User-Bin Allocation Methods for Adaptive-OFDM Downlinks of Mobile Transmissions

Mihaly Varga, Vasile Bota, Zsolt Polgar

***Abstract*— The paper analyzes the throughput performances ensured by the Frequency-Hopping and Optimal-Allocation user-bin allocation methods in an OFDM access scheme for the downlink connection of the 4G air-interface that employs adaptively a set of LDPC-coded or non-coded QAM modulations on a Rayleigh-faded multipath mobile channel. The probabilities of employing a certain modulation are evaluated both theoretically and by computer simulations. Some considerations regarding the signaling uplink traffic required by the two allocation methods are also presented.**

***Index Terms*—Frequency-Hopping bin-allocation, Optimal bin-allocation, LDPC-coded QAM constellations, Throughput vs. SINR performances**

I. INTRODUCTION

Some of the solutions proposed for the downlink connection of the 4G-air interface include the employment of Adaptive Non-coded or Coded QAM constellations in an OFDM scheme [1, 2, 3]. The user-frequency bandwidth is obtained by allocating each user a fixed number of sub-carriers C_u, out of the N_u useful sub-carriers of the OFDM scheme, along a fixed number of T_u OFDM symbol periods, generating the user-bin of $S_T = C_u$ x T_u QAM symbols. The whole available bandwidth, i.e. N_u sub-carriers, is split into $B_u = N_u/C_u$ bins that are allocated to different users. To compensate the variable attenuation of the Rayleigh fade of the mobile multipath channel, two methods that ensure the frequency diversity are considered:

- The frequency position of a user-bin is allocated according to a frequency-hopping pattern, for a time interval equaling the bin period; this method is employed in [1].
- By allocating to each user-bin the best frequency position (optimal allocation), i.e. the group of C_u sub-carriers that ensure the best SINR for the bun-period envisaged; this involves the state-prediction of all available bins, during a prediction time-horizon, which has to be performed by the user mobile station.

Since the user-bin allocation method has a significant influence upon the throughput provided by the connection and upon the signaling traffic both on the downlink and the uplink connections, this paper tries to present a performance comparison between the performances ensured by the two methods.

A. Transmission scheme

The performances of the two user-bin allocation methods are analyzed for an OFDM scheme with the following parameters:
OFDM-symbol frequency f_s = 10 kHz, guard interval G = 11 μs, leading to a payload OFDM-symbol rate f_s' = 9.09 kHz;
N_u = 1000 payload sub-carriers, carrier frequency f_c = 1.9 GHz; the number of user-bins available is B_u = 50.
The user-bin dimensions are S_T = 20 sc x 6 symbol periods with a effective bin-rate D_{bin} = 1501 bins/s;
There are M = 120 QAM-symbols/bin out of which only M_u = 108 would be payload symbols.
The structure of the scheme is similar to the one presented in [2], [3].
Two adaptive QAM sets of configurations are considered:

- a non-coded one, ANCC, with F = 8 component configurations, i.e. 2-PSK to 256-QAM, carrying $n_l = 1,\dots,8$ bits/QAM-symbol. It is listed in table 1 together with the thresholds T_i, that separate the regions within which each QAM constellation is optimum [2], [3], and the nominal bit rates D_{nl} that could be accomplished in the OFDM-scheme described above [4]

(Const. No.); n_l (bits/symb)	(1); 1	(2); 2	(3); 3	(4); 4	(5);	(6); 6	(7); 7	(8); 8
T_l [dB]	-2	8.3	13.2	16.2	20.2	23.6	26.6	29.8
D_{nl} [kbit/s]	162	324	486	648	810	972	1134	1285

Table 1 Parameters n_l., T_l, and D_{nl} of the adaptive non-coded QAM set of configurations, ANCC

- a LDPC-coded one, ACC, with F = 16 component configurations [4], having the parameters listed in table 2. In table 2, n_l denotes the number of bits/symbol, n_{cl} the number of coded bits/symbol, R_{cfgl} the coding rate of the configuration (that carries both coded and non-coded bits) and D_{ncl} the nominal payload that could be accomplished.

The nominal bit rate of the non-coded and coded configurations with index l were computed, in terms of n_l, using (1), where t = 8 is the number of CRC bits of a non-coded bin.

$$D_{nl} = D_{bin} \cdot M_u \cdot n_l \cdot (1 - \frac{t}{M_u \cdot n_l}); \quad D_{ncl} = D_{bin} \cdot M_u \cdot n_l \cdot R_{cfgl} \tag{1}$$

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(Const. No.); n_i; n_{ci}	(1); 1;1	(2); 1; 1	(3); 2; 2	(4); 2;2	(5); 2; 1	(6); 3; 2	(7); 3; 2	(8); 4; 2
R_{cfgi}	0,29	0.49	0.47	0.68	0.85	0.65	0.79	0.73
T_i [dB]	-2	-1	2.5	4.5	6.5	8	9	11
D_{ci} [kbit/s]	47	80	152	221	275	314	383	476
(Const. No.); n_i; n_{ci}	(9); 4; 2	(10); 5; 2	(11); 5; 2	(11); 6: 4	(12); 7; 2	(14); 8; 4	(15); 8; 4	(16); 8; 0
R_{cfgi}	0.84	0.79	0.87	0.86	0.85	0.82	0.90	1
T_i [dB]	13	15	16.5	18.5	21	23.5	25	30
D_{ci} [kbit/s]	545	638	707	883	962	1064	1112	1285

Table 2 Parameters n_l, T_l, D_{cl} and R_{cfgl} of the adaptive coded QAM set of configurations, ACC

The throughput Θ_l provided by one QAM modulation is computed using (2), where BinErl(SINR) denotes the bin error rate of modulation l:

$$\Theta_l(SNR) = D_{nl}(1 - BinErl(SINR)); \qquad (2)$$

The bin error rate of a non-coded scheme can be computed, as shown in [2], by using the symbol error probability of the N-QAM (p_{eNl}) and is expressed by:

$$BinErrl(SINR) = 1 - (1 - p_{eNl}(SINR))^{N_u}; \qquad (3)$$

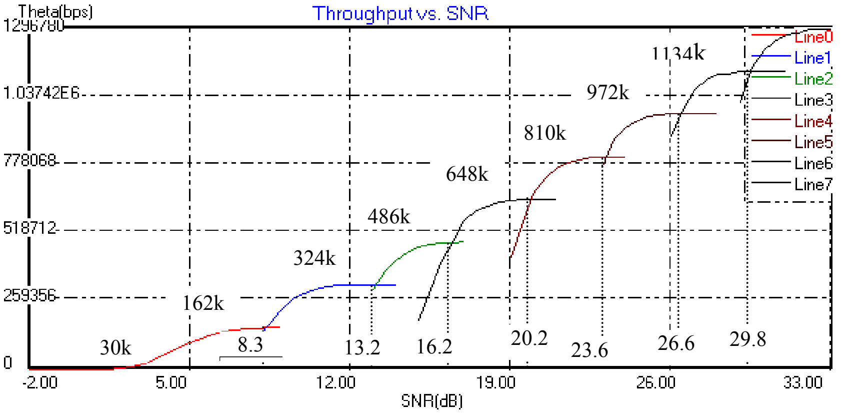

Figure 1 Θ vs. SINR of the ANCC set on a flat AWGN channel

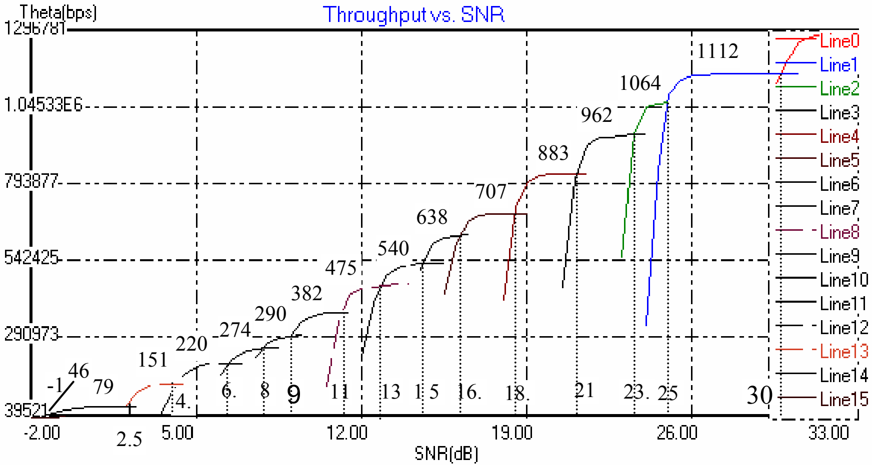

Figure 2. Θ vs. SINR of ACC set on a flat AWGN channel

As for the LDPC-coded configuration, since it employs coded and non-coded bits [4], the symbol error probability can not be computed theoretically, according to the authors' best knowledge. Therefore, for these configurations, the BinErrl and the throughput vs. SINR were determined by computer simulations, using the programs described in [5]. The throughput vs. SINR curves of the two sets of configurations are shown in figures 1 and 2, respectively.

B. Channel model

The multipath Rayleigh (MPR) channel model considered in this paper assumes W Rayleigh-faded waves. The first arrived wave is attenuated so that its average power ensures a given SINR value, with the Gaussian noise (including other interferences); the other waves are attenuated with relative attenuations a_k, referred to the first wave, and have relative delays equaling t_k [6]. The particular Rayleigh-faded channel model employed has the following parameters:

CH: 4 paths - relative attenuations of 0, 3, 6, 9 dB and relative delays of 0, 200, 400 and 600 ns, and v_{user} = 120 km/h.

II. DISTRIBUTION OF THE RATED-LEVEL OF THE RECEIVED FADED SIGNAL

The p.d.f. of i-th Rayleigh-faded wave, with an average power of A_i^2 divided by a constant σ^2, delivers the p.d.f. of the rated level x = r/σ:

$$p_i\left(\frac{r}{\sigma}\right) = \frac{r/\sigma}{A_i^2/\sigma^2}\exp\left(-\frac{r^2/\sigma^2}{2A_i^2/\sigma^2}\right) \qquad (4)$$

If the received signal is composed of W Rayleigh-faded waves, which have relative attenuations a_i, and delays τ_i, referred to the first arrived wave, the p.d.f. of rated level of the received signal transmitted on the k-th subcarrier is computed by using:

$$P_k(r) = \int_0^\infty p_1(x_1)\int_0^\infty p_2(x_2)\cdots\int_0^\infty p_{W-1}(x_{W-1})\cdot U\left(x_1, x_2, \ldots x_{W-1,} \Omega k_{1W}, \Omega k_{2W}, \ldots, \Omega k_{(W-1)W}\right)dx_{W-1}\ldots dx_2 dx_1; \qquad (5)$$

$$U\left(x_1, x_2, \ldots x_{W-1}, \Omega k_{1W}, \Omega k_{2W}, \ldots, \Omega k_{(W-1)W}\right) = \begin{cases} 0; \quad if \quad r^2 - \left(\sum\limits_{i=1}^{W-1} x_i \sin \Omega k_{iW}\right)^2 < 0; \\ p_W\left(-\sum\limits_{i=1}^{W-1} x_i \cos \Omega k_{iW} + \sqrt{r^2 - \left(\sum\limits_{i=1}^{W-1} x_i \sin \Omega k_{iW}\right)^2}\right); otherwise \end{cases} \tag{6}$$

The p.d. f. computed with (5) and (6) takes into account the phase difference Ωk_{ij} between two waves, i and j, induced by the relative delays:

$$\Omega k_{ij} = 2\pi f_k \left(\tau_i - \tau_j\right) \tag{7}$$

III. USER-BIN ALLOCATION BY FREQUENCY-HOPPING

The B_u available bins are allocated to each user according to a B_u-states pseudo-random sequence, generated by the linear congruential method [7]. The hopping frequency is the reciprocal of the bin duration and each user enters into the pseudo-random sequence with a different initial value, delivered by the base station at signing-in. The mobile station performs a prediction of the channel state across the time-horizon evaluated in [2], only for the frequency bin it will use next; it then sends to the base station only the index of the configuration (code + QAM) it should employ, from the stored set (see table 1 or 2). To evaluate the performances of the adaptive use of the modulations we need to determine the probability of the received-signal rated level to lie between a pair of threshold that correspond to two consecutive SINR thresholds, T_l and T_{l+1} that separate the SINR range where configuration l should be used. For the SINR threshold T_l of the QAM transmission, the rated level threshold J_l is:

$$J_l = \frac{A_l}{\sigma_l} = 10^{T_l/20} \tag{8}$$

The probability of the rated level, received on subcarrier k, to lie between thresholds J_l, J_{l+1} is obtained by integrating (5) between them. The selection of the employed (non)coded modulation l is made according to the average of the rated levels received on the C_u subcarriers of that bin. For this bin-allocation method, the probability of a bin to be assigned to one user is $1/B_u$. Therefore, the probability of a user to employ the modulation l is obtained by averaging, over all N_u subcarriers, the probability of the rated level of the signal, received on a subcarrier k, to lie between thresholds, J_l, J_{l+1}:

$$P_{FH}\left(J_l < r < J_{l+1}\right) = \sum_{k=1}^{N_u} \frac{1}{N_u} \int_{J_l}^{J_{l+1}} P_k(x)dx\,; \tag{9}$$

The probabilities of a user to employ a certain modulation were computed using (5) – (9), for the ANCC non-coded set (table 1 and figure 1) for the Rayleigh-channel CH, for an SINR = 16 dB and are presented in figure 3. The same probabilities were evaluated by the computer-simulating the transmission on the same environment for 10,000 transmitted bins/user; figure 4 shows the numbers of bins during which each of the 8 modulations of ANCC was employed, N_{FH}.

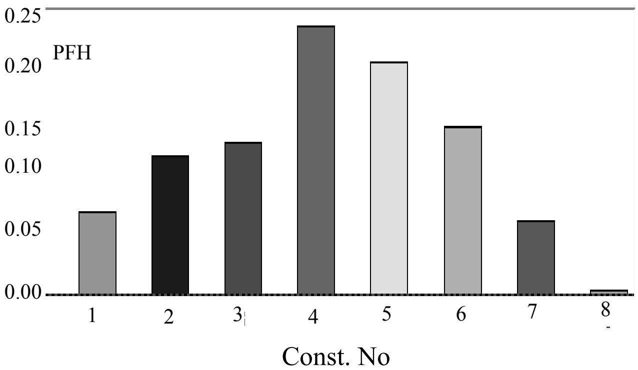

Figure 3 P_{FH} vs. Const. No. of ANCC - computed

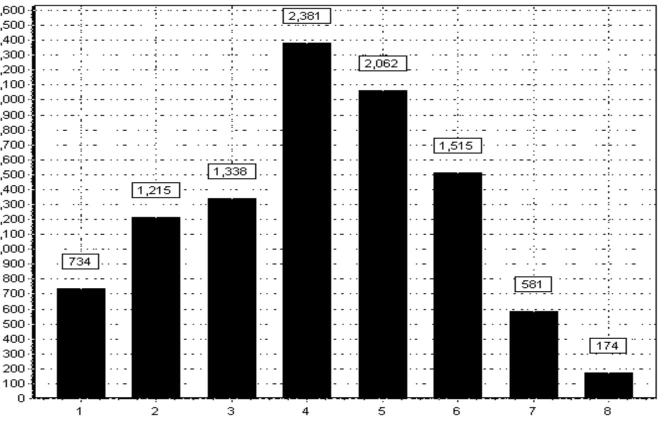

Figure 4 N_{FH} vs. Const. No. of ANCC for FH – simulated

Table 3 summarizes the values of the employment probabilities of the QAM constellation of table 1, obtained by computation and by simulation; there should be noted the small differences (bellow 1%) between the two sets of values.

Const. No - ANCC	1	2	3	4	5	6	7	8
P_{FH} -computed	0.0745	0.1228	0.1341	0.2362	0.2066	0.1488	0.0663	0.0107
N_{FH}/10000 - simulated	0.0734	0.1215	0.1338	0.2381	0.2062	0.1515	0.0581	0.0174

Table 3. Employment probabilities of the 8 constellations of ANCC for the FH allocation method - P_{FH} (SINR 16 dB)

IV. OPTIMAL ALLOCATION OF THE USER-BIN

The second user-bin allocation method attributes each user the frequency bin that ensures the highest SINR based on his channel prediction performed across the time-horizon, greater than the time response of the adaptation scheme, as established in [2], i.e. about 3 ms for the particular environment studied. Assuming that the users are placed in different positions, we may reasonably assume that each user has a different bin on which it sees a maximum SINR, on a given Rayleigh faded channel with a given average SINR.

To compute the probability $P_{OA}(J_l, J_{l+1})$ that the maximum rated level lies between thresholds J_l, J_{l+1}, we should compute the probability of the average rated level of bin m to lie in this domain multiplied with the probability that the average rated levels of all other bins to be small than the average rated level of bin m, where m ranges from 1 to B_u as expressed by:

$$P_{OA}\left(J_l < r < J_{l+1}\right) = \sum_{m=1}^{B_u} \left(\int_{J_i}^{J_{l+1}} Pa_m\left(x_m\right) \cdot \prod_{\substack{j=1 \\ j \neq m}}^{B_u} Q_{mj}\left(x_m\right) dx_m \right) \tag{10}$$

$$\text{where} \quad Pa_m(x) = \frac{1}{C_u} \sum_{q=1}^{C_u} P_{(jC_u+q)}(x); \text{ and } Q_{mj}(x) = \begin{cases} 1 & \text{if} \quad Pa_j(x) < Pa_m(x) \\ 0 & \text{otherwise} \end{cases}$$

The values of the probabilities of a user to employ a certain modulation were computed using (5) - (8) and (10), for the ANCC non-coded set (table 1 and figure 1) on the Rayleigh-channel CH, for an SINR = 16 dB and are presented in figure 5. The $P_{OA}(J_l, J_{l+1})$ probabilities were also evaluated by computer-simulation on the same environment (channel, set of SINR value, modulations, user-bin allocation pattern) for 10,000 transmitted bins/user; figure 6 shows the numbers of bins N_{OA} during which each of the 8 modulations of ANCC was employed.

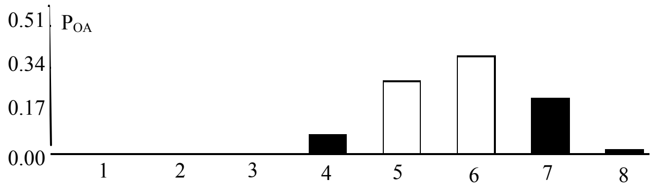

Figure 5 P_{OA} vs. Const.No. of ANCC – computed ↑

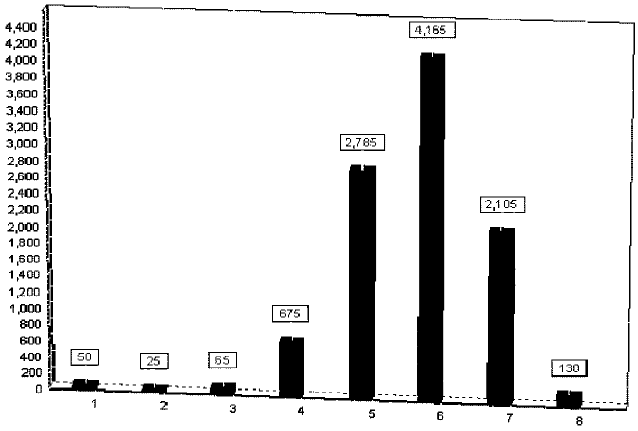

Figure 6 N_{OA} vs. Const.No. of ANCC for OpAl - simulated →

The differences between the computed values and the ones delivered by the simulation program, shown in table 4, have a maximum difference of about 2.5%, which gives a reasonable degree of reliability to the results obtained.

Const. No - ANCC	1	2	3	4	5	6	7	8
P_{OA} –computed	0.0012	0.0013	0.0076	0.0756	0.2910	0.3936	0.2158	0.0139
N_{OA}/10000 - simulated	0.0050	0.0025	0.0065	0.0675	0.2785	0.4165	0.2105	0.0130

Table 4. Employment probabilities of the 8 constellations of ANCC for the OpAl allocation method – P_{OA} (SINR 16 dB)

A comparison between the two user-bin allocation methods shows that:

The OpAl method employs most often modulations 5, 6 and 7, though according to table 1 it should employ modulation 4; this is because, by taking the bin with the highest SINR available, it takes advantage of the frequency diversity of the multipath channel in the best way possible. This leads to an average of 5.76 transmitted bits/QAM symbol.

Since its generating pattern is independent from the channel behaviour, the FH method employs in a more equally distributed manner constellations 2 to 6. Therefore the average number of bits/QAM symbol equals only 4.15.

V. THROUGHPUT PERFORMANCES OF THE PROPOSED USER-BIN ALLOCATION METHODS

The throughput ensured by the sets of coded modulations included in the OFDM scheme and using the bin-allocation methods presented above, we should multiply the probability P(l) of employing a (non) coded modulation l, with the average throughput Θ_l (2) provided by that modulation between J_l, J_{l+1}. The probability depends on the bin-allocation method as shown in (11).

$$\Theta_{av}(SINR) = \sum_{l=1}^{F} P(l) \cdot \Theta_l; \quad P(l) = \begin{cases} \mathrm{P}_{\mathrm{FH}}(J_l, J_{l+1}) & \textit{for Frequency Hopping} \\ P_{AO}(J_l, J_{l+1}) & \textit{for Optimal Allocation} \end{cases}; \tag{11}$$

Since the computed and the simulated values of P(l), present reasonable accuracy, for both allocation methods, this paper will present only the computer simulation results. The average throughput of a user was evaluated, both for the non-coded ANCC and for the LDPC-coded ACC, for SINR ranging from -2 to + 33 dB in 0.5 dB steps, transmitting 500 bins/user for each SINR value. Figure 7 shows the curve Θ_{avNC} vs. SINR of the ANCC and figure 8 shows the variation of Θ_{avC} vs. SINR of the LDPC-coded ACC respectively, both for channel CH.

Considering that the user-bin occupies a frequency bandwidth of 200 kHz, the spectral efficiencies β ensured by the two sets of configurations with the two allocation methods are shown in figures 9 and 10.

The employment of the Optimal Allocation method for the user-bin frequency position leads to a significant increase of the spectral efficiency and throughput, compared to the values ensured by the Frequency Hopping allocation method. This increase, more significant at moderate SINR ratios, may reach values between 1 – 1.7 bps/Hz, both for the non-coded and coded sets of modulations. There should be noted that the performances of the coded ACC set of modulations that are allocated with the FH method (figure 10) are comparable to the performances ensured by the non-coded ANCC set of modulations that are allocated with the OpAl method (figure 9). Comparing the corresponding curves, the differences between the spectral efficiencies are smaller than 0.5 bps/Hz, in favor of the OpAl method.

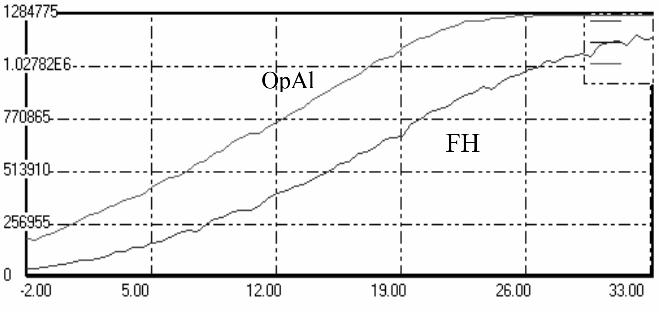

Figure 7 Θ_{avNC} vs. SINR of ANCC for FH and OpAl on CH1

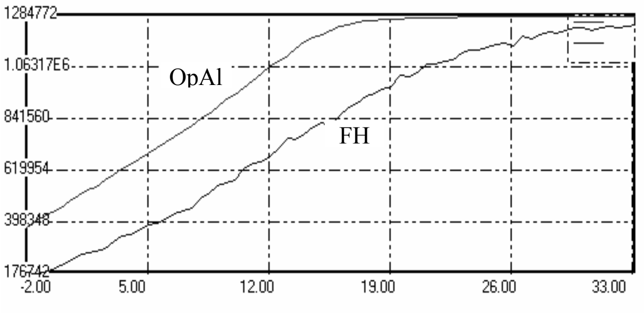

Figure 8 Θ_{avC} vs. SINR of ACC for FH and OpAl on CH1

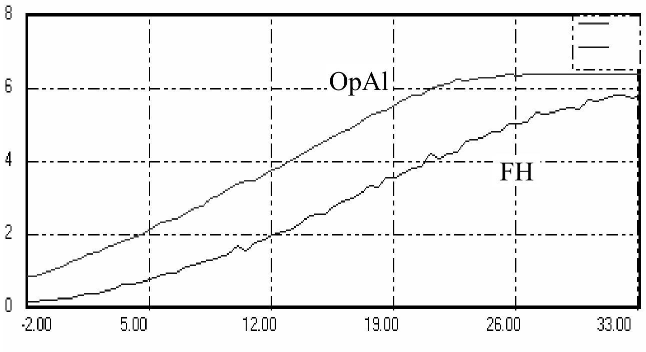

Figure 9 β_{NC} vs. SINR of ANCC for FH and OpAl on CH

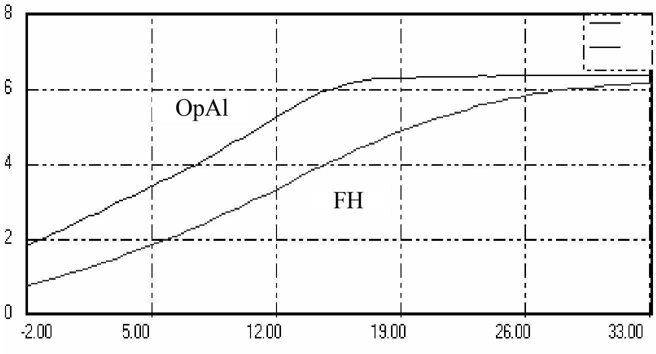

Figure 10 β_{avC} vs. SINR of ACC for FH and OpAl on CH

VI. EVALUATION OF THE UPLINK SIGNALLING BIT-RATE REQUIRED BY THE ALLOCATION METHODS

The FH bin allocation method needs to transmit on the uplink the index of the modulation to be employed in the bin that is to be transmitted after the prediction time-horizon. This index could be transmitted on maximum 4 bits, involving a signaling bit rate/user R_{SFH} = 4 bits/bin x D_{bin} = 6.67 kbps, for the transmission scheme employed.

The OpAl bin allocation method has to transmit all of the SINR values predicted for B_u available bins. These values require 8-9 bits each, for the SINR range considered, leading to a signaling bit-rate: $R_{SOA} = B_u$ x 9 x D_{bin} = 750 kbps. This high signaling bit rate might be prohibitive. An alternative, to decrease the uplink signaling traffic, would be to transmit only the M bins which are predicted to have the best SINR values; in this case the uplink message should contain the bin- index (6 bits) and the predicted SINR value (9 bits) multiplied by M. The resulted signaling rate would be $R_{SOA\text{-}M}$ = M x (6+9) x D_{bin} = 125 kbps, for M = 5.

There should be noted that the computational load, in the mobile station, required by the FH allocation method is B_u times smaller than the one required by the OpAl method.

VII. CONCLUSIONS

The theoretical and simulation-based evaluations show that the Optimal Allocation bin-allocation method ensures throughputs significantly higher than the ones provided by the Frequency Hopping method, but the uplink signaling traffic required might be prohibitive. To compensate this major drawback, two possible solutions are suggested.

A first possibility would be to decrease the amount of information transmitted by the mobile station on the uplink; this modified OpAl method could ensure about the same performances, but the signaling bit-rate would still be rather high.

A second option proposed is to employ the FH bin-allocation method in combination with the set of LDPC-coded QAM modulations; this option could offer a reasonable trade-off between the throughput provided, on one hand, and the signaling traffic and computational load in the mobile station, on the other hand.

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