Sensitivity of the Orthogonalization Methods for QO-STBC to Feedback Errors in an OFDM Environment

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Abstract

We propose two feedback based transmitter preprocessing techniques to overcome the diversity loss issue of quasi-orthogonal space-time block-codes (QO-STBCs). The algorithms are proposed for an OFDM channel with four transmit and one receive antennas. In order to decrease the feedback overhead substantially, a frequency bin partitioning method based on majority voting is proposed . The performance of the preprocessing algorithms is investigated in the presence of feedback errors. It is demonstrated that both methods provide significant gain over the open loop QO-STBC even for a moderate feedback error rate such as 10^{-2} and with reduced feedback overhead using frequency bin partitioning.

I. Introduction

Employing multiple antennas at the transmitter and the receiver is an efficient method to enhance the performance of a wireless communication system through exploitation of spatial diversity. The limitations in the mobile terminal, such as dimension, weight, battery power consumption etc., have caused considerable attraction to the transmit diversity techniques such as transmit beamforming and space-time block codes for the downlink. Although transmit beamforming provides superior downlink performance, it requires high precision and high quality channel state information at the transmitter (Tx-CSI). This makes transmit beamforming infeasible for many wireless applications due to the requirement for extensive feedbacks. Moreover, error in the Tx-CSI can result in substantial degradation in the beamformer performance.

Instead, the recently introduced space-time coding eliminates the need for Tx-CSI while preserving the diversity gain, [1]. In this study, we will focus on space-time block codes (STBCs) [2], [3] and [4]. Although STBCs require a very simple decoding algorithm and provide full diversity order, complex valued orthogonal-STBCs (O-STBCs) attain full code rate only for two transmit antennas. For more than two antennas, the code rate drops below unity, if full diversity order is to be retained. Quasi-Orthogonal (QO)-STBCs [5] have been proposed to increase the code rate to unity but they suffer from loss in the diversity order because of certain coupling between detected symbols [6].

In [7], two techniques based on partial Tx-CSI were proposed to overcome this coupling problem and to restore the full diversity order for QO-STBCs. One of the proposed methods is based on the rotation of the phase of the signals transmitted from certain antennas in a prescribed way, whereas the other one selects best two antennas out of four according to the quality of the individual channels. Both methods provide diversity order of four with very little feedback which can be as low as one or two bits per frame.

Orthogonal frequency division multiplexing (OFDM) is the most widely preferred signaling technology for future generations wireless communication systems. In this paper, we extend these closed-loop QO-STBC techniques to OFDM. Unlike a TDMA system with flat-fading channel or a CDMA system possibly with frequency selective fading channel [7] where only one or two bits feedback per frame is adequate, in OFDM, each subcarrier requires a separate feedback, resulting in a substantial increase in the feedback overheads. However, by exploiting the correlation between the response of adjacent frequency bins, the need for feedback is decreased considerably by data compression. In this study we investigate a majority voting based method for both feedback schemes. Although this method is information lossy, it provides very satisfactory performance.

In a time-division multiple access (TDMA) based transmission, Tx-CSI can be obtained from the knowledge of the reciprocal channel if the coherence time of the channel is long enough. If a frequency-division multiple access (FDMA) based transmission is employed, the CSI available at the receiver can be sent to the transmitter through a feedback channel. However, due to practical reasons, such as noise in the feedback channel, channel estimation error at the receiver, variations of the feedforward channel during CSI feedback period, etc., Tx-CSI is exposed to feedback errors. We will also investigate the robustness of the proposed methods against such feedback errors.

II. Problem Statement

Consider a communication channel with four transmit and one receive antennas. Although we consider single receive antenna for notational simplicity, it is straightforward to extend this scheme to multiple receive antennas [8]. Each subchannel, $h_i[n] = \sum_{j=0}^{L-1} h_{i,j}\delta_{j-n}$, $i = 1, ...4$, is assumed to have independent frequency-selective fading. Here, L is the multipath spread of all subchannels, the zero-mean complex-valued circularly symmetric Gaussian random variable $h_{i,j}$ represents the j-th path coefficient of the channel between the i-th transmit antenna and the receive antenna, and δ_n is the Kronecker delta function.

As it is well-known, in an OFDM scheme with a channel multipath spread of L samples, the channel can be diagonalized if a cyclic prefix of length $P > L$ is added to the data frame. Under this condition, the N subcarriers behave as frequency-flat fading sub-channels with coefficients $\begin{bmatrix}\lambda_{i,1} & \cdots & \lambda_{i,N}\end{bmatrix}^T = \mathbf{Q}\begin{bmatrix}h_{i,1} & \cdots & h_{i,L-1} & \mathbf{0}\end{bmatrix}^T$. Here, $\mathbf{Q}$ is the $N \times N$ FFT operator and $\mathbf{0}$ is the $(N-L) \times 1$ all-zero vector.

For the QO-STBC, we will employ the following $4N \times 4$ codematrix

$$\mathbf{C}_{QO} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 \\ -\mathbf{s}_2^* & \mathbf{s}_1^* & -\mathbf{s}_4^* & \mathbf{s}_3^* \\ -\mathbf{s}_3^* & -\mathbf{s}_4^* & \mathbf{s}_1^* & \mathbf{s}_2^* \\ \mathbf{s}_4 & -\mathbf{s}_3 & -\mathbf{s}_2 & \mathbf{s}_1 \end{bmatrix}$$

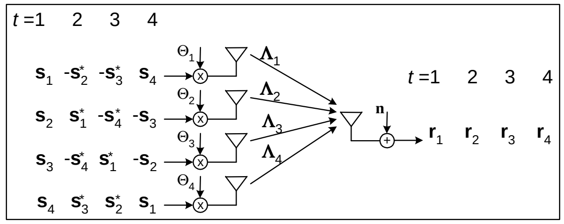

Fig. 1. The proposed system model with four transmit and one receive antennas. The QO-STBC to be transmitted is depicted on the left hand side besides the weighting of the antennas used in the proposed algorithms. The channel coeffients $\mathbf{\Lambda}_i$ are the effective flat-fading representations of $h_i[n]$ after frequency domain transformation.

where the elements of the $N \times 1$ OFDM symbol vectors $\mathbf{s}_k$, $k = 1, ..., 4$, are drawn from complex valued constellations, and N is the number of subcarriers in the OFDM symbol. The vertical axis of the matrix $\mathbf{C}_{QO}$ represents the time dimension whereas the horizontal axis represents the spatial dimension, i.e. the entry in position $[\mathbf{C}_{QO}]_{i,j}$ is transmitted from the j-th antenna during the i-th time slot. The channel is assumed to be quasi-static, i.e. the channel coefficients $h_{i,j}$, $\forall i, j$ are assumed to remain constant over a frame period consisting of four OFDM symbols. Although there exist many forms of QO-STBC, for example [5] and [6], they all possess similar properties and we chose this codematrix for its similarity to the well-known Alamouti's code [2].

In order to visualize the problem, consider the setup in Figure 1. The signal at the receive antenna over four OFDM symbol periods can be written in a vector form as

$$\begin{bmatrix}\mathbf{r}_1\\ \mathbf{r}_2^*\\ \mathbf{r}_3^*\\ \mathbf{r}_4\end{bmatrix} = \begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{\Lambda}_2 & \mathbf{\Lambda}_3 & \mathbf{\Lambda}_4\\ \mathbf{\Lambda}_2^* & -\mathbf{\Lambda}_1^* & \mathbf{\Lambda}_4^* & -\mathbf{\Lambda}_3^*\\ \mathbf{\Lambda}_3^* & \mathbf{\Lambda}_4^* & -\mathbf{\Lambda}_1^* & -\mathbf{\Lambda}_2^*\\ \mathbf{\Lambda}_4 & -\mathbf{\Lambda}_3 & -\mathbf{\Lambda}_2 & \mathbf{\Lambda}_1\end{bmatrix} \begin{bmatrix}\mathbf{s}_1\\ \mathbf{s}_2\\ \mathbf{s}_3\\ \mathbf{s}_4\end{bmatrix} + \begin{bmatrix}\mathbf{n}_1\\ \mathbf{n}_2^*\\ \mathbf{n}_3^*\\ \mathbf{n}_4\end{bmatrix}$$

$$\mathbf{r} = \mathbf{\Delta}\mathbf{S} + \mathbf{n}$$

where each entry of the $4N \times 1$ vector $\mathbf{r}$ corresponds to the signal received at each OFDM symbol period $t = 1, ..., 4$ (the second and third entries are complex conjugated without loss of generality). The $N \times 1$ vectors $\mathbf{n}_i$, $i = 1, ..., 4$ represent the zero-mean circularly symmetric additive white Gaussian noise, and the matrices Λ_i, $i = 1, ..., 4$ are diagonal matrices with the frequency response of the corresponding channel on the main diagonal, i.e. $\Lambda_i = \mathrm{diag}\{\lambda_{i,1}, ..., \lambda_{i,N}\}$. The coefficients $\mathbf{\Theta}_i$, $i = 1, ..., 4$ will be defined in the next section, but for the time being, assume that $\mathbf{\Theta}_i = \mathbf{I}$, $\forall i$.

After matched filtering [6], i.e.

$$\mathbf{r}_{mf} = \mathbf{\Delta}^H \mathbf{r} = \mathbf{\Delta}^H \mathbf{\Delta}\mathbf{S} + \mathbf{\Delta}^H \mathbf{n},$$

we observe that

$$\mathbf{\Delta}^H \mathbf{\Delta} = \begin{bmatrix}\mathbf{\Gamma} & \mathbf{0} & \mathbf{0} & \mathbf{A}\\ \mathbf{0} & \mathbf{\Gamma} & -\mathbf{A} & \mathbf{0}\\ \mathbf{0} & -\mathbf{A} & \mathbf{\Gamma} & \mathbf{0}\\ \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}\end{bmatrix}$$

where $\mathbf{\Gamma} = \sum_{i=1}^4 |\mathbf{\Lambda}_i|^2$ and $\mathbf{A} = 2\,\mathrm{Re}\{\mathbf{\Lambda}_1\mathbf{\Lambda}_4^* - \mathbf{\Lambda}_2\mathbf{\Lambda}_3^*\}$.

For O-STBCs $\mathbf{A} = \mathbf{0}$, hence there is no coupling between detected signals, and the components of $\mathbf{S}$ can be directly detected from the output of the matched filter, $\mathbf{r}_{mf}$, which is also optimum in the maximum likelihood sense. However, for QO-STBC, $\mathbf{A}$ is nonzero and causes coupling between symbols. The optimum receiver is no longer a symbol-by-symbol detector and a loss in diversity order occurs due to this coupling. Instead a joint detection algorithm has to be employed to take the coupling between the first and the fourth symbols and the second and third symbols into consideration.

In the sequel, we will propose and investigate two transmitter preprocessing methods depending on partial Tx-CSI to eliminate this coupling, so that the full diversity order is obtained and the optimum receiver becomes identical to that of O-STBCs.

III. Orthogonalization with partial Tx-CSI

Prior to transmission, assume that we multiply the signal to be transmitted from antenna i with an $N \times N$ diagonal matrix $\mathbf{\Theta}_i$. Hence we can write the modified coupling term as

$$\mathbf{A}' = 2\,\mathrm{Re}\{\mathbf{\Lambda}_1\mathbf{\Lambda}_4^*\mathbf{\Theta}_1\mathbf{\Theta}_4^* - \mathbf{\Lambda}_2\mathbf{\Lambda}_3^*\mathbf{\Theta}_2\mathbf{\Theta}_3^*\} \tag{1}$$

which will provide a general expression for the following algorithms.

A. Phase rotation algorithm

Assume that $\mathbf{\Theta}_1 = \mathbf{\Theta}_2 = \mathbf{I}_N$ and $\mathbf{\Theta}_3 = \mathbf{\Theta}_4 = e^{j\mathbf{\Phi}}$, where the entries of the diagonal matrix $\mathbf{\Phi}$ are phase angles. Hence, the coupling term $\mathbf{A}'$ becomes

$$\begin{aligned}\mathbf{A}' &= 2\,\mathrm{Re}\{(\mathbf{\Lambda}_1\mathbf{\Lambda}_4^* - \mathbf{\Lambda}_2\mathbf{\Lambda}_3^*)e^{-j\mathbf{\Phi}}\}\\ &= 2\,\mathrm{Re}\{\Xi e^{-j\mathbf{\Phi}}\}.\end{aligned}$$

where the $N \times N$ diagonal matrix $\Xi = \mathbf{\Lambda}_1\mathbf{\Lambda}_4^* - \mathbf{\Lambda}_2\mathbf{\Lambda}_3^*$. It can easily be verified that by rotating the phase of the diagonal components of the matrix $\Xi e^{-j\mathbf{\Phi}}$ to $e^{\pm j\pi/2}$ (i.e. $\mathrm{Re}\{e^{\pm j\pi/2}\} = 0$) using the matrix $\mathbf{\Phi}$, the coupling matrix $\mathbf{A}'$ can be forced to zero. Therefore for the phase rotation algorithm, we set

$$[\mathbf{\Phi}]_{n,n} = \frac{\pi}{2} - \measuredangle[\Xi]_{n,n}$$

where $\measuredangle\cdot$ is the angle operator. Observe that any entry of $\mathbf{\Phi}$ is effectively in the range $[-\pi/2, \pi/2]$ (any point on the left semicircle has a one-to-one counterpart on the right semicircle which makes the coupling zero). Since the coupling is forced to zero, the diversity order loss of QO-STBCs is also eliminated, hence, this provides full diversity.

Since transmission of the exact phase angle in $\mathbf{\Phi}$ through the feedback channel may require too much feedback overhead, we propose to feedback the quantized phase angle. Let the number of available feedback bits per subcarrier be K. Then the phase angle $[\mathbf{\Phi}]_{n,n}$ can be quantized according to the following expression

$$[\mathbf{\Phi}]_{n,n} = \underset{[\mathbf{\Phi}]_{n,n}\in\mathbf{\Omega}}{\arg\min}\ \mathrm{Re}\{[\Xi]_{n,n}e^{-j[\mathbf{\Phi}]_{n,n}}\}$$

where the set of possible angle values, $\mathbf{\Omega}$**,** is

$$\mathbf{\Omega} = \{\pm\frac{(2n-1)\pi}{2^{K+1}},\ n = 1, 2, \cdots, 2^{K-1}\}.$$

For example, for a one bit feedback this set contains $\{-\pi/4, \pi/4\}$. Hence, the algorithm chooses among these values via one feedback bit to minimize the coupling term.

B. Antenna selection algorithm

Another possible method to eliminate the coupling is the antenna selection algorithm. In this algorithm the diagonal entries of the matrices $\mathbf{\Theta}_i$, $\forall i$ assume two values, $\{0, \sqrt{2}\}$, according to the quality of the individual channels. Consider the following assignments for the k-th subcarrier

$$\left.\begin{array}{l}|\lambda_{1,k}|^2 \geq |\lambda_{4,k}|^2\\ |\lambda_{1,k}|^2 < |\lambda_{4,k}|^2\end{array}\right\} \Longrightarrow \begin{array}{l}[\mathbf{\Theta}_1]_{n,n} = \sqrt{2} \text{ and } [\mathbf{\Theta}_4]_{n,n} = 0\\ [\mathbf{\Theta}_1]_{n,n} = 0 \text{ and } [\mathbf{\Theta}_4]_{n,n} = \sqrt{2}\end{array}$$

$$\left.\begin{array}{l}|\lambda_{2,k}|^2 \geq |\lambda_{3,k}|^2\\ |\lambda_{2,k}|^2 < |\lambda_{3,k}|^2\end{array}\right\} \Longrightarrow \begin{array}{l}[\mathbf{\Theta}_2]_{n,n} = \sqrt{2} \text{ and } [\mathbf{\Theta}_3]_{n,n} = 0\\ [\mathbf{\Theta}_2]_{n,n} = 0 \text{ and } [\mathbf{\Theta}_3]_{n,n} = \sqrt{2}.\end{array}$$

This operation partitions the transmit antennas into two pairs (Ant1, Ant 4) and (Ant 2, Ant 3) separately for each subcarrier. One of the two antennas in each pair is chosen according to the above inequality test, and the total transmit power is allocated to the best two antennas while the other two antennas are switched off. The same operation is also applied to the other pair. Therefore, the total transmit power is preserved.

Since one of the component of the diagonal entries of $\mathbf{\Theta}_1\mathbf{\Theta}_4^*$ and $\mathbf{\Theta}_2\mathbf{\Theta}_3^*$ in (1) is always zero, this selection of $\mathbf{\Theta}_i$, $\forall i$ makes $\mathbf{A}'$ zero. Moreover, as it will be demonstrated in the simulations, this algorithm restores the diversity gain of QO-STBC to four.

IV. Reducing feedback overhead due to OFDM

Although the above methods require very little feedback per subcarrier, when all the N subcarriers are considered, the total amount of feedback can be excessive. However, investigating the behavior of the feedback information for a moderate wireless communication channel model, we observed a strong correlation among subcarriers so that there is a significant amount of redundancy in the feedback.

An illustrative example for the phase rotation algorithm is given in Figure 2, for an equal-power five-tap multipath fading channel. The solid line shows the actual phase value, whereas the 'stem' drawing shows the one bit quantized phase value for the feedback. Similar behavior of the 'stem' drawing is also observed for the antenna selection algorithm.

Therefore, there exists a strong correlation between adjacent subcarriers. We can exploit this correlation to decrease the amount of feedback by employing a compression algorithm.

In this paper we propose a very simple majority voting method for this purpose. The frequency range is divided into smaller partitions, each with the same length, i.e. $N/2^l$, $l = 0, 1, .., log_2 N$. Then a majority voting is applied within each partition deciding the feedback value for that particular partition. Although this is an information-lossy compression method, simulations have revealed that it yet provides a satisfactory performance-feedback overhead trade-off.

In a practical system the feedback channel is also exposed to errors. This may be due to noise in the feedback channel, channel estimation error at the receiver, variations of the feedforward channel during CSI feedback interval, etc. Therefore, in the simulations, we also investigate the effect of the feedback errors on the performance of the proposed quantized transmit diversity schemes.

V. Simulations and Results

In the simulations we considered transmission over a 4×1 MIMO channel with independent subchannels. Each subchannel has five multipath components realized with zero-mean, equal-power, independent, circularly symmetric, complex Gaussian random variables. Each OFDM subchannel has QPSK modulated 64 subcarriers and a cyclic prefix of length four is appended to each OFDM frame. For the noise model AWGN was considered.

Although matched filter detection is optimum in the ML sense for the perfect feedback scenarios, there remains some coupling when the feedback is quantized and/or exposed to errors. Hence we employed the ML algorithm explained in [5] to reduce these effects.

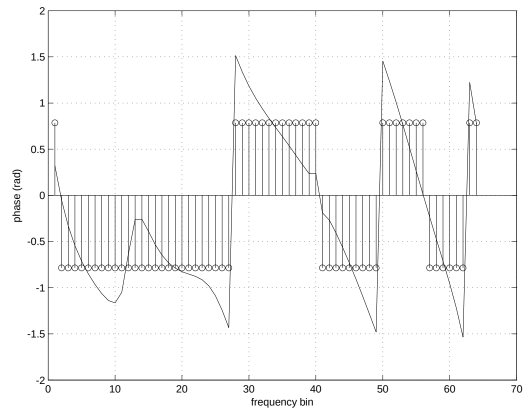

Fig. 2. An illustration of the feedback information. For the phase feedback algorithm, the actual phase values is shown with solid lines, whereas the ‘stem’ is the corresponding one-bit quantized phase value. Observe the strong correlation between adjacent bins. The same behaviour is also observed for the antenna selection algorithm.

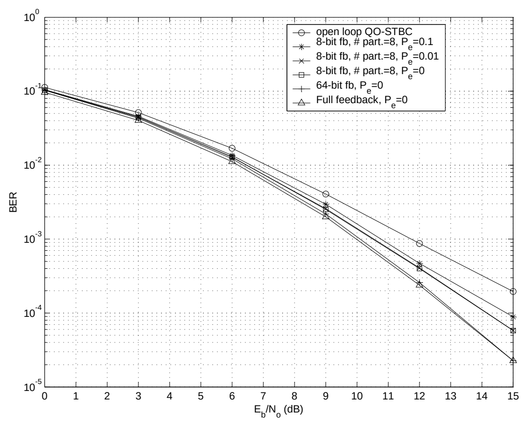

Fig. 3. BER performance comparison of the phase feedback algorithm when exposed to quantization and feedback errors. Only one bit feedback performs almost equally well as the full feedback scheme. Also the phase feedback algorithm is very robust against feedback errors and the effect of feedback error rate of 10^{-2} is negligible.

Figure 3 demonstrates the effect of quantization, partitioning and feedback errors on the phase feedback algorithm. The perfect phase feedback case provides approximately 2 dB gain in BER performance at BER=10^{-3} when compared to the open-loop QO-STBC. However this requires a feedback of 64 phase values in the range $[-\pi/2, \pi/2]$ in terms of real numbers. When the phase value is quantized to one bit per subcarrier, the loss in performance is almost negligible as seen from Figure 3 and it requires 64 bit feedback.

Even this much of feedback can introduce too much overhead to a practical application. Therefore, we divided all frequency bins into 8 partitions, each composed of 8 adjacent bins and majority voting is applied to these 8 bins to decide the ‘representative’ bit for each partition. As can be seen from Figure 3, this scheme provides a performance close to the perfect phase feedback case with an acceptable loss of approximately 0.5 dB at BER=10^{-3}. When the feedback bits are exposed to error, it is observed in Figure 3 that an error rate of $P_e \leq 10^{-2}$ has almost no effect on the performance. This makes the algorithm very robust to moderate feedback errors. Even for an error rate of $P_e = 0.1$, the loss is less than 0.3 dB.

We observe a similar immunity to feedback errors for the antenna selection algorithm as can be seen from Figure 4. The loss in performance for $P_e \leq 10^{-2}$ is negligible. However, this algorithm is more sensitive to partitioning. When the same amount of feedback bits, i.e. 8 bits, is used we can form 4 partitions, since now two feedback bits are required per partition, and the loss in performance for this case is almost 2.5 dB at BER=10^{-3} closing the gap between the open-loop performance to only 0.5 dB. Therefore, we had to increase the number of feedback bits to 16, constructing 8 frequency bin partitions. In this case, the performance gain compared to the open-loop QO-STBC is more than 2 dB.

Figure 5 demonstrates the sensitivity of both algorithms to feedback error rate. Both algorithms are very robust to error rates lower than 10^{-2}. However, as the error rate increases, their performance degrades. However, the phase rotation method is more robust than the antenna selection method.

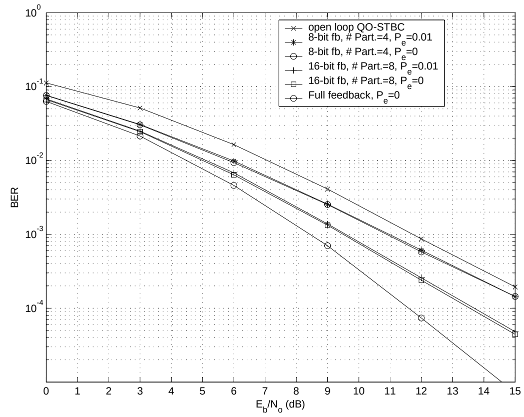

Fig. 4. BER performance comparison of the antenna selection algorithm when exposed to feedback errors. This algorithms appears to be more vulnerable to partitioning however it performs better than the phase feedback algorithm for moderate number of feedbacks, 16-bit in this simulation. The antenna selection algorithm is also very robust against feedback errors and the effect of feedback error rate of 10^{-2} is negligible.

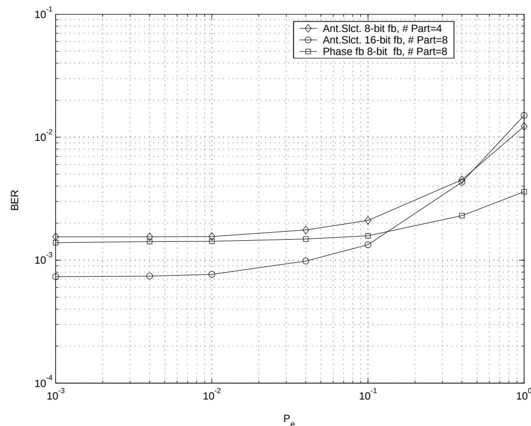

Fig. 5. BER performance comparison of the proposed algorithms with respect to the feedback error rate. Both algorithms are almost unaffected from feedback error upto P_e=10^{-2}, making them robust against feedback errors. The antenna selection algorithm appears to be more vulnerable for high P_e values compared to the phase feedback algorithm.

VI. Conclusions

The sensitivity of two transmit diversity techniques for OFDM based QO-STBC schemes against feedback quantization and feedback errors was investigated. A full feedback scheme for OFDM needs too much feedback information which may not be feasible for a practical application. It was found quantization and subcarrier partitioning can reduce the feedback overhead significantly. Our simulation studies revealed that even for a moderate feedback error rate of around 10^{-2}, the proposed algorithms retain satisfactory BER performance, demonstrating the robustness of the proposed algorithms against feedback errors.

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