MC-Cyclic Antenna Frequency Spread: A Novel Space-Frequency Spreading for MIMO-OFDM

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***Abstract*— In this paper, we investigate the effect of spreading in a multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system. A novel family of orthogonal spreading matrices –multicarrier-cyclic antenna frequency spread (MC-CAFS)– is introduced which allows equal energy distribution of all transmitted signals. These matrices make use of both the spatial and the frequency dimensions offered by a MIMO-OFDM system. We shall describe the conditions such spreading needs to fulfill in order to achieve equal energy distribution. Finally, we compare the performance of MC-CAFS with that of the unspread system and with multicarrier-code division multiplexing (MC-CDM), where the signals are only spread in the frequency direction. We consider an uncoded transmission over frequency selective time varying channels. The recurrent neural network (RNN) and the soft Cholesky equalizers (SCE) shall be applied at the receiver, where perfect channel knowledge is assumed.**

I. Introduction

Multi-carrier (MC) transmission schemes (e. g., orthogonal frequency division multiplexing OFDM) are promising candidates for 4G mobile communication systems. In the previous years, MIMO systems have gained considerable attention due to their potential of achieving very high data rates [1], [2]. In addition, MIMO-OFDM transmission schemes offer spatial, temporal, and frequency diversity that can be exploited [3].

In this paper, we will describe the discrete-time vector-valued MIMO-OFDM transmission model and look at the discrete-time channel matrix before and after spreading. In General, spreading is applied in order to distribute the energy of any signal over all or some of the available subcarriers. We present a novel family of orthogonal spreading matrices (MC-CAFS) that make use of both the frequency and spatial dimensions offered by a MIMO-OFDM system. The matrices provide **equal** energy distribution of all transmitted signals. In order to attain equal energy distribution and yet still preserve their orthogonality, these spreading matrices must satisfy certain conditions. We shall look at those conditions in Sections II and III. It was shown in [4], [5], [6] that spreading, although it increases the interference on the channel, in general improves the bit error rate (BER) performance. In Section IV, we look at the effect of spreading on the BER behavior of an uncoded transmission over time varying frequency selective channels having equal numbers of receive, n_{R}, and transmit antennas, n_{T}. The BER of MC-CAFS is compared with MIMO-MC-CDM [4], [5], [6] as well as with the unspread MIMO-OFDM system.

Since symbols transmitted at the same frequencies from different antennas interfere with each other, channel equalization at the receiver is necessary even for the unspread MIMO-OFDM system. The performance of iterative detectors, recurrent neural network (RNN) [7], [8] and soft Cholesky block decision feedback equalizer (SCE) [9], will be compared. The channel is assumed to be perfectly known at the receiver. Throughout this paper, vectors and matrices will be denoted by underlined and doubly-underlined letters, respectively. Lowercase will be used for the time domain whereas the uppercase letters for the frequency domain. Scalars will be simply designated by letters.

II. System Model

The $n_{\mathrm{T}} \times n_{\mathrm{R}}$ channel matrices, $\underline{\underline{h}}(l)$, after sampling according to the sampling theorem are given as follows:

$$\underline{\underline{h}}(l) = \begin{pmatrix} h_{11}(l) & \dots & \dots & \dots & h_{1n_{\mathrm{T}}}(l) \\ \vdots & \ddots & h_{i_{\mathrm{R}}i_{\mathrm{T}}}(l) & \ddots & \vdots \\ h_{n_{\mathrm{R}}1}(l) & \dots & \dots & \dots & h_{n_{\mathrm{R}}n_{\mathrm{T}}}(l) \end{pmatrix}, \tag{1}$$

where $h_{i_{\mathrm{R}}i_{\mathrm{T}}}(l)$ is the l-th tap of channel impulse responce (CIR) between transmit antenna i_{T} ($i_{\mathrm{T}} = 1 \dots n_{\mathrm{T}}$) and receive antenna i_{R} ($i_{\mathrm{R}} = 1 \dots n_{\mathrm{R}}$), and ($l = 1 \dots L$). We assume that the length of the cyclic prefix (CP) is large enough to maintain orthogonality among the subcarriers and that the channel is perfectly known at the receiver. We will concentrate on spreading in the frequency and spatial directions.

In the frequency domain, each transmit antenna can transmit over N OFDM subcarriers, with these being the same for all transmit antennas. One OFDM symbol, $\underline{x}$, is thus a vector of length $N \times n_{\mathrm{T}}$. Modulation and demodulation can be efficiently implemented through the use of inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT) respectively. Accordingly, the discrete-time channel matrix, $\underline{\underline{R}}_{\mathrm{MO}}$, for this MIMO-OFDM system can be described by the following equation [4], [5]:

$$\underline{\underline{R}}_{\mathrm{MO}} = \frac{1}{n_{\mathrm{R}}} \underline{\underline{H}}^H \, \underline{\underline{H}}, \tag{2}$$

where $\underline{\underline{H}}$ is an $(Nn_{\rm R}) \times (N\,n_{\rm T})$ matrix containing the transfer functions of the channel impulse responses between all receive and transmit antennas and ^{H} denotes its conjugate transpose. $\underline{\underline{H}}$ is given by

$$\underline{\underline{H}} = \begin{pmatrix} \underline{\underline{H}}_{11} & \cdots & \cdots & \cdots & \underline{\underline{H}}_{1n_{\rm T}} \\ \vdots & \ddots & \underline{\underline{H}}_{i_{\rm R}i_{\rm T}} & \ddots & \vdots \\ \underline{\underline{H}}_{n_{\rm R}1} & \cdots & \cdots & \cdots & \underline{\underline{H}}_{n_{\rm R}n_{\rm T}} \end{pmatrix}, \quad (3)$$

The submatrices $\underline{\underline{H}}_{i_{\rm R}i_{\rm T}}$ are $N \times N$ diagonal matrices and are given by:

$$\underline{\underline{H}}_{i_{\rm R}i_{\rm T}} = {\rm diag}({\rm DFT}(\underline{h}_{i_{\rm R}i_{\rm T}})). \quad (4)$$

where $\underline{h}_{i_{\rm R}i_{\rm T}} = [h_{i_{\rm R}i_{\rm T}}(1) \ldots h_{i_{\rm R}i_{\rm T}}(l) \ldots h_{i_{\rm R}i_{\rm T}}(L)]$.

A. Spreading, Signal power and Interference

OFDM transmission schemes are sensitive to the frequency-selective behavior of the channel which may cause one or more of the subcarriers to totally fade out [10], [11]. Although MIMO-OFDM is more robust to this selective behavior, some signals may still fade stronger than others. This can be better explained by looking at the diagonal elements of $\underline{\underline{R}}_{\rm MO}$, $r_{i_{\rm T},k}$, which can be expressed as follows

$$r_{i_{\rm T},k} = \frac{1}{n_{\rm R}} \sum_{i_{\rm R}=1}^{n_{\rm R}} |H_{i_{\rm R}i_{\rm T}}(k)|^2, \quad (5)$$

where $k = 1 \ldots N$ and $i_{\rm T} = 1 \ldots n_{\rm T}$. Note that the indices of the $r_{i_{\rm T},k}$ are seperated by a comma since they only refer to the diagonal elements of $\underline{\underline{R}}_{\rm MO}$, which are grouped according to transmit antenna and frequency. For instance, the first N diagonal elements belong to the first transmit antenna, the second set of N diagonal elements belong to the second antenna and so forth. Equation 5 is the sum over all receive antennas, thus it represents the maximum ration combining (MRC) technique for a signal transmitted from antenna $i_{\rm T}$ at frequency k.
The off-diagonal elements, $r_{{\rm off},k}$, represent the effect of the interference of signals transmitted at the same frequency, k, from different antennas. Since $n_{\rm R} << \infty$, the diagonal elements $r_{i_{\rm T},k}$ are not equal and $r_{{\rm off},k} \neq 0$. Accordingly, spreading is also necessary in the MIMO case. Spreading at the transmitter can be represented through postmultiplication by a spreading matrix, $\underline{\underline{\rm U}}$, and despreading at the receiver through premultiplication by the despreading matrix, $\underline{\underline{\rm U}}^H$, as follows [10], [11]

$$\underline{\underline{R}}_{\rm S} = \underline{\underline{U}}^H\, \underline{\underline{R}}_{\rm MO}\, \underline{\underline{U}}, \quad (6)$$

where $\underline{\underline{R}}_{\rm S}$ is the resulting channel correlation matrix. In the ideal case, $n_{\rm R} \rightarrow \infty$, $\underline{\underline{R}}_{\rm MO}$ is an identity matrix, $\underline{\underline{I}}$, i.e parallel transmission of all signals without interference. The $\underline{\underline{R}}_{\rm MO}$ has been normalized such that the average power, $\bar{r}$, in each subchannel $\bar{r} = \mathcal{E}(r_{i_{\rm T},k}) = 1$ which translates to

$$\mathrm{Tr}(\underline{\underline{R}}_{\rm MO}) = \sum_{i=1}^{n_{\rm T}N} \lambda_i(\underline{\underline{R}}_{\rm MO}) = n_{\rm T}N, \quad (7)$$

where λ_i is the i-th eigenvalue of $\underline{\underline{R}}_{\rm MO}$ and Tr denotes its trace. Since only orthogonal spreading matrices are considered, the eigenvalues of $\underline{\underline{R}}_{\rm MO}$ do not change after spreading. That is, eqn. 7 still applies to $\underline{\underline{R}}_{\rm S}$. The eigenvectors do however change.
The received OFDM symbol $\tilde{\underline{x}}$ can thus be given by

$$\tilde{\underline{x}} = \underline{\underline{R}}\,\underline{x} + \tilde{\underline{n}}_c, \quad (8)$$

where $\underline{\underline{R}}$ is either the unspread, $\underline{\underline{R}}_{\rm MO}$, or spread channel matrix, $\underline{\underline{R}}_{\rm S}$, and $\tilde{\underline{n}}_c$ the colored noise of variance $2N_o\underline{\underline{R}}$.

B. Spreading Criteria

In order to explain the requirements a spreading matrix needs to fulfill, we need to take a closer look at the diagonal elements of the channel matrix $\underline{\underline{R}}$, before, $r_{i_{\rm T},k}$, and after spreading, ${}^{\rm s}r_{i_{\rm T},k}$. We shall start by expressing the transfer function between transmit antenna $i_{\rm T}$ and receive antenna $i_{\rm R}$ at frequency k as follows

$$H_{i_{\rm R}i_{\rm T}}(k) = \sum_{m=1}^{L} h_{i_{\rm R}i_{\rm T}}(m) e^{-j2\pi(\frac{(k-1)(m-1)}{N})}. \quad (9)$$

Equation 9 represents the k-th diagonal element of eqn.4. Accordingly, $|H_{i_{\rm R}i_{\rm T}}(k)|^2$ can be written as

$$|H_{i_{\rm R}i_{\rm T}}(k)|^2 = \sum_{m=1}^{L} |h_{i_{\rm R}i_{\rm T}}(m)|^2 + \sum_{m=1}^{L}\sum_{l>m}^{L} 2Re\{h_{i_{\rm R}i_{\rm T}}(m) h^*_{i_{\rm R}i_{\rm T}}(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\}. \quad (10)$$

Equations 5, 7 and 10 give a guideline for selecting the appropriate spreading. First, spreading should be applied such that the diagonal elements of $\underline{\underline{R}}_S$, ${}^{\rm s}r_{i_{\rm T},k}$, satisfy the following equation

$${}^{\rm s}r_{i_{\rm T},k} = \sum_{k\subset\mathcal{S}_1, i_{\rm T}\subset\mathcal{S}_2} |w_{i_{\rm T},k}|^2 r_{i_{\rm T},k}, \quad (11)$$

where the $\mathcal{S}_1$ and $\mathcal{S}_2$ are sets containing all frequencies and transmit antennas, repsectively, i. e. $\mathcal{S}_1 = \{1 \ldots N\}$ and $\mathcal{S}_2 = \{1 \ldots n_{\rm T}\}$. Since we are trying to achieve both spatial and frequency diversities, the subsets chosen from $\mathcal{S}_1$ and $\mathcal{S}_2$ can not be empty. The $|w_{i_{\rm T},k}|^2$ are weighting constants dependent on the subsets of $\mathcal{S}_1$ and $\mathcal{S}_2$ and should satisfy

$$\sum_{k\subset\mathcal{S}_1, i_{\rm T}\subset\mathcal{S}_2} |w_{i_{\rm T},k}|^2 = 1. \quad (12)$$

They ensure that equation 7 is satisfied after spreading. For the spreading matrices considered here, the absolute values for $w_{i_{\rm T},k}$ are the same for all $i_{\rm T}$ and k, i. e. $|w_{i_{\rm T},k}| = |w|$. Equation 11 imposes the requirement that the off-diagonal elements of $\underline{\underline{R}}_{\rm MO}$ do not affect the diagonal elements of $\underline{\underline{R}}_{\rm S}$ [5]. Physically, this means that a signal should not interefere with itself after spreading.

Second, the subsets of $\mathcal{S}_1$ and $\mathcal{S}_2$ should be chosen such that

$$\begin{aligned} \sum_{k\subset\mathcal{S}_1, i_{\mathrm{T}}\subset\mathcal{S}_2} |w_{i_{\mathrm{T}},k}|^2 \sum_{i_{\mathrm{R}}=1}^{n_{\mathrm{R}}} \sum_{m=1}^{L} \sum_{l>m}^{L} 2A = \\ |w|^2 \sum_{k\subset\mathcal{S}_1, i_{\mathrm{T}}\subset\mathcal{S}_2} \sum_{i_{\mathrm{R}}=1}^{n_{\mathrm{R}}} \sum_{m=1}^{L} \sum_{l>m}^{L} 2A = 0, \end{aligned} \tag{13}$$

where

$$A = Re\{h_{i_{\mathrm{R}}i_{\mathrm{T}}}(m) h^{*}_{i_{\mathrm{R}}i_{\mathrm{T}}}(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\}. \tag{14}$$

Equation 13 is also satisfied, if only the following sum is zero,

$$\sum_{k\subset\mathcal{S}_1} \sum_{m=1}^{L} \sum_{l>m}^{L} 2Re\{h_{i_{\mathrm{R}}i_{\mathrm{T}}}(m) h^{*}_{i_{\mathrm{R}}i_{\mathrm{T}}}(l) e^{-j2\pi(\frac{(k-1)(l-m)}{N})}\} = 0. \tag{15}$$

Thus, if, through spreading, the subsets for i_{T} and k in eqns.11 and 13 (or 15) are adequately chosen, the diagonal elements of $\underline{\underline{R}}_{\mathrm{S}}$ become

$$\begin{aligned} {}^{\mathrm{s}}r_{i_{\mathrm{T}},k} &= \frac{1}{n_{\mathrm{R}} n_{\mathrm{T}}} \sum_{i_{\mathrm{R}}=1}^{n_{\mathrm{R}}} \sum_{i_{\mathrm{T}}=1}^{n_{\mathrm{T}}} \sum_{l=1}^{L} |h_{i_{\mathrm{R}}i_{\mathrm{T}}}(l)|^2 \\ &= \frac{1}{n_{\mathrm{R}} n_{\mathrm{T}}} ||\underline{\underline{h}}||_F^2 = 1.0, \end{aligned} \tag{16}$$

for all i_{T} and k. The square norm $||\underline{\underline{h}}||_F^2$ is defined as follows

$$||\underline{\underline{h}}||_F^2 = \sum_{l=1}^{L} ||\underline{\underline{h}}(l)||_F^2, \tag{17}$$

where $||\underline{\underline{h}}(l)||_F^2$ is the squared frobenius norm of $\underline{\underline{h}}(l)$. That is, the diagonal elements of $\underline{\underline{R}}_{\mathrm{S}}$ are equal and in the ideal case, when all interference has been removed (i. e. the matched filter bound: MFB), the received OFDM symbol $\underline{\tilde{x}}$ can be expressed by

$$\begin{aligned} \underline{\tilde{x}} &= \frac{1}{n_{\mathrm{R}} n_{\mathrm{T}}} ||\underline{\underline{h}}||_F^2 \underline{\underline{I}}_{n_{\mathrm{T}}N} \underline{x} + \underline{\tilde{n}} \\ &= \underline{\underline{I}}_{n_{\mathrm{T}}N} \underline{x} + \underline{\tilde{n}}, \end{aligned} \tag{18}$$

where $\underline{x}$ is the transmit OFDM symbol, $\underline{\underline{I}}_{n_T N}$ the identity matrix of size $n_{\mathrm{T}}N$ and $\underline{\tilde{n}}$ is the noise.

III. **MC-CAFS** SPREADING

In this section, we introduce a new family of spreading matrices that make use of the frequency as well as the spatial dimension offered by the MIMO system. The proposed spreading matrices spread each signal over all transmit antennas. In addition, it spreads each signal over a set of frequencies for each transmit anntenna. These frequency sets are different for each antenna and thus the name **MC-CAFS**: multi-carrier cyclic antenna frequency spread. We shall show how this family of spreading matrices can satisfy eqns. 11 through 16. As mentioned earlier, we shall only consider orthogonal spreading matrices. That is,

$$\underline{\underline{U}}^H \underline{\underline{U}} = \underline{\underline{I}}. \tag{19}$$

The spreading matrix, $\underline{\underline{U}}_{MC-CAFS}$, is defined by,

$$\underline{\underline{U}}_{MC-CAFS} = \left(\begin{array}{cccc} s_{11}\underline{\underline{I}} & s_{12}\underline{\underline{I}} & \cdots & s_{1(B_{n_{\mathrm{T}}})}\underline{\underline{I}} \\ \vdots & \vdots & \vdots & \vdots \\ s_{B1}\underline{\underline{I}} & s_{B2}\underline{\underline{I}} & \cdots & s_{B(B_{n_{\mathrm{T}}})}\underline{\underline{I}} \\ \hline s_{(B_1+1)1}\underline{\underline{I}}_{p_1} & s_{(B_1 1)2}\underline{\underline{I}}_{p_1} & \cdots & s_{(B_1+1)(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{p_1} \\ \vdots & \vdots & \vdots & \vdots \\ s_{(B_2)1}\underline{\underline{I}}_{p_1} & s_{(B_2)2}\underline{\underline{I}}_{p_1} & \cdots & s_{(B_2)(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{p_1} \\ \hline s_{(B_2+1)1}\underline{\underline{I}}_{p_2} & s_{(B_2+1)2}\underline{\underline{I}}_{p_2} & \cdots & s_{(B_2+1)(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{p_2} \\ \vdots & \vdots & \vdots & \vdots \\ s_{(B_3)1}\underline{\underline{I}}_{p_2} & s_{(B_3)2}\underline{\underline{I}}_{p_2} & \cdots & s_{(B_3)(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{p_2} \\ \hline \vdots & \vdots & \vdots & \vdots \\ \hline s_{(B_{n_{\mathrm{T}}-1}+1)1}\underline{\underline{I}}_{pn} & \cdots & \cdots & s_{(B_{n_{\mathrm{T}}-1}+1)(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{pn} \\ \vdots & \vdots & \vdots & \vdots \\ s_{(B_{n_{\mathrm{T}}})1}\underline{\underline{I}}_{pn} & \cdots & \cdots & s_{(B_{n_{\mathrm{T}}})(B_{n_{\mathrm{T}}})}\underline{\underline{I}}_{pn} \end{array} \right)$$

where B is the number of blocks per transmit antenna and represents the number of frequencies over which each signal is spread at each transmit antenna (the horizontal lines show the seperation between the transmit antennas), $B_m = mB$, and s_{ij} are elements of the orthogonal matrix $\underline{\underline{S}}$ of size $(n_{\mathrm{T}}B) \times (n_{\mathrm{T}}B)$. $\underline{\underline{I}}$ is identity matrices of size $(N/B) \times (N/B)$ and $\underline{\underline{I}}_{p_i}$ is its i-th permutation. The permutations imply that each signal is spread over different frequencies at each transmit antenna and ensure that eqn.11 is satisfied. That is, the off-diagonal elements of $\underline{\underline{R}}_{\mathrm{MO}}$ do not affect the diagonal elements after spreading [5]. In addition, the number of allowed permutations, n, must satisfy $n \leq (n_{\mathrm{T}} - 1)$ (i. e. no permutation should repeat) else $\underline{\underline{U}}_{MC-CAFS}$ will not be orthogonal. This condition translates to $(N/B) \geq n_{\mathrm{T}}$ or equivalently $B \leq N/n_{\mathrm{T}}$.

To insure that eqn. 15 (and consequently eqn. 13) is satisfied, $B \geq L$ (this inequality will be explained through the following example). To summarize, the number of blocks, B, must satisfy the following criteria

$$L \leq B \leq \frac{N}{n_{\mathrm{T}}}, \tag{20}$$

to insure that ${}^{\mathrm{s}}r_{i_{\mathrm{T}},k} = 1$ for all i_{T} and k (eqn.16). Note that for a given N, n_{T} and L, there is a set of B values (i. e. set of spreading matrices) that would satisfy eqns. 11 to 16. The B values must however be chosen such that N/B is an integer.

A. Example

Equation 20 can be better explained by looking at the following example. In this example, we consider a 2×2 MIMO system with $N = 8$. We consider two cases, $L = 2$ and $L = 3$. By spreading over several frequencies of the same antenna, eqn. 13 can be satisfied. However, the choice of those frequencies is important and is dependent on L. In other words, the choice of the subsets

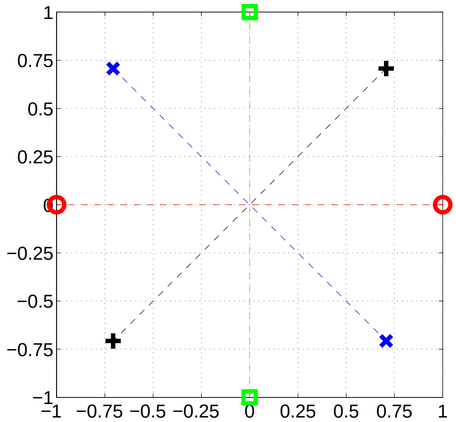

Fig. 1. E(k)=$e^{-j2\pi(k-1)\frac{(l-m)}{N}}$, $N=8$, $k=1\dots N$, $(l-m)=1$

k	$E(k)$ $(l-m)=1$, $N=8$		$E(k)$ $(l-m)=2$, $N=8$	
1	$1.0+j0.0$	$\circ$	$1.0+j0.0$	$\diamond$
2	$\sqrt{2}-j\sqrt{2}$	$\times$	$0.0-j1.0$	$*$
3	$0.0-j1.0$	$\square$	$-1.0+j0.0$	$\diamond$
4	$-\sqrt{2}-j\sqrt{2}$	$+$	$0.0+j1.0$	$*$
5	$-1.0-j0.0$	$\circ$	$1.0+j0.0$	$\diamond$
6	$-\sqrt{2}+j\sqrt{2}$	$\times$	$0.0-j1.0$	$*$
7	$0.0+j1.0$	$\square$	$-1.0+j0.0$	$\diamond$
8	$\sqrt{2}+j\sqrt{2}$	$+$	$0.0+j1.0$	$*$

TABLE I
VALUES FOR $E(k)$ FOR $(l-m)=1$ AND $(l-m)=2$

($k \subset \mathcal{S}_1$) is crucial. This can be illustrated by analysing fig. 1, where the values of $E(k) = e^{-j2\pi(k-1)\frac{(l-m)}{N}}$ for $N = 8$, $k = 1\dots N$, $(l-m) = 1$ are shown. The 8 points lie on a circle of radius one. For every point on the circle, there is another point $E(k+c) = -E(k)$, where c is a constant and is equal to 4. Those pairs are joined by dashed lines and are denoted by the same symbol in table I. If $L = 2$, then the maximum value of $(l-m)$ is 1. Thus, to insure that eqn. 15 is satisfied, any one symbol must be spread over 2 frequencies, k and $k+c$, $c = 4$, of transmit antenna, t. This can be easily satisfied by choosing $B = 2$, i. e. $N/B = 4$. Thus each subset would contain 2 frequencies, k and $k+c$ where $c = 4$. If $L = 3$, then the points for $(l-m) = 2$ need also to be considered (see table I). In this case, each symbol must now be spread over 4 frequencies, $E(k+c) = -E(k)$ with $c = 2$. That is, $B = 4$ and $N/B = 2$. Note that in this case, the condition for $(l-m) = 1$ is automatically satisfied.

Based on this anaylsis, it is clear that for eqn. 13 to be satisfied, it is enough to insure that any one symbol is spread over the adequate number of frequencies of its corresponding transmit antenna, i. e. it is enough to satisfy eqn.15. The minimum number of frequencies is dependent on the longest delay on the channel, L. The choice of the subset $i_{\mathrm{T}} \subset \mathcal{S}_2 = \{1\dots n_{\mathrm{T}}\}$ does not affect the equality in eqn. 13. However, it affects validity of eqn. 16. By choosing $i_{\mathrm{T}} = \mathcal{S}_2$, eqn. 16 is satisfied. That is, althought the symbols need not be spread over all frequencies, they must be spread over all antennas. This could also be directly deduced from eqn. 16 (the sum is over all n_{T}).

IV. SIMULATION RESULTS

We look at a two path, $L = 2$, time varying 4×4 MIMO system with $N = 32$ subcarriers. Each path is assumed to be complex gaussian, $CN(\mu, \sigma^2)$, with mean, $\mu = 0$ and variance $\sigma^2 = 0.5$. The channel is assumed to be perfectly known at the receiver. The transmitted symbols in the frequency domain are chosen from a 4 PSK alphabet. At the receiver, we employ two iterative detectors for symbol equalization in the frequency domain: the RNN and SCE equalizers [7], [8], [9]. Both equalizers employ soft decision functions, with that of the SCE being more complex. In addition, the SCE employes a whitening filter and thus has an increased complexity compared to the RNN equalizer. For both equalizers, the number of iterations was set to 5.

In this section, the BER performance of MC-CAFS is compared with the unspread MIMO-OFDM and MC-CDM using the above mentioned detectors. The spreading matrix for MC-CDM is given by [4], [5], [6]:

$$\underline{\underline{U}}_{MC-CDM} = \begin{pmatrix} \underline{\underline{S}} & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{S}} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{S}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{S}} \end{pmatrix}, \tag{21}$$

where $\underline{\underline{S}}$ is an $N\times N$ orthogonal spreading matrix, and $\underline{\underline{0}}$ are zero matrices of the same size. In this case, the energy of each symbol is equally spread over N subcarriers of its corresponding transmit antenna. Note that although the MC-CDM spreading matrix satisfies eqn. 15, the subset $i_{\mathrm{T}} \subset \mathcal{S}_2$ in eqn.11 is empty and thus full diversity can not be achieved. In other words, the diagonal elements ${}^{\mathrm{s}}r_{i_{\mathrm{T}},k}$ are not equal for all i_{T} and k. For both the MC-CDM and MC-CAFS spreading matrices, we chose $\underline{\underline{S}}$=Walsh-Hadamard matrix, $\underline{\underline{WH}}(Z)$, of the appropriate size, Z.

Based on eqn. 20, and the given values for N, n_{T} and L, $2 \leq B \leq 8$. For the simulations, $B = 4$. This corresponds to $N/B = 8$, and for $n_{\mathrm{T}} = 4$, only 3 permutations were necessary. For the MC-CDM, $\underline{\underline{S}} = \frac{1}{\sqrt{32}}\underline{\underline{WH}}(32)$ (i. e. $|w_{i_{\mathrm{T}},k}|^2 = 1/32$) while for MC-CAFS, $\underline{\underline{S}} = \frac{1}{\sqrt{16}}\underline{\underline{WH}}(16)$ (i. e. $|w_{i_{\mathrm{T}},k}|^2 = 1/16$).

Figure 2 shows the BER for SCE for the unspread MIMO-OFDM system, MC-CDM and MC-CAFS with $B = 4$. The SCE has the lowest BER, about 1.5 dB away from the AWGN curve at BER $= 10^{-3}$ and about 3 dB better than MC-CDM and 5 dB better than the unspread MIMO-OFDM system.

Figure 3 shows the BER for RNN for the unspread MIMO-OFDM system, MC-CDM and MC-CAFS with $B = 4$. Similar results as for the SCE can be observed with the RNN equalizer. MC-CAFS results in the best BER performance especially at very high E_b/N_o where the MC-CDM and the unspread MIMO-OFDM experience an error flow at BER $= 5 \times 10^{-3}$ and 1.5×10^{-2} respectively. The RNN is more sensitive to interference which can lead to such behavior.

As expected, the BER performance of the RNN equalizer is worse than that of the SCE. The SCE leads to better BER than the RNN (more than 2.5 dB at BER $= 10^{-3}$ for MC-CAFS) and shows no error flow. It is thus more suitable for channels where intereference is high. However, as mentioned earlier, this comes at the cost of higher complexity.

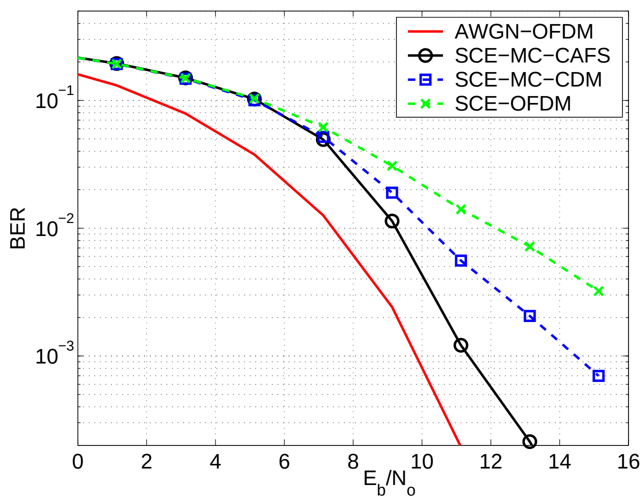

Fig. 2. SCE BER for time varying MIMO-OFDM, MC-CDM and MC-CAFS ($B = 4$)

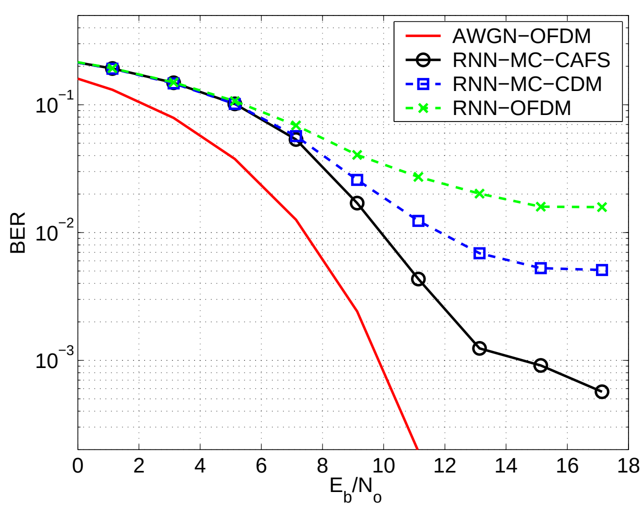

Fig. 3. RNN BER for time varying MIMO-OFDM, MC-CDM and MC-CAFS ($B = 4$)

V. Conclusions and Outlook

We have looked at the MIMO-OFDM discrete channel model and introduced a new family of spreading matrices, MC-CAFS, that allow for equal energy distribution of the transmitted symbols. The performance of MC-CAFS was compared with that of the unspread MIMO-OFDM and MC-CDM using two iterative equalizers, RNN and SCE. The results have shown that the MC-CAFS leads to the lowest BER. The SCE was shown to outperform the RNN, however at the cost of increased complexity. For future work, the performance of MC-CAFS shall be tested for different values of B. For although, for all values of B satisfying eqn.20, the diagonal elements of $\underline{\underline{R}}_{\mathrm{S}}$ are equal, the off diagonal elements, $r_{\mathrm{off},k}$, have different distributions and peak values for the different B values (i. e. the eigenvectors are different). We shall also look at MIMO-OFDM systems where $n_{\mathrm{R}} > n_{\mathrm{T}}$. In [6], it was shown that by using more receive than transmit antennas, the BER performance is improved. This also expected for the spreading proposed here. In addition, we shall look at coded transmission with iterative detection and decoding at the receiver. Previous simulations ([5], [6]) have shown that, MC-CAFS with $B = 1$ outperforms MC-CDM and the unspread system in case of coded transmission. Similar results are to be expect for $B > 1$.

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References

[1] I. Telatar, “Capacity of multi-antenna gaussian channels,” *ATT Bell Laboratoires*, 1995.

[2] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multiple antennas,” *Bell Labs Tech. J.*, pp. 41–59, Autumn 1996.

[3] H. Bölcskei and A. Paulraj, “SPACE-FREQUENCY CODED BROADBAND OFDM SYSTEMS,” in *IEEE WCNC 2000*, (Chicago (IL)), Sept. 2000.

[4] M. A. Dangl, D. Yacoub, U. Marxmeier, W. G. Teich, and J. Lindner, “Performance of Joint Detection Techniques for Coded MIMO-OFDM and MIMO-MC-CDM,” in *Proc. COST 273 Workshop on Broadband Wireless Local Access*, (Paris/France), pp. 17/1–17/6, May 2003.

[5] D. Yacoub, M. A. Dangl, U. Marxmeier, W. G. Teich, and J. Lindner, “Comparison of Iterative Detection Schemes for MIMO Systems with Spreading based on OFDM,” in *4th International Workshop on Multi-Carrier Spread Spectrum (MC-SS)*, (Oberpfaffenhofen / Germany), pp. 101–111, Sept. 2003.

[6] D. Y. Yacoub, E. Msechu, W. G. Teich, and J. Lindner, “Effect of Spreading in an Outdoor MIMO-OFDM System,” in *9th International OFDM-Workshop*, (, Dresden/ Germany), Sept. 2004.

[7] W. G. Teich and M. Seidl, “Code division multiple access communications: Multiuser detection based on a recurrent neural network structure,” in *Proc. IEEE ISSSTA*, vol. 3, (Mainz/Germany), pp. 979–984, Sept. 1996.

[8] A. Engelhart, W. G. Teich, J. Lindner, G. Jeney, S. Imre, and L. Pap, “A Survey of Multiuser/Multisubchannel Detection Schemes Based on Recurrent Neural Networks,” *Wireless Communications and Mobile Computing*, vol. 2, pp. 269–284, May 2002.

[9] J. Egle, C. Sgraja, and J. Lindner, “Iterative soft cholesky block decision feedback equalizer – a promising approach to combat interference,” in *Proc. IEEE VTC*, vol. 3, (Rhodes/Greece), pp. 1604–1608, May 2001.

[10] J. Lindner, “MC-CDMA in the context of general multiuser/multisubchannel transmission methods,” *European Trans. Telecomm.*, vol. 10, pp. 351–367, July/Aug. 1999.

[11] J. Lindner, *Informationsübertragung: Grundlagen der Kommunikationstechnik.* Springer, first ed., 2004.