# OFDM Systems and related Multiple Access Schemes

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***Abstract*— The Orthogonal Frequency Division Multiplexing (OFDM) transmission technique can efficiently deal with multi-path propagation effects especially in broadband radio channels. It also has a high degree of system flexibility in multiple access schemes by combining the conventional TDMA, FDMA and CDMA approaches with the OFDM modulation procedure which is particularly important in the uplink of a multi-user system. In OFDM-FDMA schemes carrier synchronization and the resulting subcarrier orthogonality plays an important role to avoid any multiple access interferences (MAI) in the base station receiver. An additional technical challenge in system design is the required amplifier linearity to avoid any non-linear effects caused by a large peak-to-average ratio (PAR) of an OFDM transmission signal.**

**A specific OFDM-FDMA uplink procedure is proposed and will be designed in this paper which can be seen as a combination of a specific subcarrier spreading scheme and subcarrier selection process. The resulting transmit signal consists of a periodic extension and multiple repetition of all modulation symbols and leads therefore to an extremely low computation complexity in the transmitter. Furthermore, the transmit signal shows simultaneously a constant envelope to avoid any non-linear effects in the amplification process. This uplink scheme can be considered as a trade off between low computation complexity and system performance.**

***Keywords*—OFDM, FDMA, CDMA, Uplink, PAR, OFDMA DFT Spreading**

## I. Introduction

THE broadband radio channel is characterized by its frequency selective fading due to multi-path propagation. In mobile communication applications the radio channel is additionally time-variant due to the movement of the mobile terminal. The OFDM transmission technique can cope with these two different interference effects of frequency selectivity and time variance with a low implementation and computation complexity. Inter symbol interferences (ISI) can completely be avoided by subdividing the total system bandwidth into a large number of spectrally overlapping but mutually orthogonal, non-frequency selective narrowband subchannels and by introducing an additional guard interval as cyclic prefix into the OFDM symbol. All these orthogonal and narrowband subchannels are called the set of subcarriers.

Even at the output of a frequency-selective and time-invariant radio channel the subcarrier orthogonality can be maintained, because all subcarrier signals are Eigenfunctions of the radio channel. Equalization of each subchannel is then reduced to a single complex-valued multiplication per subchannel [1] which is a one-tap equalization procedure. There are several system proposals published for an OFDM-based downlink procedure for broadcast and communication systems, respectively. But by designing an OFDM uplink transmission scheme some important and additional technical questions will come up. Therefore, OFDM-based uplink systems are still under consideration and research [2]. As a contribution to this topic, an OFDM based multi user uplink system with $M$ different users inside a single cell is considered in this paper, see Figure 1.

Each user shares the entire bandwidth with all other users inside the cell by allocating exclusively a deterministic subset of all available subcarriers inside the considered OFDM system. This user-specific subcarrier selection process allows to share the total bandwidth in a very flexible way between all mobile terminals. Hence, as a relevant multiple access scheme an OFDM-FDMA structure is considered, in which each user claims the same bandwidth or the same number of subcarriers inside the total bandwidth respectively. Due to the assumed perfect carrier synchronization and resulting subcarrier orthogonality in the receiver, any multiple access interference (MAI) between different users can be avoided. The subcarrier allocation process can either be designed to be non-adaptive or adaptive in accordance with the current radio channel state information (CSI).

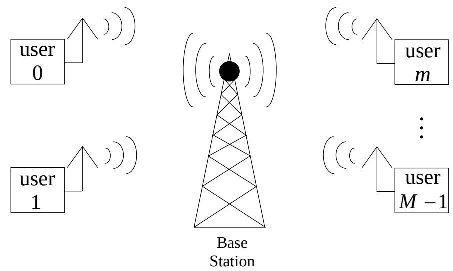



Fig.1. Multi user environment for an OFDM-FDMA based uplink scheme

Since the OFDM transmission signal results from the superposition of a large number of independent data symbols and subcarrier signals, the envelope of the complex valued baseband time signal is in general not constant but has a large peak-to-average ratio (PAR). The largest output power value of the amplifier will therefore limit the maximum amplitude in the transmit signal. Additionally, non-linear distortions due to clipping and amplification effects in the transmit signal will lead to both in-band interferences and out-of-band emissions [3]. Therefore, in the downlink case each base station will spend some effort and computation power to control the transmit signal amplitude and to reduce the PAR. The objective is in this case to minimize the resulting non-linear effects or even to avoid any interferences.
But for the uplink case it is especially important to design a transmit signal with low PAR to reduce computation complexity in the mobile terminal and to avoid any interference situation caused by non-linear effects of the amplification process.
It will be shown in this paper that an OFDM-FDMA system based on an equidistant subcarrier selection procedure combined with an additional subcarrier spreading technique will result in a constant envelope signal and therefore reduce the resulting PAR significantly [4] for the uplink procedure. Furthermore, this proposal will lead to a transmitter structure which becomes technically very simple where the transmit signal consists of a periodic extension and multiple repetition of all modulation symbols. This is the result of the duality between multi carrier CDMA and single-carrier transmission technique as described in [4].

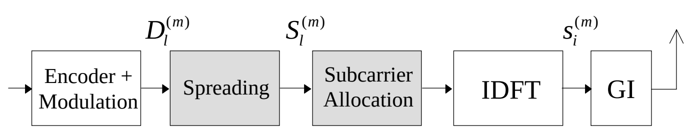



Fig. 2. Block diagram of a multi user OFDM-FDMA uplink system

Figure 2 shows the general structure of an OFDM uplink signal processing in the mobile terminal, which will be considered in this paper. In this block diagram there are two main components in the OFDM-based modulation scheme which will be treated in the design process of a multi user uplink system: The subcarrier selection technique and a user specific spreading scheme applied to the user's selected subcarrier subset, respectively.
The last two blocks in the block diagram show the characteristic IDFT processing and the guard interval (GI) insertion, that are common in all OFDM-based transmitter schemes.

## II. OVERALL SYSTEM STRUCTURE

The uplink system model of a multi user, OFDM-FDMA based scheme is shown in Fig. 1 and 2. In this case an arbitrary number of $M$ different users are considered inside a single cell and each user allocates exclusively $L$ different subcarriers which are considered inside the entire system bandwidth for data transmission. The total number of all considered subcarriers inside the system bandwidth of the transmission scheme is therefore $N_C = L \cdot M$ .

The input data stream for each mobile user terminal $m$, $m = 0, \ldots, M-1$, is convolutionally encoded in a first step. Afterwards, the bit sequence is mapped onto a modulation symbol vector

$\vec{D}^{(m)} = \left(D_0^{(m)}, D_1^{(m)}, \ldots, D_{L-1}^{(m)}\right)$ of $L$ complex valued symbols $D_l^{(m)}$ from a given modulation alphabet with $2^Q$ different modulation symbols inside the constellation diagram. An example for such a modulation alphabet is given in Fig. 3 for a 16-QAM.

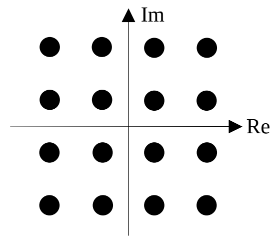



Fig. 3. 16-QAM as an exemplary alphabet with modulation symbols $D_l^{(m)}$.

In this paper, a non-differential, higher level modulation scheme is assumed for the uplink case. Each user transmits $L \cdot Q$ bits per OFDM symbol. It is assumed in this paper without any loss of generality that each user transmits the same data rate or the same number of modulation symbols per OFDM signal respectively.

### *A. Subcarrier Allocation Process*

The first important question in the OFDM-FDMA multi user uplink system design is the user specific subcarrier selection scheme. This process is responsible for sharing the bandwidth between $M$ different users, see Figure 4. There is a large degree of freedom in this system design step to allocate exclusively a subset of $L$ specific subcarriers to each user. This can either be done by a random or a deterministic allocation scheme. Alternatively there are proposals made for adaptive subcarrier selection schemes to increase the resulting system capacity [5], [6], [7].

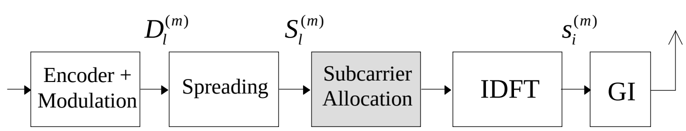



Fig.4. Block diagram for an OFDM-FDMA based system with subcarrier selection process

In this paper a very specific non-adaptive subcarrier selection pattern is proposed. In this case the allocated subcarrier subset is equidistantly located on the frequency axis over the entire system bandwidth. This approach is shown in Fig. 5 and will be pursued in the following.

In this multi user uplink system each user $m$ allocates exclusively in total $L$ subcarriers which are in each case placed in an equidistant way on the frequency axis. The selected $L$ subcarriers are modulated with $L$ complex valued transmit symbols $S_l^{(m)}$, described and denoted by the transmit symbol vector $\vec{S}^{(m)}$. The proposed non-adaptive subcarrier pattern and modulation process does not need any radio channel state information (CSI) at the transmitter side.

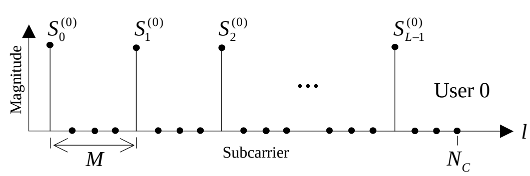



Fig. 5. Equidistantly allocated subset of $L$ subcarriers
for a single user in a multi user environment

Due to this specific subcarrier pattern, based on equidistantly located subcarriers on the frequency axis, the resulting OFDM uplink transmit time signal $s_i^{(m)}$ of any user has a periodic structure with period-length $L$ and consists in any case of an $M$ -times repetition time signal, see Figure 6.

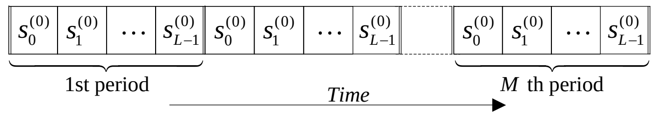



Fig. 6. OFDM-FDMA based periodic transmit time signal with period length $L$ and $M$ times repetition

Equation (1) describes the relation between the subcarrier transmit symbols $S_l^{(0)}$ and a single period of the resulting OFDM transmit time signal $s_i^{(0)}$ for user 0 analytically.

$$s_i^{(0)} = \frac{1}{\sqrt{L}} \sum_{l=0}^{L-1} S_l^{(0)} e^{j2\pi il/L} \qquad \text{for } i = 0,1,\ldots,L-1 \tag{1}$$

This relation in equation (1) is simply an IDFT applied to the transmit symbols $S_l^{(0)}$, as shown in (2).

$$\begin{pmatrix} s_0^{(0)} \\ s_1^{(0)} \\ \cdots \\ s_{L-1}^{(0)} \end{pmatrix} = \text{IDFT} \begin{pmatrix} S_0^{(0)} \\ S_1^{(0)} \\ \cdots \\ S_{L-1}^{(0)} \end{pmatrix} \tag{2}$$

Because the subcarrier subset of a single user is assumed to be allocated equidistantly over all $N_C$ subcarriers inside the entire bandwidth (Fig. 5), it can be shown that an $N_C$ -IDFT processing of the subcarrier transmit symbols $S_l^{(0)}$ inside the OFDM transmitter leads to the same $M$- times repetition of the user time signal $s_i^{(0)}$ as shown in Figure 6. The periodicity of the transmit signal is directly related to the selection process of equidistantly located subcarriers on the frequency axis. The transmit time signal calculated in (2) and shown in Figure 6 has in general a large PAR due to the superposition of many subcarrier signals. In order to reduce the PAR dramatically and to develop a transmit signal with constant envelope an additional spreading technique will be considered in the next paragraph.

### B. *Subcarrier Spreading Technique*

This paragraph addresses the second design element of an OFDM-FDMA-based system: a spreading technique applied to the user's selected subcarriers, see Figure 7. There are several well known spreading techniques, which can be integrated into an OFDM-based transmission technique, [8],[9].

Analogous to other MC-CDMA systems, described in [8] and [9], the vector $\vec{D}^{(m)}$ of $L$ modulation symbols (see Fig. 3) is spread in this case over $L$ subcarriers which are exclusively allocated to user *m* applying an unitary spreading matrix $[C]$.

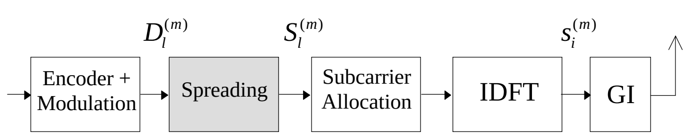



Fig. 7. Block diagram of a multi user OFDM-FDMA uplink system with additional spreading technique

This results in a transmit subcarrier symbol vector $\vec{S}^{(m)} = \left(S_0^{(m)}, S_1^{(m)}, \ldots, S_{L-1}^{(m)}\right)$ consisting of $L$ complex-valued transmit symbols $S_l^{(m)}$, *l = 0,…,L-1*. The spreading operation can be denoted mathematically by the following matrix multiplication (3) where each complex-valued transmit symbol $S_l^{(m)}$ is calculated by the sum of $L$ user-specific modulation symbols $D_l^{(m)}$ weighted by $L$ orthogonal code vectors $\vec{C}_l = \left(C_{l,0}, C_{l,1}, \ldots, C_{l,L-1}\right)$ with $l = 0,\ldots,L-1$:

$$\begin{pmatrix} S_0^{(m)} \\ S_1^{(m)} \\ \vdots \\ S_{L-1}^{(m)} \end{pmatrix} = \begin{bmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,L-1} \\ C_{1,0} & \ddots & & C_{1,L-1} \\ \vdots & & & \vdots \\ C_{L-1,0} & \cdots & \cdots & C_{L-1,L-1} \end{bmatrix} \cdot \begin{pmatrix} D_0^{(m)} \\ D_1^{(m)} \\ \vdots \\ D_{L-1}^{(m)} \end{pmatrix} \tag{3}$$

The spreading Matrix $[C]$ consists of $L$ orthogonal spreading codes. It can be designed e.g. by a Walsh-Hadamard matrix like in [8] and [9] or by a DFT matrix respectively as described in [3], [4]. Both matrix types fulfill the requirements for unity and orthogonality. Examples for these matrices are shown in Fig. 8. In the considered multi-user uplink system, only DFT-matrix-based spreading technique will be used, because of the resulting benefits in combination with an equidistant subcarrier pattern. The subcarrier-specific transmit symbols $S_l^{(m)}$ are then mapped onto $L$ subcarrier signals which are exclusively allocated to user $m$. In principle, the user specific subcarrier subset can be composed of any $L$ out of $N_C$ subcarriers that have not been assigned to another user.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\pi/2} & e^{-j\pi} & e^{-j\frac{3}{2}\pi} \\ 1 & e^{-j\pi} & e^{-j2\pi} & e^{-j3\pi} \\ 1 & e^{-j\frac{3}{2}\pi} & e^{-j3\pi} & e^{-j\frac{9}{2}\pi} \end{bmatrix}$$

Fig. 8. Examples for Walsh-Hadamard (left) and DFT spreading matrix

### C. *Combination of spreading and subcarrier allocation*

As explained in the previous paragraph, the spreading technique applied to the modulation symbols $D_l^{(m)}$ is considered in a way, that a DFT-Matrix can be used as spreading matrix $[C]$. Therefore, the relation between modulation symbols $D_l^{(0)}$ and subcarrier transmit symbols $S_l^{(0)}$ is described analytically by (4):

$$\begin{pmatrix} S_0^{(0)} \\ S_1^{(0)} \\ \cdots \\ S_{L-1}^{(0)} \end{pmatrix} = \begin{bmatrix} \\ & [C] & \\ \\ \end{bmatrix} \cdot \begin{pmatrix} D_0^{(0)} \\ D_1^{(0)} \\ \cdots \\ D_{L-1}^{(0)} \end{pmatrix} = \mathrm{DFT} \begin{pmatrix} D_0^{(0)} \\ D_1^{(0)} \\ \cdots \\ D_{L-1}^{(0)} \end{pmatrix} \tag{4}$$

If this DFT-based spreading technique is combined with the earlier explained, equidistant subcarrier selection pattern the transmit time signal $s_i^{(0)}$ can be calculated directly by the $M$ times repetition of modulation symbol vector $\vec{D}^{(0)}$ which consists of $L$ complex-valued modulation symbols, see Fig. 6. Therefore, it is needless to process the DFT spreading matrix and the IFFT in the OFDM system structure explicitly, which reduces the computation complexity in the mobile terminal (5).

$$\begin{pmatrix} s_0^{(0)} \\ s_1^{(0)} \\ \vdots \\ s_{L-1}^{(0)} \end{pmatrix} = \mathrm{IDFT} \begin{pmatrix} S_0^{(0)} \\ S_1^{(0)} \\ \vdots \\ S_{L-1}^{(0)} \end{pmatrix} = \mathrm{IDFT} \left( \mathrm{DFT} \begin{pmatrix} D_0^{(0)} \\ D_1^{(0)} \\ \vdots \\ D_{L-1}^{(0)} \end{pmatrix} \right) = \begin{pmatrix} D_0^{(0)} \\ D_1^{(0)} \\ \vdots \\ D_{L-1}^{(0)} \end{pmatrix} \tag{5}$$

Hence, a single period of the resulting time signal $s_i^{(0)}$ is directly given by the calculated modulation symbols $D_l^{(0)}$.

In almost all OFDM systems, a cyclic prefix of length $N_G$ will be added to the transmit time signal $s_i^{(0)}$ to avoid any ISI. Therefore, the so-called guard interval is also an integral part of the multi-user uplink system described in this paper.

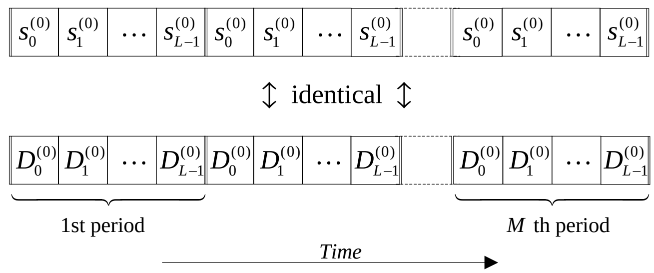



Fig. 6. Periodic transmit signal for the multi user uplink system.
Symbols $s_i^{(0)}$ and modulation symbols $D_l^{(0)}$

Thus, the structure of the OFDM-FDMA multi-user uplink system depicted in Fig. 8 can be simplified. Figure 8 shows the functionality of the overall system in detail. It becomes clear, that because of the cancellation of DFT spreading and IDFT calculation these components can be completely removed in the technical realization. They are replaced by a simple repetition process of the considered user specific modulation symbols $D_l^{(0)}$ .

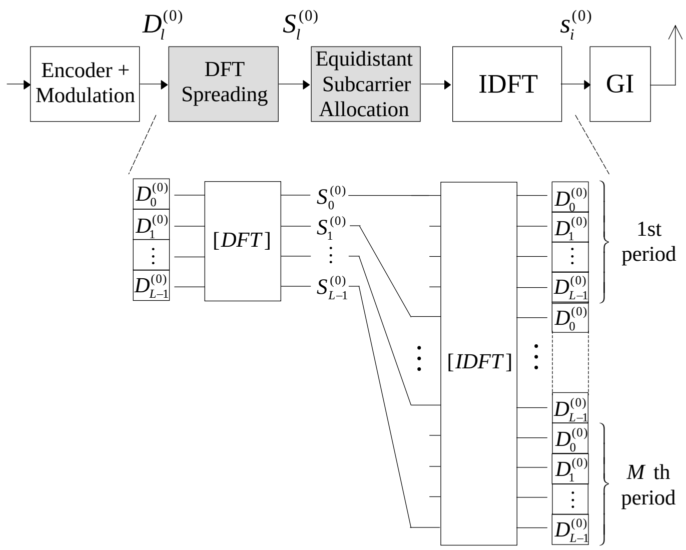



Fig. 8. OFDM-FDMA based uplink system including a DFT spreading matrix
applied to a set of equidistant subcarriers.

## III. Multi User Case

The extension from a single to an arbitrary user $m$ is straight forward and will be described in the following. Another user $m$ also allocates an equidistantly spaced subset of all sub-carriers which is shifted in the frequency space by $m$ subcarriers, see Figure 9.

Any frequency shift results in a multiplication of the transmit time signal $s_i^{(m)}$ with a complex valued signal $e^{j2\pi im/N_C}$, see equation (6).

$$S_l^{(m)} * \delta(l-m) \quad \bullet\!-\!\circ \quad s_i^{(m)} \cdot e^{j2\pi im/N_C} \tag{6}$$

This yields a phase rotation of the transmit time symbols $s_i^{(m)}$ with the constant frequency $f_0 = m/N_C$ .

But this has no significant impact on the complexity of the transmitter structure. Also, the signal envelope of a single OFDM symbol is still constant. The simplified synthesis of the transmit time signal for the multi-user case is depicted in Fig. 10.

First, the vector of $L$ modulation symbols $\vec{D}^{(m)}$ is calculated and repeated $M$ -times on the time axis.

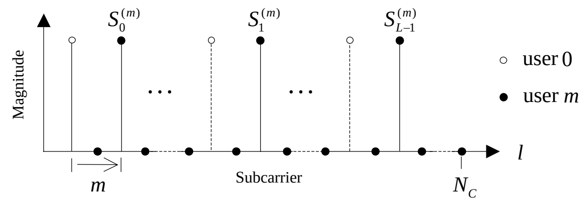



Fig. 9. Shifting the total subcarrier subset in a multi user environment

Then, the time sequence is element-wise multiplied by the user-specific phase rotating sequence $e^{j2\pi im/N_C}$. In the last step, the guard interval is added. Figure 10 describes the simple transmit signal processing and the low computation complexity at the transmitter side and in the mobile terminal for the uplink case.

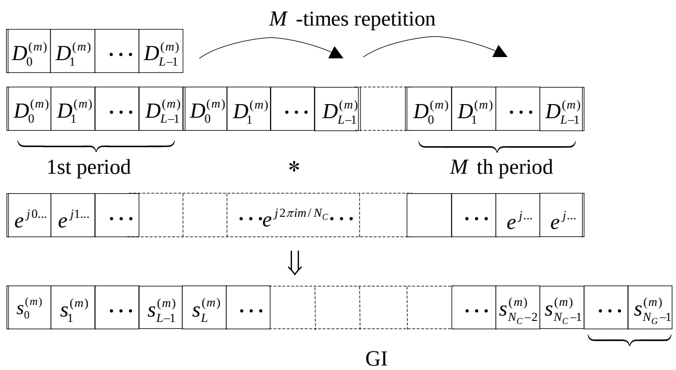



Fig. 10. OFDM-FDMA uplink transmit signal for an arbitrary user $m$

Fig. 9 and 10 show that the time signal of the OFDM FDMA-based uplink scheme with DFT spreading can be considered as a blockwise single-carrier periodic transmission system where a cyclic prefix is integrated into a single block as a guard interval. Therefore the signal envelope is nearly constant and additional techniques like $\pi/4-$QPSK can be employed to even reduce the resulting small PAR for this single-carrier system.

## IV. Receiver Structure

At the base station (BS) and in a multi-user environment the received time signal consists of the superposition of all $M$ user signals transmitted over different radio channels. After time and carrier synchronization and after A/D conversion the time discrete receive signal is processed by removing the guard interval, calculating the Discrete Fourier Transform (DFT) which splits the receive signal into the $N_C$ orthogonal subcarrier channels, and finally separating the $M$ different user signals by selecting the user-specific $L$ different equidistantly arranged subcarrier signals described by the complex valued symbols $R_l^{(m)}$ at the DFT output respectively. The user specific subcarrier subset is located at subcarrier $k = m, m+M, \ldots, m+(L-1)M$ for user $m$. The complex valued symbols $R_l^{(m)}$ at the DFT output can analytically be described as

$$R_l^{(m)} = H_{m+lM}^{(m)} S_l^{(m)} + N_{m+lM}^{(m)} . \tag{7}$$

where $H_k^{(m)}$ denotes the complex-valued channel transfer factor on subcarrier $k$ of user $m$ and $N_k^{(m)}$ the complex-valued Gaussian noise with variance $N = \sigma_N^2$. The general multi-user receiver structure in each BS is depicted in Fig. 11.

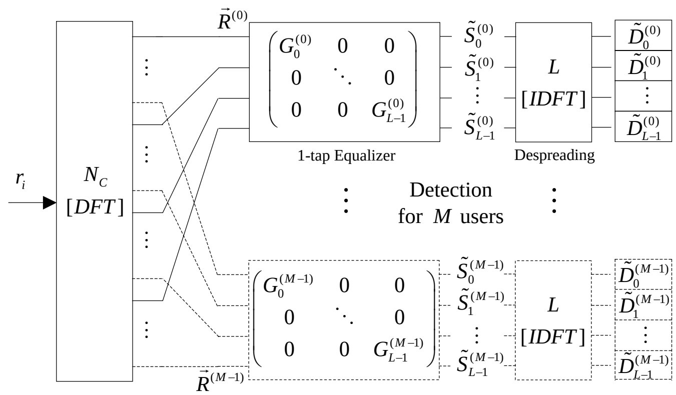



Fig. 11. Block diagram of OFDM receiver with IDFT-despreading

For a perfectly time- and carrier-synchronized system the received signals are not influenced by ISI and ICI at all. A data symbol estimation $\tilde{D}_l^{(m)}$ can then be calculated independently for each user by applying classical detection schemes.

## V. Equalization Technique and BER Performance

The discussed OFDM-FDMA-based multi-user uplink scheme with equidistant subcarrier pattern and user-specific DFT spreading technique can alternatively be considered as a blockwise processed single-carrier transmission signal. This is due to the transmit signal development procedure which is based on an M-times repetition of the user-specific modulation symbols to a periodic transmit signal, see Fig. 10. Additionally a guard time is integrated into the transmit signal as a cyclic prefix. From a pure single-carrier modulation technique with blockwise transmission and guard interval integration viewpoint the receiver structure with DFT, one-tap equalization and IDFT procedure can alternatively be interpreted as a frequency domain equalizer (FDE). It is shown in [10], that this resulting single-carrier-FDE with MMSE-equalization is optimal in the sense of minimized BER compared to other OFDM systems with matrix-spreading.

But the described uplink scheme with one-tap equalization process can also be considered as a pure OFDM-FDMA system with spreading. Therefore the classical one-tap equalization and demodulation schemes are applied at the BS as for any OFDM transmission system. A block diagram of the base station multi-user receiver with linear equalization and user-specific despreading technique is depicted in Fig. 11. From an OFDM-FDMA system viewpoint, the DFT matrix spreading scheme over equidistant arranged subcarriers on the frequency axis is despread in the receiver by applying the IDFT of length $L$ after the equalization step. Due to the duality between OFDM-FDMA schemes and single-carrier transmission technique based on FDE the same bit error rate will be expected.

The one-tap equalization procedure will be carried out by the Zero-Forcing (ZF) approach or alternatively by the MMSE technique at the DFT output for each subcarrier signal independently, based on the user specific radio channel information. Due to the guard interval, an ISI-free transmission is assumed and therefore the channel influence can be perfectly reversed by multiplying the received signal in the frequency domain with the inverted radio channel coefficients (8). This approach is known as the ZF-technique.

$$G_{k,ZF}^{(m)} = \frac{1}{H_k^{(m)}} \qquad k = 0,1,\ldots,L-1 \tag{8}$$

The processing of a received signal vector $\vec{R}$ for an arbitrary user in the equalizer is shown in (9). This equalization procedure is shown in the upper branch of Fig. 11. Equations (8) and (9) also reveal the main drawback of the ZF equalizer: If the channel suffers from deep spectral fading, the inverted channel coefficients will cause a high noise amplification.

$$\begin{pmatrix} \tilde{S}_0^{(m)} \\ \vdots \\ \tilde{S}_{L-1}^{(m)} \end{pmatrix} = \begin{pmatrix} G_0^{(m)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & G_{L-1}^{(m)} \end{pmatrix} \begin{pmatrix} R_0^{(m)} \\ \vdots \\ R_{L-1}^{(m)} \end{pmatrix} \qquad (9)$$

To avoid this problem, an MMSE-Equalizer will be considered alternatively in the following. The Equalizer coefficients $G_k$ with $k = 0, \ldots, L-1$ are derived in this case from the well known MMSE-criterion and are given in (10).

$$G_{k,MMSE}^{(m)} = \frac{H_k^{(m)*}}{\left|H_k^{(m)}\right|^2 + \frac{1}{SNR}} \qquad (10)$$

The equation shows, that the noise amplification for the $k$ th subcarrier in case of deep fading is limited to a constant factor.

Since the spreading technique was based on a DFT matrix, the despreading procedure can be realized by applying an IDFT after the equalization scheme to the equalized symbol sequence $\tilde{S}_l^{(m)}$, see (11). This procedure has significant influence on the noise power distribution over estimated

$$\tilde{D}_l^{(m)} = \text{IDFT}\left(\tilde{S}_l^{(m)}\right) \qquad (11)$$

modulation symbols $\tilde{D}_l^{(m)}$ at the despreader output. For the considered IDFT-despreading scheme it can be shown, that the noise power $\sigma_l^2$ at the IDFT output is constant for all estimated modulation symbols $\tilde{D}_l^{(m)}$

$$\sigma_{ZF}^2 = \frac{1}{L} \sum_{k=0}^{L-1} \frac{N}{\left|H_k^{(m)}\right|^2} \qquad (12)$$

when a ZF-equalizer is used. This means, that the overall noise power from all subcarriers is distributed uniformly over the estimated modulation symbols $\tilde{D}_l^{(m)}$. This is due to the specific despreading procedure which is based on the IDFT-matrix. Still, after despreading, the modulation symbols might suffer from noise amplification. With an MMSE-equalizer, the noise power is also averaged over all symbols, which is shown in (13).

$$\sigma_{MMSE}^2 = \frac{1}{L} \sum_{k=0}^{L-1} \frac{N}{\left|H_k^{(m)}\right|^2 + N} \qquad (13)$$

It is assumed in this case that the average power of the transmit signal is normalized to one. The above expression shows clearly, that in deep spectral fades the noise amplification in the equalizer is limited to a constant value on a subcarrier basis. Therefore, the MMSE-Equalizer is also advantageous for the considered transmission system with IDFT despreading.

### *A. BER Performance*

To evaluate the system performance for ZF- and MMSE-equalization, a Monte-Carlo simulation based on a stochastic channel model has been developed. We consider an OFDM FDMA system with $N_C = 256$ subcarriers and a guard interval of $N_G = 64$ shared between $M = 16$ users. The system bandwidth is 20 MHz. The modulation symbols $D_l^{(m)}$ are based on 16-QAM signals. No channel coding is employed in the system. After the symbol repetition and phase rotation process in the transmitter each transmit signal is distorted independently by a WSSUS radio channel. The channel parameters are as follows:

- Exponentially decreasing power delay (0 - 3.2μs)
- 30 uncorrelated paths
- Rayleigh fading
- No Doppler-shift

Figure 12 shows the BER performance curves of an arbitrary user for ZF- and MMSE-equalization. The advantage of the
MMSE-receiver in such a frequency-selective channel is clearly visible. Generally, all users experience the same average performance.

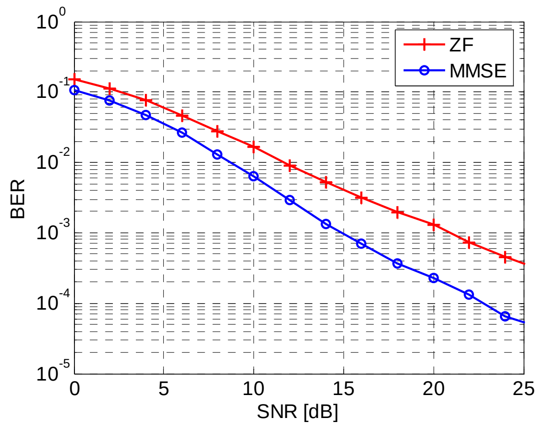



Fig. 12. Performance results for ZF- and MMSE- equalization for the described uncoded system (DFT spreading and equidistant subcarrier pattern)

## VI. NON-LINEARITIES

If the subcarrier allocation process is applied by a random subcarrier selection the resulting transmit signal will show a large peak-to-average ratio (PAR) in the transmit signal envelope where the signal amplitudes are nearly Gaussian distributed.

The maximum peak-to-average ratio (PAR), defined by

$$PAR = 10\log_{10}\left(\frac{\max\left(\left|s_i^{(m)}\right|^2\right)}{\sigma_s^2}\right) \quad [dB] \tag{14}$$

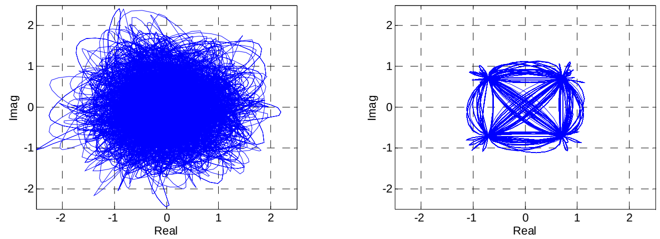
where $\sigma_s^2$ denotes the average transmit signal power, occurs only if the identical modulation symbol $S_l$ is transmitted on all used subcarriers. In this case the PAR is determined by the number of used subcarriers $L$.

In order to avoid non-linear distortions even in the case of a highly linear power amplifier a sufficient input-backoff (IBO) of the transmit signal to the amplifier has to be used. Different techniques have been developed which reduce the PAR of the OFDM signal by means of a modified channel coding [11], an additive [12] or multiplicative [13] correction function or a selective mapping of modulation symbols to subcarriers [14]. The majority of these techniques has a high computational complexity due to the fact that they analyze the generated transmit signal and modify it either in the frequency- or time domain to reduce its peak amplitudes.

For that reason it is extremely important to design a transmit signal in the mobile terminal which already has a constant envelope signal. That has been shown in this paper.

To show the reduced signal dynamic of the considered system, the complex envelope of the transmit signal is depicted in Fig. 13b. For comparison, the envelope of another spreaded OFDM-System is shown in Fig. 13a. In the latter case, also DFT spreading is used, but the subcarriers for transmission are allocated in a randomized manner and not equidistantly.

Figure 13 shows clearly the reduction in the signal envelope for the proposed uplink system. And the signal dynamic can be reduced even further by applying common techniques as pulseshaping or PAR-reducing modulation schemes like $\pi/4-$QPSK or constant phase modulation (CPM).




(a) Random Allocation (b) Equidistant Allocation

Fig. 13. Signal envelope of a DFT-spreaded transmission system with a) random and b) equidistant subcarrier allocation

The preceding considerations demonstrate that the main benefit of the proposed OFDM-FDMA uplink system with equidistant subcarrier allocation and DFT spreading is the resulting constant envelope signal and the low computation complexity.

## VII. Summary

In this paper an OFDM-based multi-user uplink communication system has been proposed and described in detail. It consists of an OFDM-FDMA approach with equidistant subcarrier allocation for each user and an integrated spreading technique based on a DFT spreading matrix. The transmit signal can be calculated just by periodic extension of all modulation symbols $D_l^{(m)}$. This greatly simplifies the structure of the transmitter for the proposed uplink system. Moreover, the PAR of the transmit signal and the computational effort are reduced dramatically in the mobile terminal.

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