An Overview of Symbol Predistortion Techniques for PAPR Reduction in OFDM and OFDMA Systems

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***Abstract*—In this paper, we give an overview of peak-to-average power ratio (PAPR) reduction techniques based on symbol predistortion for OFDM transmission. Existing methods are discussed and compared with some recently proposed algorithms. Using the QPSK signal constellation, performance is investigated for one-shot and iterative PAPR reduction.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has recently become very popular in wireless communications. Indeed, the IEEE 802.11a specifications for wireless local area networks (LANs) are based on OFDM (with TDMA for multiple access) and the IEEE 802.16 specifications for broadband wireless access at frequencies below 11 GHz include both OFDM/TDMA and OFDMA. Strong of these recent successes, OFDM also appears today as a potential candidate for 4G mobile cellular systems.

One of the main problems of OFDM, which somewhat counter balances its flexibility and other attractive features, is its high peak-to-average power ratio (PAPR), which is substantially higher than in single-carrier transmission. The increased PAPR requires backing off the transmit power amplifier from its output saturation point by a substantial amount, and this leads to a very inefficient use of the available power. This problem is particularly important on the uplink, because of the stringent low-cost and power consumption requirements on user terminals.

A number of techniques are available in the literature for peak power reduction in OFDM systems. This includes coding [1], phase optimization [2], selective mapping [3], partial transmit sequences [4], and others. Coding reduces the useful data rate and other techniques require the transmission of side information to the receiver, both of which are undesirable. More recently, peak power reduction techniques were developed which are based on modifying the signal constellation, introducing new constellations, or inserting pilot signals in some unused subcarriers [5] – [8].

A simple metric-based symbol predistortion technique to reduce OFDM peak power was recently introduced by the present authors in [9] and [10]. In this technique, a metric is computed for each input symbol, which essentially measures how much the peak values of the inverse DFT output samples can be reduced by predistorting that particular symbol.

In this paper, we give an overview of PAPR reduction in OFDM using symbol predistortion and describe several different variants. The technique newly introduced by the present authors will be compared to previously published PAPR reduction techniques. The paper is organized as follows: In Section 2, we give a brief review of the PAPR problem and the general PAPR reduction techniques. Section 3 describes the present symbol predistortion techniques as well as the proposed single and multilevel amplitude predistortion methods for PAPR reduction. In Section 4, we present our simulation results and the relevant comparisons using the quaternary phase-shift keying (QPSK) signal constellation. Finally, we give our conclusions in Section 5.

II. Papr in OFDM

A. The PAPR Problem

In OFDM transmission, the complex data symbol block $\mathbf{a} = (a_0, a_1, \ldots.., a_{N-1})$ is passed through an N-point inverse fast Fourier transform (IFFT) to obtain the discrete time-domain samples to be transmitted. The transmitted signal samples can be written as

$$b_n^i = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} a_m^i e^{j2\pi nm/N} , \tag{1}$$

where i is the OFDM symbol index and a_m^i is the data symbol transmitted over the mth subcarrier. For convenience, the symbol index will be omitted in the sequel. The data symbols (the a_m 's) are i.i.d. random variables, and from the central limit theorem, with a large number of subcarriers, the time-domain samples at the IFFT output can be modeled as truncated Gaussian random variables with zero mean. Thus, most of the magnitudes will be small, but a very small percentage of them will have a very large magnitude. This results in the problem of large peak-power from which multicarrier systems suffer considerably.

Although peak-power is the main concern for power amplifiers, PAPR is taken as a figure of merit in the literature. The PAPR of the time-domain sample sequence $\mathbf{b} = (b_0, b_1, \ldots.., b_{N-1})$ is defined as

$$PAPR(\mathbf{b}) = \frac{\|\mathbf{b}\|_\infty^2}{E\left[\|\mathbf{b}\|_2^2\right]/N}, \quad (2)$$

where $\|\cdot\|_p$ denotes the p-norm of the enclosed vector. In the literature, the most common way to evaluate the performance is to investigate the probability that the PAPR of a block is larger than a certain level γ^2. This is represented by the Complementary Cumulative Distribution Function (CCDF) of the $PAPR(\mathbf{b})$, which is a random variable, as

$$CCDF(PAPR(\mathbf{b})) = \Pr(PAPR(\mathbf{b}) > \gamma^2)\ . \quad (3)$$

The samples to be transmitted are passed to a parallel-to-serial (S/P) converter, and then a cyclic prefix (CP) is inserted. The resulting signal samples are filtered and frequency up-converted, and the RF signal is sent to a high-power amplifier (HPA) with nonlinear characteristics. The efficiency of this amplifier is directly related to the dynamic range of the input signal. Indeed, this amplifier must be significantly backed off from its saturation power if the input signal has a large dynamic range and a high PAPR. In addition, a high PAPR will require an increased number of bits in the A/D converter at the receiver to keep the same level of quantization noise.

B. PAPR Reduction Techniques

PAPR reduction of OFDM signals has been investigated for more than a decade and a considerable amount of work has been published. However, it still remains an important research topic, because existing techniques are either too complex, are of limited use, or give unsatisfactory performance. Existing methods can be grouped into two categories depending on whether they distort the input signal or not [5]. Methods with distortion basically apply clipping while coding, phase optimization, constellation shaping, tone reservation/injection and symbol predistortion can be classified as distortionless techniques. Below, we give an overview of these PAPR reduction techniques.

1) Clipping

This technique is the simplest and perhaps the most intuitive method for PAPR reduction. The distortion resulting from clipping increases the BER, which is undesirable. The performance degradation is generally compensated by error correcting codes, with a reduction in the overall data rate. Clipping can be made digitally, at the HPA, or at both stages. The first leads to nonlinear effects in the signal band, while the second causes out-of band spectral leakage.

2) Phase Optimization

The primary reason of large PAPR is the phase alignment of data symbols together with the FFT which constructively forms peaks in the output samples. Hence, a satisfactory reduction can be achieved by changing the phases of the input symbols. Existing phase optimization techniques use structured or optimized phase vectors in an iterative manner. Examples for structured phase vectors that lead to lower PAPR are the Newman Phases, the Shapiro and Rudin phases, the Golay complementary sequences and the Narahashi phases ([2], [4], [11]). These phases may result in a very low PAPR but at the expense of an unacceptable loss in useful data rate. Methods based on phase optimization generally involve a large number of computations. Some simplified approaches exist which do phase manipulations and chose a proper phase vector with acceptable complexity ([3], [4]). However, these techniques generally require the reliable transmission of side information to the receiver, which is either inapplicable or undesirable.

3) Coding

Coding techniques perform like structured phase codes. In the literature, well known block codes have been applied for PAPR reduction and good results were obtained, but again at the expense of a considerable loss in data rate. Since channel coding already exists against transmission problems, it is better not to use coding for PAPR reduction in order to avoid a further decrease of the date rate.

4) Tone reservation

In some applications, some of the subcarriers are not used for data transmission and are available for other usages. Tone reservation methods utilize such unused subcarriers and design a signal that reduces PAPR ([5], [8]). This can be a good solution in systems like ADSL, where high frequencies are not used for data transmission, but tone reservation is not desirable in wireless systems, where all the subcarriers are occupied.

5) Constellation Shaping

These recently proposed methods use shell mapping techniques in an N-dimensional complex space to design N-dimensional hyperspherical and hyperdiamond constellations [6]. Although these methods may result in an attractive PAPR reduction, they are too complex to be used in practical implementations.

6) Symbol Predistortion

This class of PAPR reduction techniques introduces new constellations for some symbols or extends the available symbols without reducing the minimum distance. An interesting feature of these techniques is that they are fully transparent to the receiver. Examples of this method are the tone injection [5] and active constellation extension method (ACE) ([7], [12]), which extend the QAM constellation in such a way that the PAPR is minimized at the IFFT output. The method proposed in [9], which we call amplitude predistortion, falls in this class. A common property of these techniques is that they increase the average signal power. In the sequel, we describe these techniques and give a performance comparison.

III. Symbol Predistortion Techniques

Recently, the attention for PAPR reduction has been turned to schemes which tend to play with the constellation intelligently (see, e.g. [5] and [7]). These techniques lead to large improvements compared to the previous ones without having to transmit any side information to the receiver. Symbol predistortion consists of modifying the transmitted data symbol values without affecting the minimum distance and consequently the system bit error rate (BER). These methods actually increase the transmitted average signal power, but this increase can be easily controlled.

Predistorting the constellation needs some optimization strategy to obtain the right parameters minimizing the PAPR. Basically, the predistortion can be managed with simple addition or multiplication on the information symbol vector. The methods proposed recently use the addition of a vector which extends the symbol constellation properly and needs quadratically-constrained quadratic programming (QCQP) for the optimum solution. Suboptimum versions are possible and will be presented here for comparison. Also, it is possible to obtain satisfactory reduction in peak power by just simple multiplication, which we call amplitude predistortion. This was described in [9] and [10], where it was shown that even a constellation shaping based on a simple scalar multiplication can achieve satisfactory results.

We will first discuss the previously proposed ACE method with its two variants, and then we will describe the methods based on amplitude predistortion.

A. ACE-POCS and ACE-SGP

ACE-based methods rely on the change of the constellation without changing the minimum distance. The optimum solution of ACE requires complex optimization and it is not practical. Two practical variants of ACE are the ACE-POCS [12] and ACE-SGP [7]. The ACE-POCS method is based on projection onto convex sets (POCS) algorithm ([12], [13]), and this method theoretically approaches the target PAPR. Its convergence is very slow and may take a large number of iterations to obtain the required PAPR.

In the POCS algorithm, the IFFT output samples which are above a predefined threshold value are clipped as

$$\bar{b}_n = \begin{cases} b_n, & |b_n| \le A_1 \\ A_1 e^{j\theta_n}, & |b_n| > A_1 \end{cases}, \tag{4}$$

where $b_n = |b_n| e^{j\theta_n}$. Then, the clipped vector $\bar{\mathbf{b}}$ is passed to an FFT to obtain the new input vector $\bar{\mathbf{a}}$. Next, the ACE constraints are enforced on $\bar{\mathbf{a}}$ and all the inner points which decrease the minimum distance are restored to their original values. This procedure is iterated until the target PAPR is achieved or no points remain for clipping.

Because of the slow convergence of the ACE-POCS method, a faster ACE-SGP method has been proposed in [7]. Basically, the algorithm takes a clipping vector defined as

$$\mathbf{c}_{clip} = \bar{\mathbf{b}} - \mathbf{b}^i . \tag{5}$$

Then, this vector is transformed to frequency domain to determine the possible extensions. Using the ACE constraints, the update vector $\mathbf{c}^i$ is obtained at the *i*th iteration and the iterative signal update is performed as

$$\mathbf{b}^{i+1} = \mathbf{b}^i + \mu \mathbf{c}^i . \tag{6}$$

where μ is the gradient step size, which makes the algorithm differ from the POCS method. The data symbols are extended on the allowed regions, and once a symbol is extended to a point it is not reversed in the subsequent iterations. Although the algorithm is not optimal, large PAPR reduction can be obtained at the first iterations by a proper choice of μ. Depending on the linearization of the quadratic balancing equation, the step size is obtained as

$$\mu_n = \frac{\max_n |b_n^i| - |b_n^i|}{c_n^{proj} - c_{n_{\max}}^{proj}} \tag{7}$$

where c_n^{proj} is the projection of c_n^i on b_n^i. Equation (7) is evaluated for $c_n^{proj} > 0$ and the proper step size is chosen as the minimum of these values. The algorithm is iterated until μ is negative valued.

B. Amplitude Predistortion

In the scheme proposed in [9], a metric is computed for each input data symbol which measures how this symbol contributes to the IFFT output samples with large values. This metric gives an idea for the scaling of the input data symbols. In its general form, such a metric is defined as

$$\mu_m = \sum_n w(n) f(n,m) \tag{8}$$

where $f(n,m)$ is a function which gives an appropriate measure of the contribution of symbol a_m on the output sample b_n and $w(n)$ is a weighting function of b_n. An appropriate definition of $f(n,m)$ is

$$f(n,m) = -Cos(\varphi_{nm}), \tag{9}$$

where φ_{nm} is the angle between $a_m e^{j2\pi nm/N}$ and b_n. The idea here is to predistort a data symbol if this operation is likely to reduce the peak values of the output block. Note that the $f(n,m)$ function has its maximum for $\varphi_{nm} = \pi$ and decreases monotonically around this phase value. Consequently, a large value of this function indicates that $a_m e^{j2\pi nm/N}$ and b_n are almost in opposite phase and symbol a_m can be predistorted to reduce the magnitude of b_n without reducing minimum Euclidean distance and degrading performance.

The weighting function $w(n)$ is defined to reflect the importance of the output samples on the metric μ_m. The peak power may be expressed as the $\|\mathbf{b}\|_p$ for large *p*, which we call the power shaping factor. Thus, a reasonable weighting function is $w(n) = |b_n|^p$ with a proper value of *p*. Using the definition $Cos(\varphi_{nm}) = \mathrm{Re}\{b_n a_m^* e^{j2\pi nm/N}\} / |b_n a_m|$, the final metric becomes

$$\mu_m = \frac{-1}{K|a_m|} \sum_{n \in S_K} |b_n|^{p-1} \mathrm{Re}\{b_n a_m^* e^{-j2\pi nm/N}\}, \tag{10}$$

where K is the number of output samples larger than a predetermined threshold value A_2. Once the metric is computed for all input symbols of the block, the symbols with positive metrics are selected. Then symbols are sequentially predistorted in the decreasing order of their metrics. The procedure stops when the peak power at the IFFT output stops decreasing.

The proposed algorithm involves five steps and can be summarized as follows:

1. Obtain the output sequence **b** via IFFT of the input data symbol block $\mathbf{a} = (a_0, a_1, \ldots, a_{N-1})$.
2. For K largest samples of the output sequence define a weighting function $w(n)$, which is an increasing monotone function of its power.
3. For each input data symbol a_m compute the metric μ_m that is required for predistortion.
4. Predistort the L symbols with largest positive metrics using a scaling factor $d_m > 0$.
5. Finally, for $n = 0, \ldots, N-1$ update the IFFT output as

$$\overline{b}_n = b_n + \tfrac{1}{\sqrt{N}} \sum_{m \in S_L} d_m a_m e^{j2\pi nm/N}, \tag{11}$$

where S_L is a set of size L whose elements are the indices of the expanded symbols in the input sequence. The number L is determined by observing the output peak power reduction on average. In case of multiple iterations, the above steps are repeated in the same order.

Two variants of this symbol predistortion will be presented: simple amplitude predistortion [9] and multilevel amplitude predistortion [10].

1) Simple Amplitude Predistortion:

For the simple case, the scaling factor d_m is a constant which we denote α. This constant scaling factor is determined by observing the PAPR reduction on average. We pick the α value for which we observe maximum PAPR decrease. The peak output power is indeed reduced using the described procedure up to some value of L, but reduction may stop at that point and increasing L beyond that value may increase peak output power. The L parameter is accordingly selected such that peak power reduction is sufficiently large. The metric is evaluated as in (5) with K denoting the number of output samples greater than the threshold value A_2. Then, we obtain the output samples as

$$\overline{b}_n = b_n + \tfrac{\alpha}{\sqrt{N}} \sum_{m \in S_L} a_m e^{j2\pi nm/N} \; . \tag{12}$$

During the iterations, the scaling factor is multiplied by the ratio of the initial average output power to the average output power at the current iteration, i.e.,, $E\left[\left|b_n^0\right|^2\right] / E\left[\left|b_n^i\right|^2\right]$, in order to take account of the power increase.

2) Multilevel Amplitude Predistortion:

The multilevel predistortion presented in [10] involves the expansion with a scaling factor d_m which differs from symbol to symbol. Since the metric (10) has been introduced as a measure of power, symbols are predistorted with the square root of its value. The resulting predistortion of a symbol a_m consists of transmitting:

$$\overline{a}_m = (1 + \alpha \sqrt{\mu_m^+}) a_m \, , \tag{13}$$

where α is a positive real number. The notation $(\cdot)^+$ is used to indicate the positive values of the metric. The choice of the α parameter has a strong impact on the PAPR reduction performance of this technique. The procedure for determining α and L are the same as in the previous case together with the normalization at each iteration. Finally, the output samples are updated as

$$\overline{b}_n = b_n + \tfrac{\alpha}{\sqrt{N}} \sum_{m \in S_L} \sqrt{\mu_m^+} a_m e^{j2\pi nm/N} \; . \tag{14}$$

It is worth noting that, the parameters A_2, α and L introduced in the proposed amplitude predistortion techniques are system dependent and can be set beforehand. Hence, they do not bring any additional complexity to the system. Moreover, compared to the ACE based methods there is no need to check for constellation constraints.

IV. Simulation Results

In this section, performance of the PAPR reduction techniques described above is investigated using QPSK signaling and N = 256 subcarriers. We fixed a target PAPR level of 6 dB for all methods and applied them when the PAPR is larger than that value. For the ACE-POCS and ACE-SGP methods, the clipping value A_1 was set to 4.86 dB above the average signal power, as mentioned in [7]. In the amplitude predistortion method, a threshold level A_2 of 3.5 dB above the average power was used and we applied the procedure to the K largest output samples having power above that level. Note that in PAPR calculations, the ratio of the achieved peak power to the initial average power was taken into consideration.

In Fig. 1, performance of the ACE-POCS and ACE-SGP algorithms is shown. The convergence of the ACE-POCS algorithm is very slow and a large number of iterations are needed to obtain a reasonable PAPR reduction. The dashed curves correspond to the 5th, 10th, 15th, 25th and the 60th iterations. Even at the 60th iteration, the target level was not reached. In the ACE-SGP method, convergence is faster, and at a probability of 10^{-5} a gain of 4.5 dB is obtained at the first iteration. The second iteration provides 0.25 dB gain, but further iterations do not bring any noticeable further reduction.

Fig. 2 presents the results of simple amplitude predistortion. The value of α was taken as 0.6. For amplitude predistortion, the optimum values of α and L depend on the application whether the algorithm is applied as a one-shot procedure or iteratively. We chose the parameters such that the PAPR reduction is large at the first iteration as well as at the following iterations. At the first iteration, we have a gain of 2.3 dB at the probability of 10^{-5}. The second iteration brings an additional 1.6 dB gain, which is also significant. As compared with the ACE-SGP method, this simple algorithm gives better results after the first iteration.

V. Conclusions

We have given an overview of OFDM PAPR reduction techniques based on constellation extension. These techniques do not require the transmission of any side information to the receiver. After describing the optimization techniques available in the literature, we described an amplitude predistortion technique recently proposed by the present authors. This technique employs an appropriately defined metric for each input symbol that measures its contribution to the output signal samples of large magnitude. The metrics are used to define the predistortion of the input symbols, and predistortion can be implemented either as a one-shot process or as an iterative algorithm.

Using the QPSK signal constellation, performance of the described techniques was investigated, and it was found that simple amplitude predistortion gives comparable results to more complex constellation extension schemes. In both cases, the gain in terms of PAPR CCDF was found to exceed 4 dB after a few iterations. Further work on symbol predistortion will investigate the extension of the described symbol technique to joint amplitude and phase predistortion and to higher-level quadrature amplitude modulation (QAM) schemes. In addition, performance evaluation with nonlinear amplifier characteristics is needed to assess the real gain achieved by PAPR reduction in terms the required amplifier back-off and the total SNR degradation.

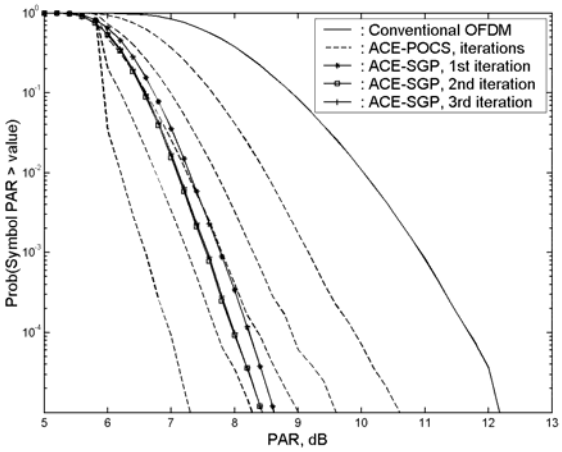
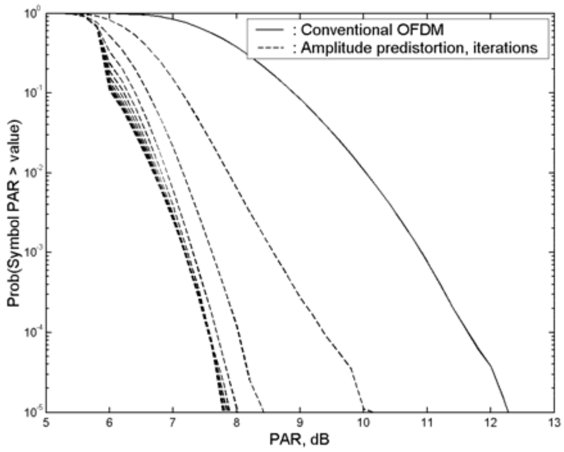


Fig. 1. CCDF of PAPR for the ACE-POCS and ACE-SGP methods



.Fig. 2. CCDF of PAPR for conventional OFDM and the simple amplitude predistortion scheme.

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