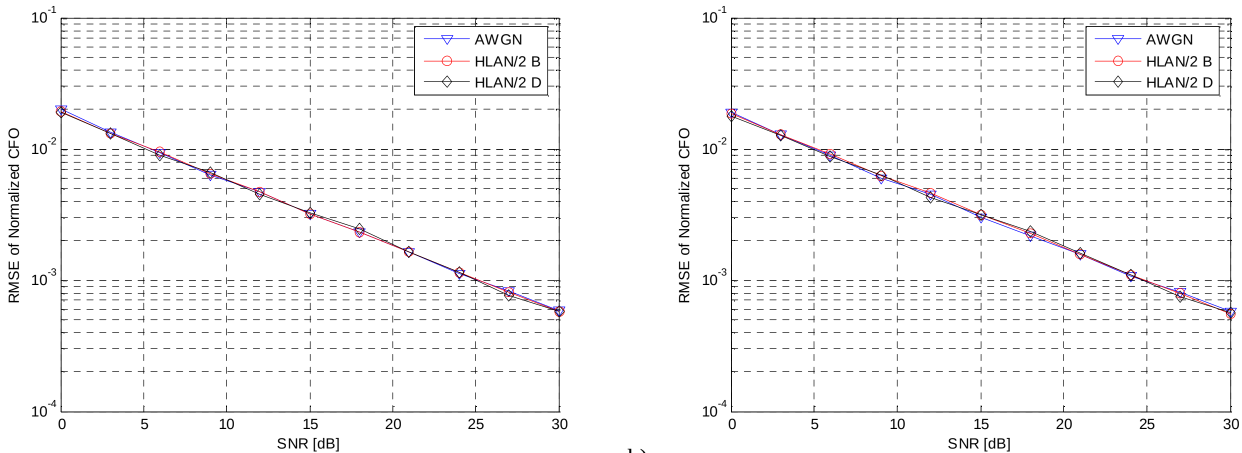
In this section we first give simulation setup and then simulation results. We assume OFDMA system with N=2048 carriers with 20 MHz bandwidth in 5 GHz band. Sampling period is 50 ns. Length of CP is 256, one half of it compensates maximal channel delay and the other half, free of inter block interference, is used as T=Q+1 observation vector, so there can be Q active users in the system. In one cell there are Q=16 channels and P=128 subcarriers in each. Subcarriers are assigned to the users according to interleaved scheme. We use AWGN, HiperLAN/2 B and D channels that are constant during one OFDMA block. B channel is NLOS and D channel is LOS with Rice factor of 10 dB [14]. Since our sampling frequency is not the same as in [14] we resample power delay profile using the following algorithm. Power of each tap in the original HiperLAN/2 delay profile is shared between two taps in the resampled profile that correspond to the previous and following sampling period. Ratio of shared powers is inversely proportional to the absolute time difference between original delay and the delay of each of two resampled taps. Power delay profile of original and resampled Hiperlan/2 channels are shown on Fig. 1.



a) b)

Fig. 1. Power delay profile of original and resampled channel. a) HiperLAN/2 B, b) HiperLAN/2 D.

For fair comparison with AWGN channel we normalize sum of tap powers of HLAN channel to one. First, we evaluate channel type effects on root mean square error (RMSE) of estimated normalized CFO. Each user has the same channel type to easy the simulation experiment. We perform 50 Monte Carlo simulations with K=16 users so averaging is done on 800 estimations for each SNR. SNR is defined in time domain (per user) as $SNR^{(k)} = E\{|r^{(k)}(n)|^2\}/s^2$. In each Monte Carlo test the normalized CFO and the channel of each user are randomly generated. Maximal absolute value of normalized CFO is 0.2, which is about 2000 Hz of frequency shift. All carriers were modulated with independent QPSK symbols. Results from simulation for RMSE are given on Fig. 2.



a) b)

Fig. 2. RMSE of normalized CFO vs. SNR in various channel type.
All K=16 users have the same channel type. a) MUSIC algorithm, b) Modified MUSIC.

The same results (for both algorithms) are obtained for RMSE averaged among even users in the case of near-far effect, when odd users are 20 dB stronger. Such big ratio is taken because synchronization is done prior the power control. We notice that RMSE is not changed in the case of multipath channels. It is because the signal power is almost not changed thanks to maximal frequency diversity of interleaved scheme. We may conclude that estimation error of *k*th user depends on $SNR^{(k)}$ value, and in the case of many interleaved subcarriers assigned to one user it is independent on channel type.

For low SNR values number of peaks in the cost function could be less than the number of users, such situation is called underestimation. It is especially frequent for 100% loaded system, we analyze, and close effective CFOs. In this scenario the estimation algorithm fails to give separated peaks. Cost function shape is shown on Fig. 3 for both MUSIC and MMUSIC.

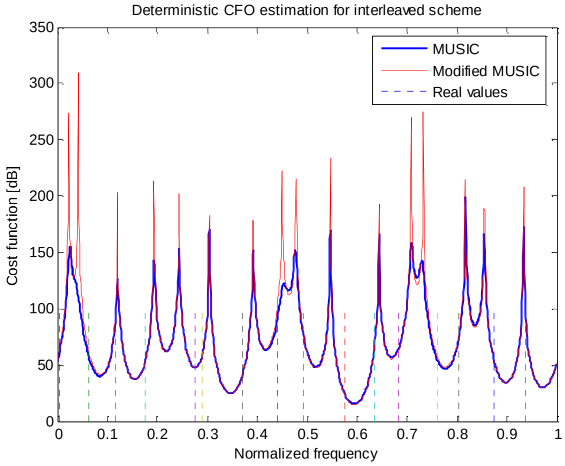


Fig 3. Effective CFO cost function shape and real values for the case of MUSIC underestimation. Maximal absolute normalized CFO is 0.4. SNR = 0 dB. Channel type is AWGN.

Uncompensated CFO of only one user introduces multiple access interference in the system and thus such situation is to be avoided. Monte Carlo tests were performed to quantify underestimation occurrence in three channel types for both algorithms. We notice from the results given in Table 1. that MMUSIC has significantly lower underestimation occurrence than MUSIC in all channel types. Since amount of computations is almost the same, as well as accuracy, it is clear that MMUSIC algorithm is better to use.

Table 1. Results of underestimation occurrence test for MUSIC and Modified MUSIC on the same observation matrix. Maximal absolute value of normalized CFO is 0.4.

SNR = 0 dB, 500 MC tests	Channel type for underestimation test		
	AWGN	HiperLAN/B	HiperLAN/D
MUSIC	57	55	49
Modified MUSIC	5	9	2

In [11] diversity techniques are proposed to lower the underestimation occurrence and also the RMSE of normalized CFO, if it is possible.

V. Conclusion

Performances of algorithm for multi-user CFO estimation for OFDMA uplink in various channel types are assessed in this paper. The method recently is proposed in [10]. Channel type does not influence the estimation of CFO because the algorithm is relying on received signal (user) power. Even in frequency selective fading channel, the signal power is not significantly lowered due to its interleaved structure and many subcarriers in one channel. RMSE of normalized CFO achieves 0.01 for SNR of 6 dB. That corresponds to SNR per carrier of 18 dB. In [15] is reported that CFO of 1% limits maximal SNR per carrier to 30 dB, since we have a margin of 12 dB there will be no BER degradation due to estimation error with higher QAM constellation. Underestimation occurrence is possible when SNR is low, system is fully loaded, or user’s signals are to close. It is very important to lower as much as possible its probability using the modified algorithm or/and by diversity techniques. Finally, the estimation method is near-far robust because it is based on MUSIC algorithm.

VI. References

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