Distributed Spatial Multiplexing in Wireless Networks

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***Abstract*— This paper describes the idea of distributed spatial multiplexing in wireless multihop networks and gives some preliminary results. We consider a set of single-antenna nodes with one source-destination pair and several relay nodes that transport the data of the source node to the destination node in a multihop fashion [1], [2]. In order to achieve high throughput and spectral efficiency we split the data stream into several sub-streams and transmit the sub-streams simultaneously over spatial sub-channels, whereas the sub-channels are created in a distributed fashion by the network itself (distributed spatial multiplexing). We compare two schemes: First the relays operate in an amplify-and-forward (AF) mode and establish a point-to-point MIMO channel between source and destination (*MIMO tunnel*). The second scheme operates with decode-and-forward (DF) relays where the sub-streams are transmitted over different network paths (multi-path routing). We show that the MIMO tunnel with AF relays obtains almost the performance of an idealized (interference-free) multi-path routing scheme with DF relays and is much better when interference from other routes is considered.**

I. Distributed Spatial Multiplexing with Amplify-and-Forward Relays

Consider a wireless network where a single-antenna source wants to send a data packet of length T_{P} to a single-antenna destination over $L+1$ hops, whereas L denotes the number of clusters containing K intermediate nodes that act as single-antenna *amplify-and-forward* (AF) relays. In Fig. 1 two amplify-and-forward relays jointly forward in each time step. The nodes $r_1^{(1)}$ and $r_2^{(1)}$ cooperate such, that $r_1^{(1)}$ forwards the first half of the data packet and $r_2^{(1)}$ the second half. As a result the packet duration at intermediate hops is $T_{\mathrm{P}}/2$. Intermediate nodes simply store and forward the received packets simultaneously without further cooperation. Note that the sub-packets forwarded by $r_1^{(2)}$ and $r_2^{(2)}$ are linear superpositions of the sub-packets forwarded by $r_1^{(1)}$ and $r_2^{(1)}$. In the last hop the nodes $r_1^{(4)}$ and $r_2^{(4)}$ cooperate such, that the destination receives two sub-packets sequentially in time. This establishes a distributed 2×2 MIMO channel between source and destination which we name *MIMO tunnel*. With suitable space-time signal processing at the source the destination is able to decode the information bits. As the packet length in the MIMO tunnel is reduced to $T_{\mathrm{P}}/2$, the multi-hop delay drops accordingly. Neglecting the time for signal propagation, the multi-hop delay τ is in general given by $\tau = T_{\mathrm{P}}\left(2+\frac{L-1}{K}\right)$, where K is the number of forwarding relays in a cluster.

Signal and Channel Model. All signal arrivals and departures are perfectly synchronized. The nodes cannot transmit and receive simultaneously and transmitted signals only reach the next cluster. The equivalent complex baseband signals received by the relays in the first cluster (entrance to the MIMO tunnel) are stacked into a vector $\mathbf{y}_1 = \mathbf{H}_1\mathbf{x} + \mathbf{w}_1$, where $\mathbf{x} = (x_1,\ldots,x_K)^{\mathrm{T}}$ is the data vector sent by the source with average power constraint $\mathrm{E}\left[|x_i^2|\right] = P/K$. Note that x_i is the symbol which will be forwarded from relay i in the first cluster. $\mathbf{H}_1 = \mathrm{diag}(h_{11},\ldots,h_{K1})$ is a $K\times K$ diagonal matrix with $h_{i1}\sim\mathcal{CN}(0,1)$ the complex path gain from source to relay i in the first forwarding cluster and $\mathbf{w}_1 = (w_1^{(1)},\ldots,w_K^{(1)})^{\mathrm{T}}$ with $w_i^{(l)}\sim\mathcal{CN}(0,\sigma_{\mathrm{R}}^2)$ describes the additive white Gaussian noise at the relays. The symbols $x_1,\ldots,x_K$ are launched into the MIMO tunnel via orthogonal channels (for example with time division multiplexing, TDM) that are described by $\mathbf{H}_1$.

The signals received in the lth cluster are given as $\mathbf{y}_l = \mathbf{H}_l g_l \mathbf{y}_{l-1} + \mathbf{w}_l$, with $\mathbf{y}_l = (y_1^{(l)},\ldots,y_K^{(l)})^{\mathrm{T}}$. $\mathbf{H}_l$ with $l = 2,\ldots,L$ is a $K\times K$ channel matrix with elements $h_{ij}^{(l)}\sim\mathcal{CN}(0,1)$ describing the complex path gain from the jth relay in cluster $l-1$ to the ith relay in cluster l. The scaling factor g_l is chosen such that the average transmit power of every

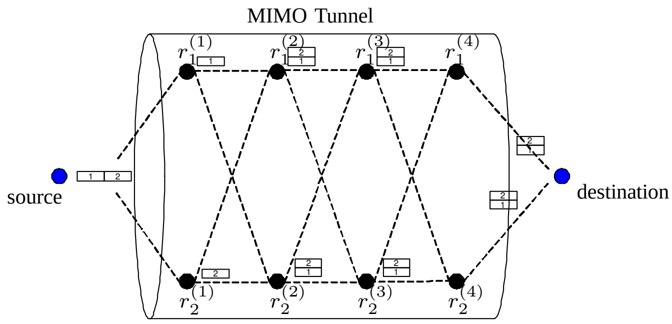

Fig. 1. A Distributed 2×2 Multi-Hop MIMO System with amplify-and-forward relays: *MIMO Tunnel*

node equals P/K. The vector received by the destination is then given by

$$\mathbf{y} = \underbrace{\mathbf{H}_{L+1}\prod_{l=1}^{L} g_l\mathbf{H}_l}_{\mathbf{H}}\,\mathbf{x} + \underbrace{\sum_{m=1}^{L}\prod_{n=m}^{L} g_n\mathbf{H}_{n+1}\mathbf{w}_m + \mathbf{w}}_{\mathbf{n}}, \qquad (1)$$

with

$$g_l^2 = \begin{cases} \frac{P/K}{P/K+\sigma_{\mathrm{R}^2}} & ; l=1, \\ \frac{P/K}{P+\sigma_{\mathrm{R}^2}} & ; 2\le l\le L. \end{cases}$$

$\mathbf{H}_{L+1} = \mathrm{diag}(h_{11},\ldots,h_{1K})$ is again a $K\times K$ diagonal matrix with $h_{1i}\sim\mathcal{CN}(0,1)$ the complex path gain from relay i in the last forwarding cluster to the destination, i.e., the signals from the last cluster (exit of the MIMO tunnel) are transmitted via orthogonal channels to the destination (e.g. TDM). Note that the noise in (1) is not white and the covariance matrix $\mathbf{R}$ is given as

$$\mathbf{R} = \sum_{m=1}^{L}\prod_{n=m}^{L} g_n^2\mathbf{H}_{n+1}\left(\prod_{n=m}^{L}\mathbf{H}_{n+1}\right)^{\mathrm{H}}\sigma_{\mathrm{R}}^2\mathbf{I}_K + \sigma^2\mathbf{I}_K.$$

where σ^2 denotes the noise power at the destination.

Capacity. Assuming perfect knowledge of the compound channel matrix $\mathbf{H}$ and the noise covariance matrix $\mathbf{R}$ at the destination the mutual information of the MIMO tunnel measured in bit per link use, i.e., (L+1) consecutive uses of K channels simultaneously, is

$$I_{\mathrm{AF}}(\mathbf{x};\mathbf{y}|\mathbf{H},\mathbf{R}) = \log_2\det\left(\mathbf{I}_K + \frac{P}{K}\mathbf{R}^{-1}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right), \qquad (2)$$

where P denotes the total power injected into the MIMO tunnel. The ergodic capacity of the MIMO tunnels follows by averaging (2) over the fading statistics: $C_{\mathrm{AF}} = \mathrm{E}\left[I_{\mathrm{AF}}(\mathbf{x};\mathbf{y}|\mathbf{H},\mathbf{R})\right]$.

II. Distributed Spatial Multiplexing with Decode-and-Forward Relays

We compare the performance of the MIMO tunnel with a multipath routing approach where the source sends its data $\mathbf{x} = (x_1,\ldots,x_K)^{\mathrm{T}}$ simultaneously over K network routes. The first symbol x_1 is transmitted along the route $r_1^{(1)}\to r_1^{(2)}\to\cdots\to r_1^{(L)}$, the second symbol x_2 along $r_2^{(1)}\to r_2^{(2)}\to\cdots\to r_2^{(L)}$ and so on. Every route consists of $L+1$ *decode-and-forward* (DF) SISO links. The signal received in path k in layer l is given as

$$y_k^{(l)} = h_k^{(l)}x_k^{(l-1)} + \sum_{i=1,i\neq k}^{K} h_i^{(l)}x_i^{(l-1)} + w_k^{(l)} \qquad (3)$$

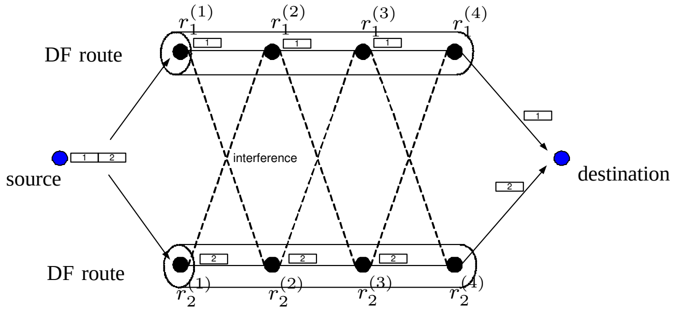

Fig. 2. Multi-path routing in a multi-hop wireless network with decode-and-forward relay nodes

where $h_k^{(l)}$ denotes the channel gain from the kth relay in cluster $l-1$ to the kth relay in cluster l. $x_k^{(l-1)}$ is the re-encoded source signal x_k, the sum in (3) describes the interference from the other routes and $w_k^{(l)}$ denotes additive white Gaussian noise with variance σ_{R}^2. All channel coefficients are drawn according to $\sim \mathcal{CN}(0,1)$. We refer to orthogonal multi-path routing when $\sum_{i=0,i\neq k}^{K} h_i^{(l)} x_i^{(l-1)} = 0$ (no interference) and to non-orthogonal multi-path routing when interference is considered.

a) $T_{\text{coh}} \gg T_{\text{P}}$ **.** For the case that the network coherence time T_{coh} (roughly spoken the time interval where all channel coefficients remain constant) is much larger than the length of a data packet T_{P} (codeword) usually one has to sidestep to capacity-versus-outage notion [3]. However, when we assume channel state information at the source node the source may adapt the code rate to the instantaneous channel conditions (in order to avoid outage events). Hence, the source has to know the channel coefficients of all links which are involved in the communication between source and destination. The capacity of one path (consisting of $L+1$ SISO links) is then determined by the capacity of the weakest link in that path. The source chooses for every link a code rate that matches the capacity of the weakest link. The total capacity between source and destination for that particular signaling scheme is then upper bounded by

$$C_{\text{Sum,a}} \leq \sum_{k=1}^{K} \min_{l} \left\{ \log_2 \left(1 + \frac{P}{K\left(\sigma^2 + P_{\text{I}}\right)} \right) |h_k^{(l)}|^2 \right\}, \quad (4)$$

where $P_{\text{I}} = \left| \sum_{i=1,i\neq k}^{K} h_i^{(l)} \right|^2 \cdot \frac{P}{K}$ is the interference power caused by the other routes. For the case that the nodes are able to transmit and receive simultaneously the bound in (4) approaches equality. The average capacity is obtained by taking the expectation of (4) with respect to the fading statistics

$$\overline{C}_{\text{Sum,a}} = \text{E}\left[C_{\text{Sum,a}}\right] = \sum_{k=1}^{K} \text{E}\left[\min_{l} \left\{ C_{\text{a},l} \right\} \right].$$

b) $T_{\text{coh}} \ll T_{\text{P}}$ **.** If we assume that the network coherence time is small compared to the length of one data packet, the nodes can achieve ergodic capacity by having codewords that averages out the channel fluctuations [3]. The advantage is that no channel knowledge at the source is necessary to achieve the ergodic capacity. The capacity of one route (path) is then determined by the lowest ergodic SISO link capacity in that route. The total ergodic capacity between source and destination is then given as

$$\overline{C}_{\text{Sum,b}} = \sum_{k=1}^{K} \min_{l} \left\{ \text{E}\left[\log_2 \left(1 + \frac{P}{K\left(\sigma^2 + P_{\text{I}}\right)} \right) |h_k^{(l)}|^2 \right] \right\}$$
$$= \sum_{k=1}^{K} \min_{l} \left\{ \text{E}\left[C_{\text{b},l}\right] \right\}.$$

Due to convexity properties of the $\min\{\cdot\}$ operator and by the use of Jensen's inequality we see that case b) performs better than case a):

$$\overline{C}_{\text{Sum,a}} = \sum_{k=1}^{K} \text{E}\left[\min_{l} \left\{ C_{\text{a},l} \right\} \right] \leq \sum_{k=1}^{K} \min_{l} \left\{ \text{E}\left[C_{\text{b},l}\right] \right\} = \overline{C}_{\text{Sum,b}}.$$

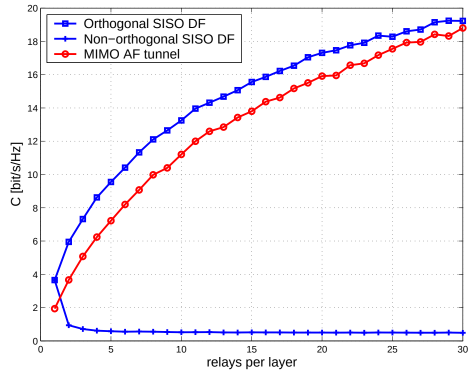

Fig. 3. Capacity vs. number of relays per layer for distributed MIMO AF transmission and multi-path SISO DF routing, L=4

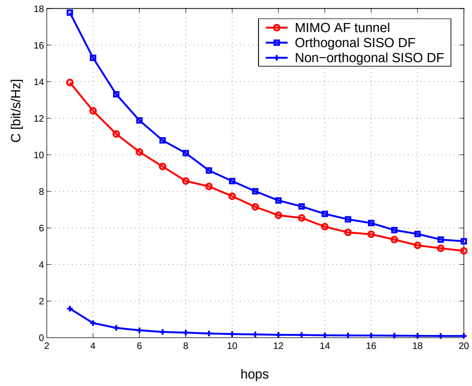

Fig. 4. Capacity vs. number of hops for distributed MIMO AF transmission and multi-path SISO DF routing, K=10

III. Numerical Examples and Conclusion

Due to space constraints we compare here only the performance of the MIMO tunnel with the multi-path routing under the assumption that $T_{\text{coh}} \gg T_{\text{P}}$. In Fig. 3 we see that the average capacity $C_{\text{AF}} = \text{E}\left[I_{\text{AF}}\right]$ of the MIMO tunnel approaches almost the average capacity of the orthogonal (interference-free) multi-path routing, whereas the MIMO tunnel clearly outperforms the case where interference between the network routes is considered. The reason for the large degradation of the non-orthogonal multi-path routing here is our simplified channel model, where the interfering signals are roughly equally strong as the desired signal. It's interesting to see that the interference from neighbor nodes helps us in the MIMO tunnel to establish a rich scattering environment which is necessary to obtain significant spatial multiplexing gains, whereas the interference in the multi-path routing approach limits this signaling scheme strongly. In Fig. 4 we compare the two schemes with respect to the number of hops between source and destination. We see that both schemes decrease as the number of hops increases. The reason for the performance degradation of the MIMO tunnel is simply the observation that the noise is accumulated from cluster to cluster and finally transferred to the destination. In the case of the multi-path routing approach every additional hop increases the probability of a weaker link in a path and therefore the probability that the path capacity will be smaller by adding a new link (hop) increases.

References

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