

Sensitivity of the Orthogonalization Methods for QO-STBC to Feedback Errors in an OFDM Environment



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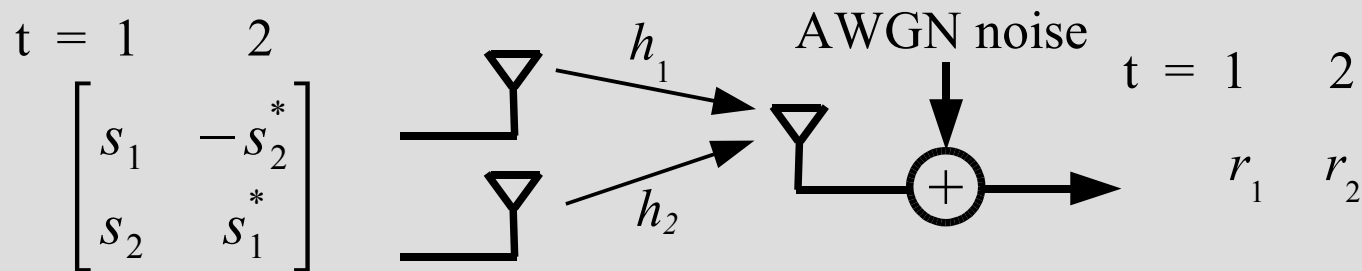
Outline

- Orthogonal STBCs,
- Quasi-Orthogonal STBCs,
- Orthogonalization of QO-STBCs,
- Extension to OFDM,
- Simulations and Results,
- Conclusions.



Orthogonal Space Time Block Codes

- Exploit spatial diversity with no Tx-CSI,
- Full diversity order, n_T , but code rate < 1 for more than 2 Tx ant.s ,
- Alamouti's STBC for 2 Tx antennas:



$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \quad \longrightarrow \quad \mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$$



ML Detection for O-STBCs

- For AWGN channel ML detection for a given channel $\{h_i\}$ is

$$\hat{s} = \underset{\forall s}{\operatorname{arg\,min}} |r - Hs|^2$$

- Equivalently, after matched filtering:

$$\hat{s} = H^H r = \underbrace{H^H H}_{\gamma} s + H^H n$$

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

$$\gamma = h_1^2 + h_2^2$$

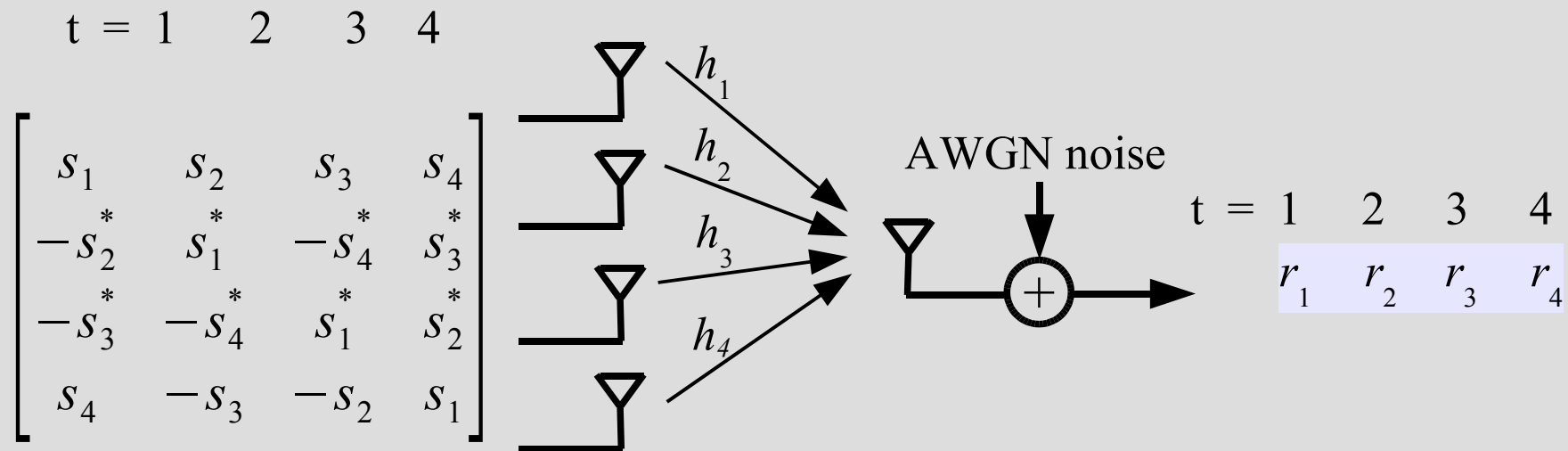
- $\hat{s}_i = \gamma s_i + \tilde{n}_i$: no coupling between symbols, symbol-by-symbol detection is optimum in the ML sense,

- $SNR = \frac{\gamma^2}{\gamma} = h_1^2 + h_2^2$: diversity order four, full diversity order.



Quasi-Orthogonal STBCs

- Extension of Alamouti's code to four Tx antennas



$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_3^* & -h_4^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \rightarrow \mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$$



ML Detection for QO-STBCs

- Matched filtering:

$$\hat{\mathbf{s}} = \mathbf{H}^H \mathbf{r} = \underbrace{\mathbf{H}^H \mathbf{H}}_{\Gamma} \mathbf{s} + \mathbf{H}^H \mathbf{n}$$

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \alpha \\ 0 & \gamma & -\alpha & 0 \\ 0 & -\alpha & \gamma & 0 \\ \alpha & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix}$$

$$\gamma = h_1^2 + h_2^2 + h_3^2 + h_4^2$$

$$\alpha = 2 \Re \{ h_1 h_4^* - h_2 h_3^* \}$$

- for ex., $\hat{s}_1 = \gamma s_1 + \alpha s_4 + \tilde{n}_1$: coupling between symbols, symbol-by-symbol detection is no longer optimum in the ML sense,
- $SNR = \frac{\gamma^2 - \alpha^2}{\gamma}$: diversity order can be shown to be two, loss in diversity order,
- Full code rate, i.e., four symbols transmitted in four time slots.



Orthogonalization of QO-STBC

- The coupling term α must be made zero for full diversity, i.e.,

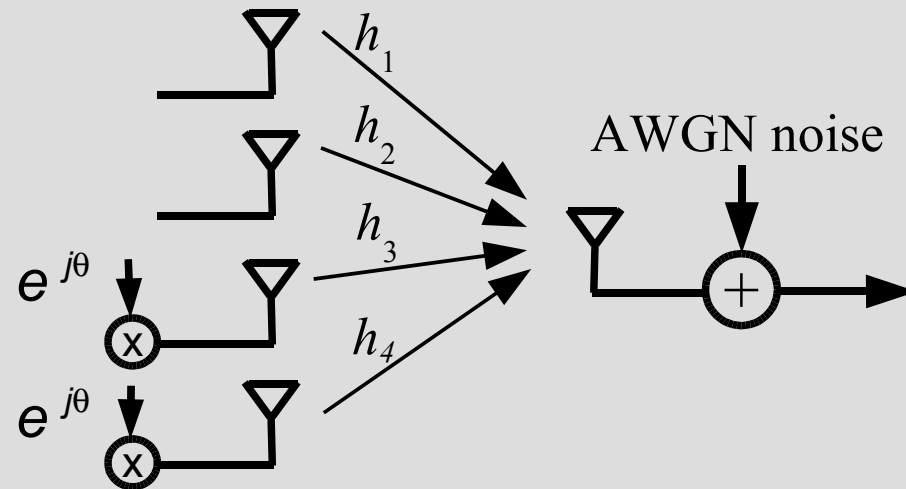
$$\alpha = 2 \Re \{ h_1 h_4^* - h_2 h_3^* \}$$

- We propose two methods based on (partial) Tx-CSI for this purpose:
 - transmitted signal phase rotation,
 - transmit antenna selection.



Phase rotation algorithm

- Multiply the signals transmitted from the third and the fourth antennas by a phasor over four time slots:



- This is equivalent to multiplying the corresponding channel coef.s:

$$\alpha' = 2 \Re \{ (h_1 h_4^* - h_2 h_3^*) e^{-j\theta} \}$$

- Let $\Xi = h_1 h_4^* - h_2 h_3^*$, then, if $\theta = \pi/2 - \text{angle}\{\Xi\}$, then $\alpha' = 0$ and coupling is eliminated,
- $\theta \in [-\pi/2, \pi/2]$, quantization is required in a practical application.



Phase rotation algorithm

- For quantization set θ as

$$\theta = \arg \min_{\theta \in \Omega} \Re \{ \Xi e^{-j\theta} \}$$

where $\Omega = \left\{ \frac{\pm(2n-1)\pi}{2^{K+1}}, n=1, 2, \dots, 2^{K-1} \right\}$.

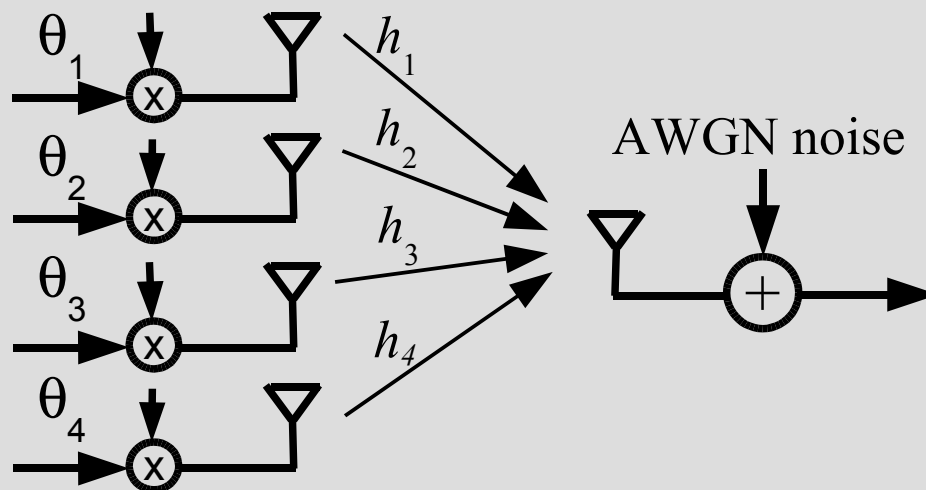
- For example for a one bit feedback, i.e. $K=1$, the set of angles is

$$\Omega = \{ \pm\pi/4 \}$$



Antenna selection algorithm

- Multiply the signals transmitted from all antennas with coef.s θ_i :



- Assign the coefficients according to channel qualities

$$|h_1|^2 \geq |h_4|^2 \rightarrow \theta_1 = \sqrt{2}, \theta_4 = 0$$

$$|h_1|^2 < |h_4|^2 \rightarrow \theta_1 = 0, \theta_4 = \sqrt{2}$$

$$|h_2|^2 \geq |h_3|^2 \rightarrow \theta_2 = \sqrt{2}, \theta_3 = 0$$

$$|h_2|^2 < |h_3|^2 \rightarrow \theta_2 = 0, \theta_3 = \sqrt{2}$$

Total transmit power is unaffected.

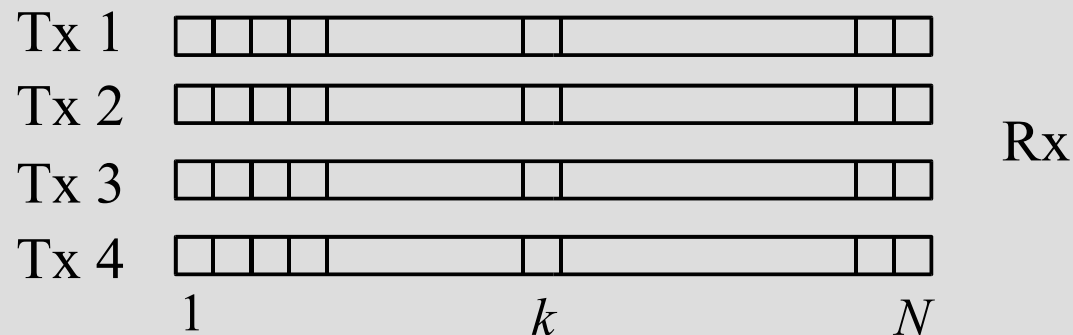
Two bit feedback is required.

$$\alpha' = 2 \Re \{ h_1 h_4^* \theta_1 \theta_4 - h_2 h_3^* \theta_2 \theta_3 \}$$



Extension to OFDM

- With a proper cyclic prefix OFDM transforms frequency-selective fading channels to flat-fading subchannels.
- For all four subchannels from the transmit antennas to the receive antennas, let there be N frequency bins,



- Since each frequency bin experiences flat-fading the explained algorithms directly applies to these bins separately,
- The coupling term for the k -th frequency bin is

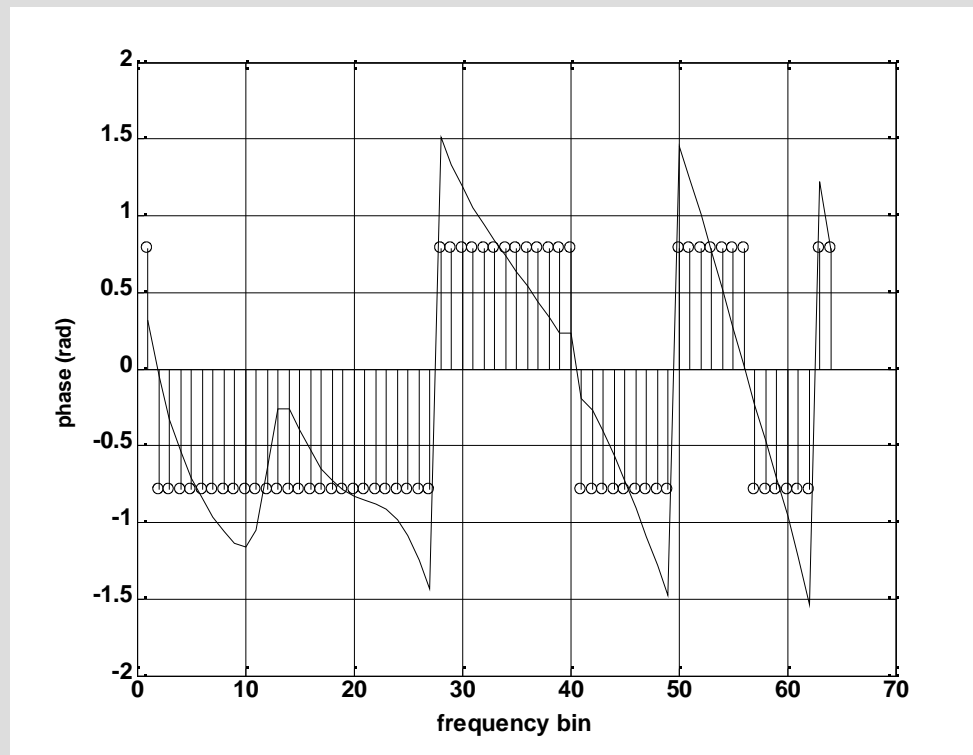
$$\alpha_k = 2 \Re \{ \lambda_{1,k} \lambda_{4,k}^* - \lambda_{2,k} \lambda_{3,k}^* \}$$

- N feedback channels are needed for the orthogonalization of each frequency bin, which may not be feasible.



Reducing feedback overhead

- Observing that the feedback information for adjacent frequency bins are correlated, i.e.

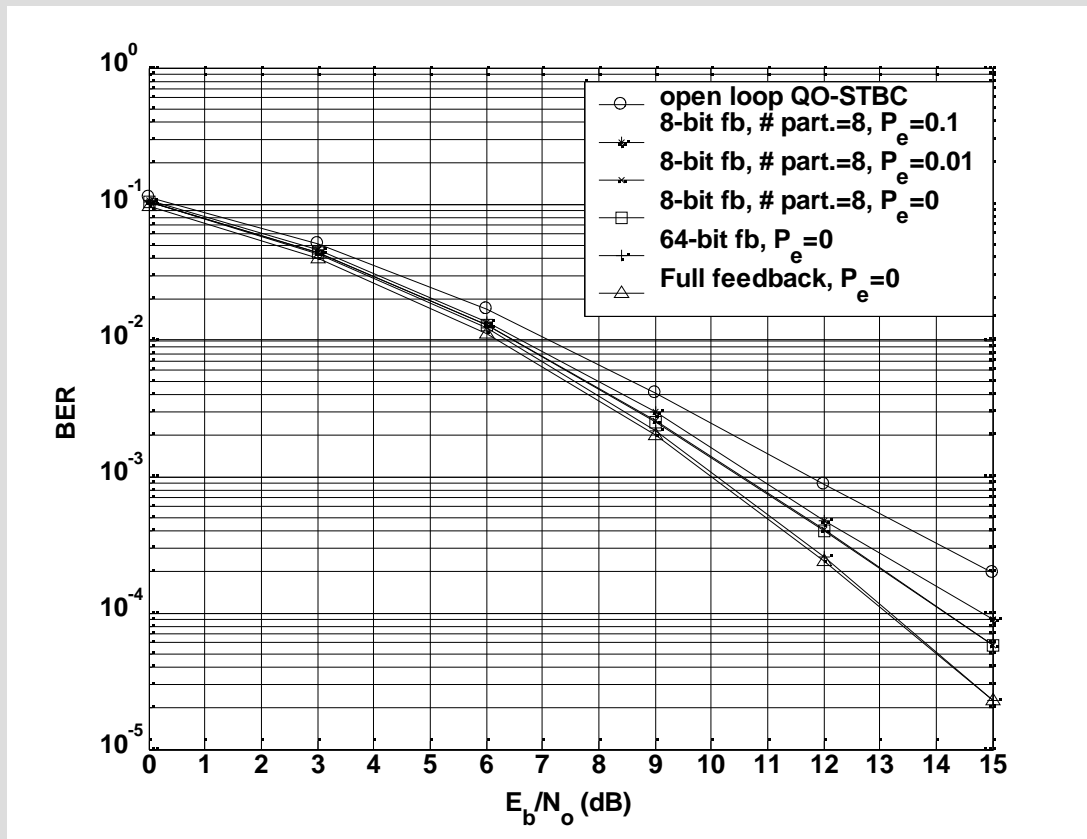


Full precision (solid line)
and quantized (stem)
phase feedback.

we propose to group feedback bits according to majority voting.
For this example, 64 bins are grouped into 8 groups with 8 bins each.



Results (Phase feedback)



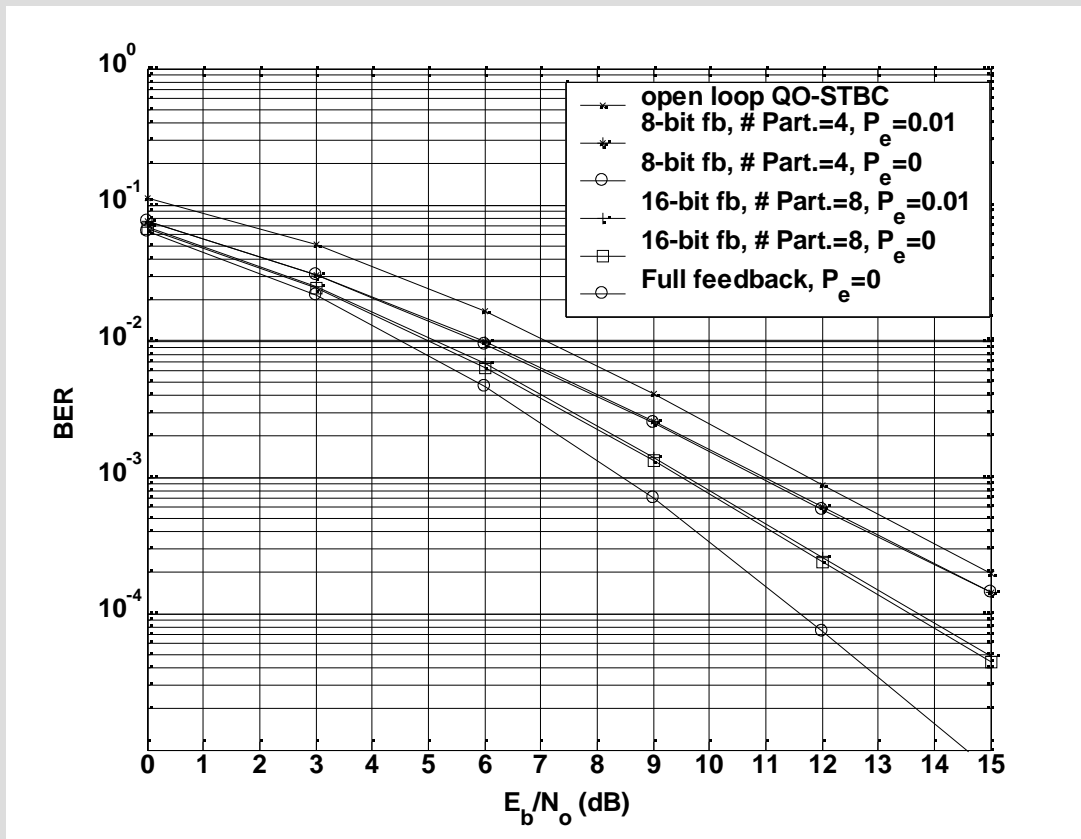
- Full precision phase feedback is compared with quantized and partitioned feedback with/without feedback errors.

- 64 frequency bins, QPSK
4x1 MISO channel,
Channel Length=5 taps,
AWGN, ML detection

- Algorithm is very robust against feedback errors,
Reasonable loss when partitioned.



Results (Antenna Selection)



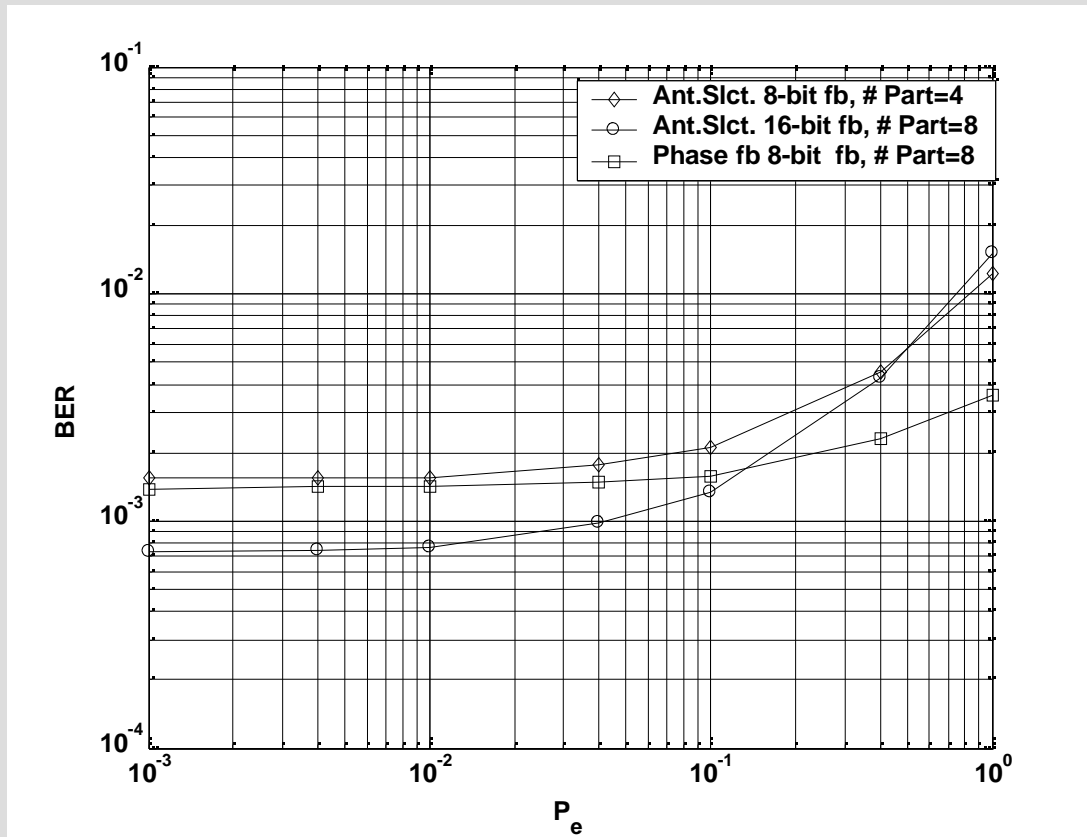
- Antenna selection with full feedback is compared with partitioned feedback with/without feedback errors.

- 64 frequency bins, QPSK
4x1 MISO channel,
Channel Length=5 taps,
AWGN, ML detection

- Algorithm is very robust against feedback errors,
Better performance than phase feedback for good feedback channels,
More vulnerable to partitioning than phase feedback.



Results



Both algorithms are almost unaffected for feedback error rate $\leq 10^{-2}$,

Antenna selection algorithm is more sensitive to partitioning compared to phase feedback algorithm.



Conclusions

- Performance of two partial Tx-CSI based algorithms for the orthogonalization of QO-STBC is investigated.
- Both the phase feedback and the antenna selection algorithms are very robust to quantized feedback and feedback errors.
- Feedback overhead is an important issue for OFDM systems. Both algorithms require separate feedback for each frequency bin, which can be unfeasible.
- Correlation between adjacent bins is exploited to reduce the amount of feedback by partitioning the frequency range.
- Phase feedback algorithm is found to be robust to partitioning also, whereas antenna selection algorithm worsens for higher degree of partitioning.
- The antenna selection algorithm performs better for high amount and high quality of feedback, whereas phase feedback algorithm is more robust to relatively bad channels with limited amount of feedback.