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Using Frequency-Offset Estimation Schemes for Acquisition

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Outline:

- Acquisition in Systems with Large Frequency Offsets
- Comparison of Acquisition and Frequency Estimation Schemes
- Advanced Frequency Offset Estimators (Time Domain)
- Simulation Results
- Conclusions and Outlook

System Model

- No data modulation considered
- Received samples $r(k)$ given by:

$$r(k) = \sqrt{E_s} e^{j(2\pi \Delta f k T + \Phi)} + n(k), \quad 0 \leq k \leq L-1$$

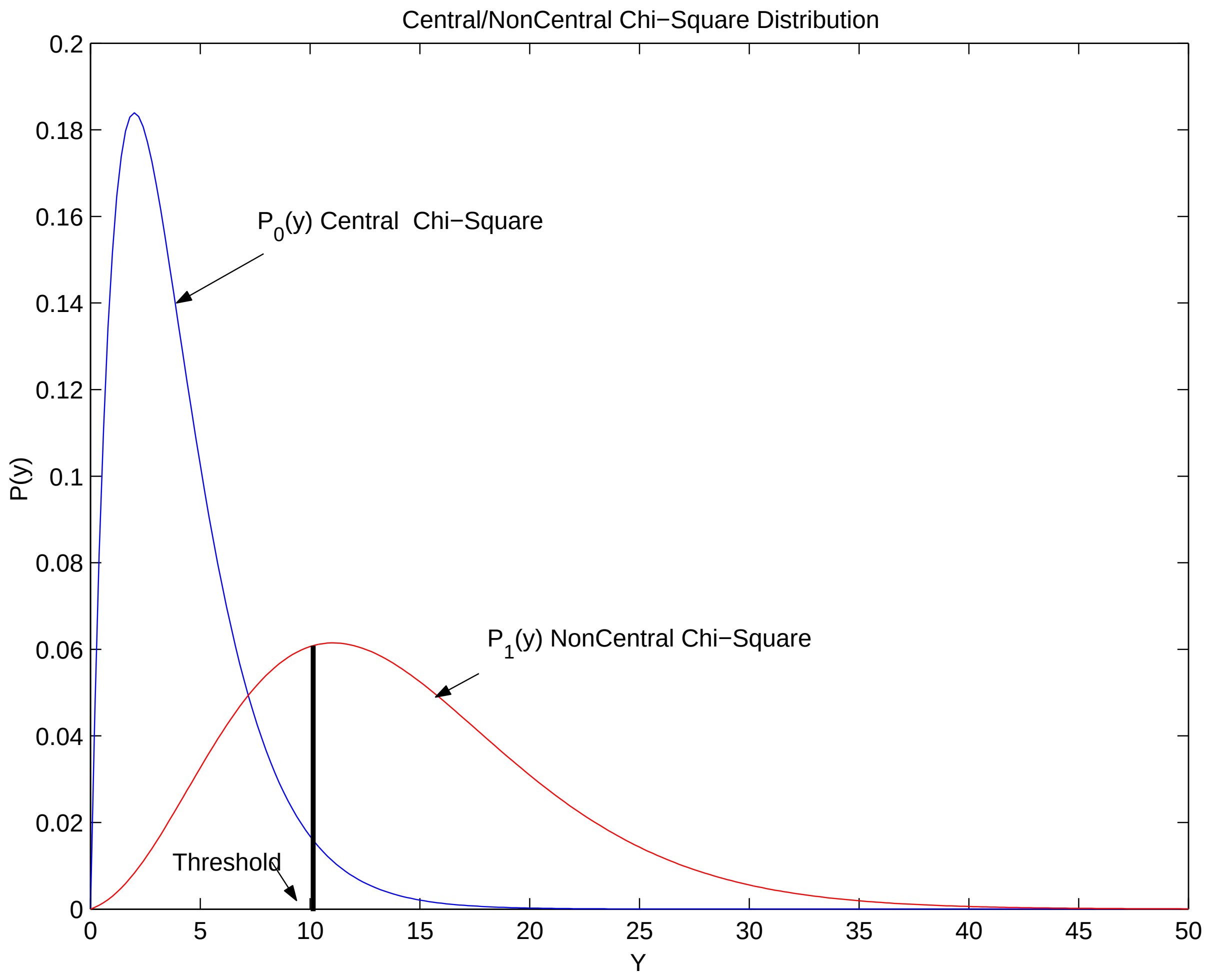
  - k: time index
  - L: length of observed sequence
  - E_s: symbol energy
  - T: symbol interval
  - f, ϕ: deterministic, unknown, frequency- and phase-offset
  - $n(k)$: additive white Gaussian noise

Acquisition - No Frequency Offsets

- Form Decision variable as: $Z_C = |\sum_{k=0}^{L-1} r(k)|^2$
- Compare Z to threshold θ:
  - If $Z < \theta$ declare preamble not present
  - If $Z > \theta$ declare preamble present
    - If preamble present, successful detection ($\rightarrow P_D$)
    - If preamble not present, false alarm ($\rightarrow P_{FA}$)
- Employ power-scaled threshold, which yields a constant false alarm rate P_{FA}:

$$\theta = -(\sum_{k=0}^{L-1} |r(k)|^2)\ln(P_{FA})$$

Acquisition - Probability Density Functions



- Example for resulting probability density functions

Acquisition - Effect of Frequency Offsets

- Frequency Offset Δf causes degradation $D(\Delta f)$ for sequence of length L:

$$D(\Delta f) = \left(\frac{\sin(\pi L \Delta f T)}{\pi L \Delta f T} \right)^2$$

- Use frequency dependant decision variable:

$$Z(f) = \left| \sum_{k=0}^{L-1} r(k) \exp\left(-j 2\pi k f T\right) \right|^2$$

- Possible implementation via FFT, which evaluates $Z(\frac{m}{LT})\,,\; 0 \leq m < L$

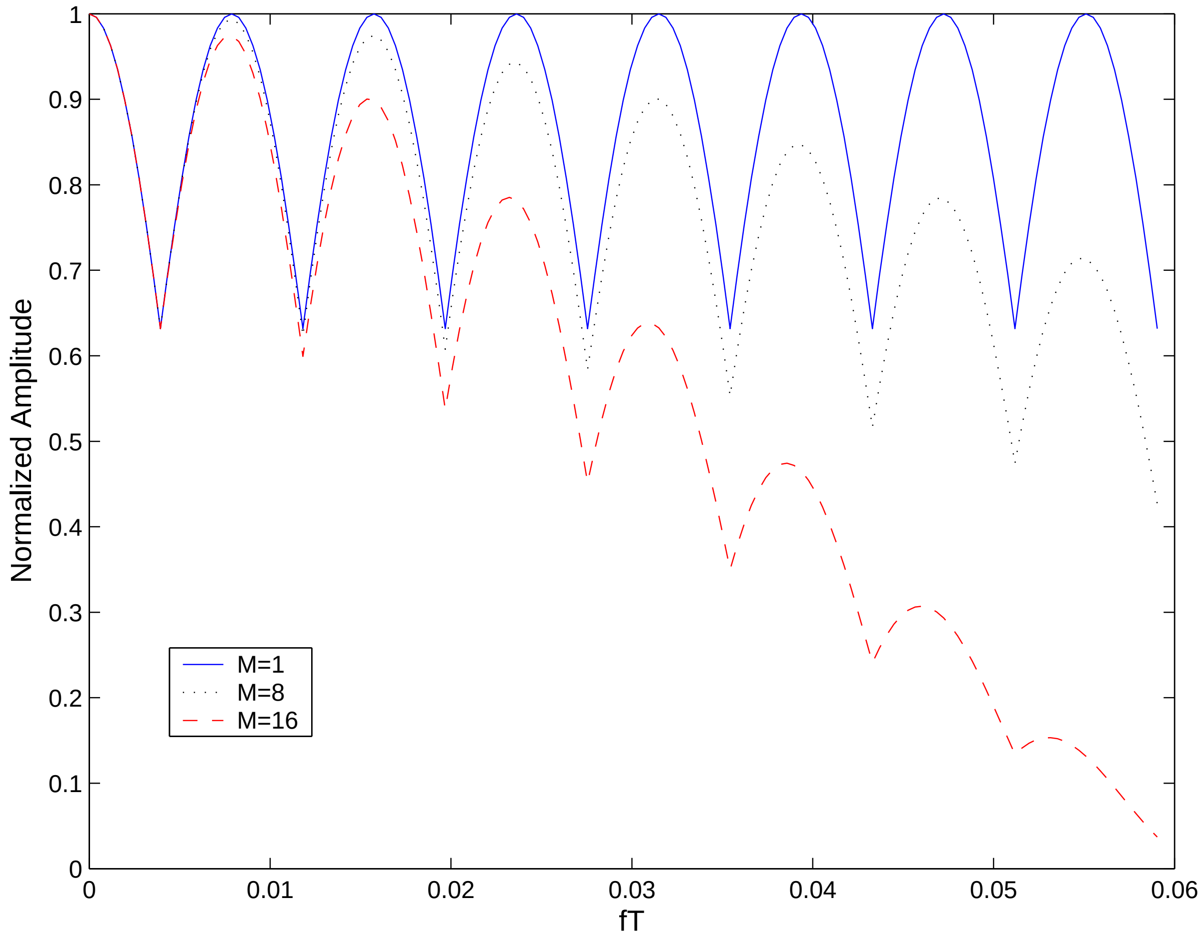
Acquisition - Coherent Sample Combining

- In most applications frequency offset small compared to the signal bandwidth $\Rightarrow$ phase rotation between succeeding samples is small
- Small frequency offsets allow to perform sample combining in original sequence, to obtain a new reduced length sequence $r_c(l)$:

$$r_c(l) = \sum_{k=0}^{M-1} r(k + lM)$$

- Reduced sequence length means that a reduced length FFT is evaluated

Acquisition - Effect of Coherent Sample Combining



- Signal degradation due to offset relative to bins and increasing combining rate M.

Acquisition - Post Detection Integration (PDI)

- Non-coherent Post Detection Integration

$$Z_{NC} = \sum_{k=0}^{\frac{L}{M}-1} |r_c(k)|^2$$

- Differential Post Detection Integration - Absolute

$$Z_{DA} = |\sum_{k=1}^{\frac{L}{M}-1} r_c(k) r_c^*(k-1)|^2$$

- Differential Post Detection Integration - Real

$$Z_{DR} = Re\{\sum_{k=1}^{\frac{L}{M}-1} r_c(k) r_c^*(k-1)\}$$

Frequency Offset Estimation in Time Domain(1)

- The Kay-Estimator:
  Idea is to evaluate phase rotation between succeeding samples:

$$\Delta\hat{f} = \arg\{\sum_{k=1}^{L-1} w(k)r(k)r^*(k-1)\}$$

- Alternatively, could evaluate the sum over the phase values
- Many advanced estimators exist that are based on the same principle
- Many different FFT-based estimators exist

Frequency Offset Estimation in Time Domain (2)

- Idea of advanced schemes is to evaluate phase rotation between samples of several delays:

$$R(l) = \frac{1}{L-l} \sum_{k=l}^{L-1} r(k) r^*(k-l)$$

- Combine several $R(l)$ to generate estimate
- Alternatively, the difference between phase angles could be evaluated

Frequency Offset Estimation in Time Domain (3)

- M&M-Estimator:
  Exploits several sample correlation values, but maintains estimation range by evaluating phase-differences between sample correlations:

$$R_1(l) = R(l)R^*(l-1)$$

The decision variable is then obtained by:

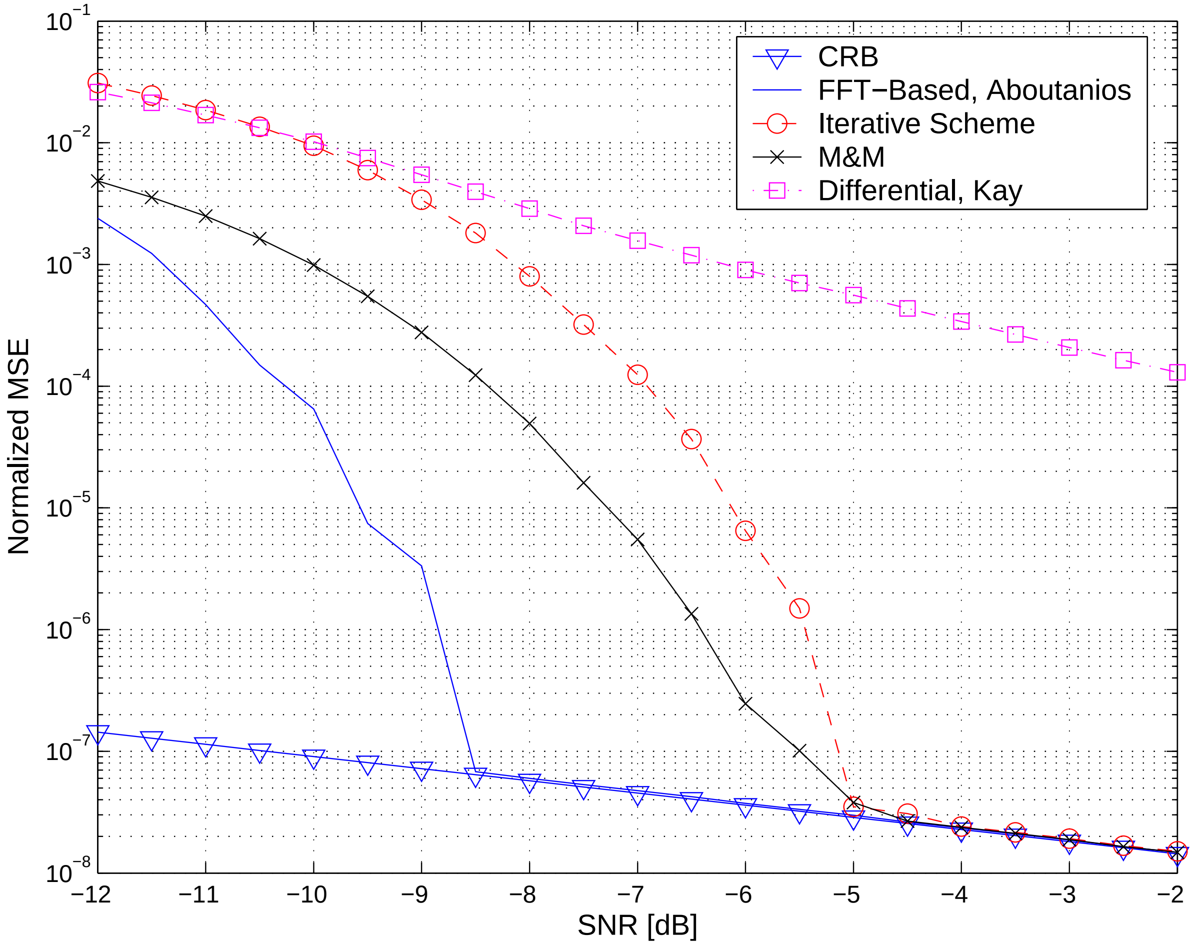
$$Z_{MM} = \left| \sum_{l=1}^{N} \beta(l) R_1(l) \right|^2$$

where $\beta(l)$ are real coefficients that sum to unity

Iterative Frequency Offset Estimation

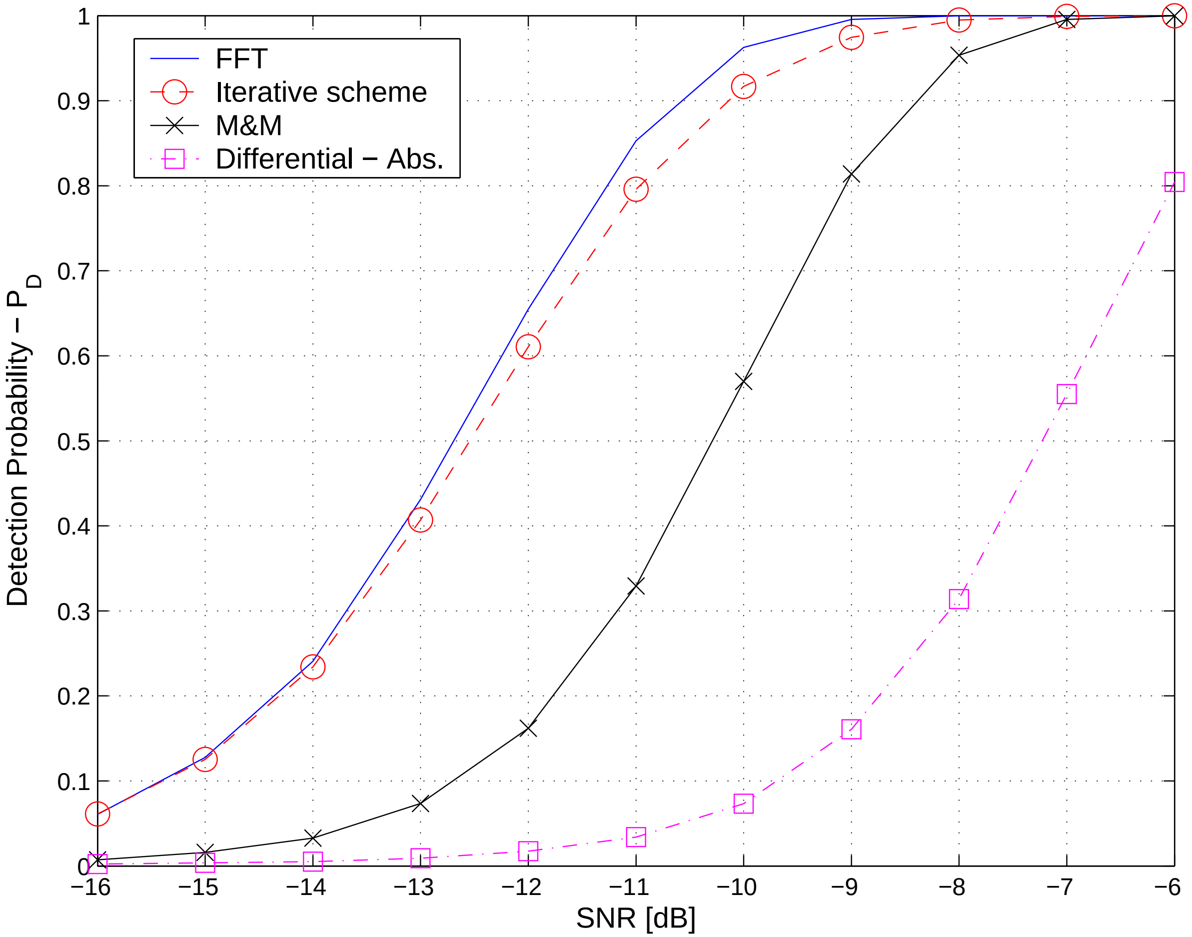
- Iterative Estimation:
  1. Estimate the frequency offset without (or with little) combining.
  2. Correct the frequency offset in the sequence using the current estimate from step 1).
  3. Combine a number of samples to obtain a new sequence.
  4. Continue with 1).
- To use the scheme for acquisition, just combine all remaining elements in the last iteration $\Rightarrow$ Same decision variable as in classical acquisition theory

MSE of Different Frequency Offset Estimators



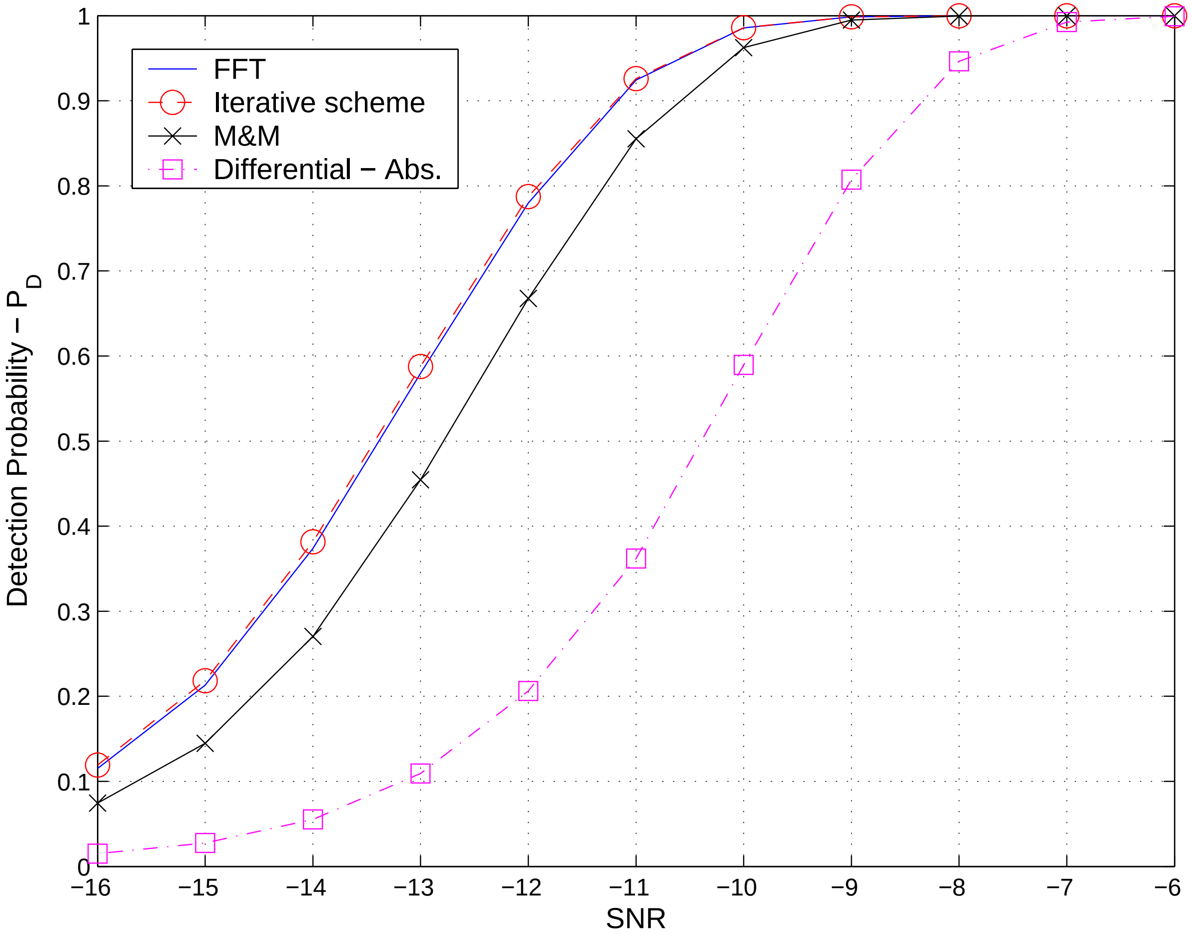
- Sequence length $L = 256$, $N = 128$ for the M&M-estimator
- Estimators have very different SNR-thresholds

Detection Probabilities without Combining



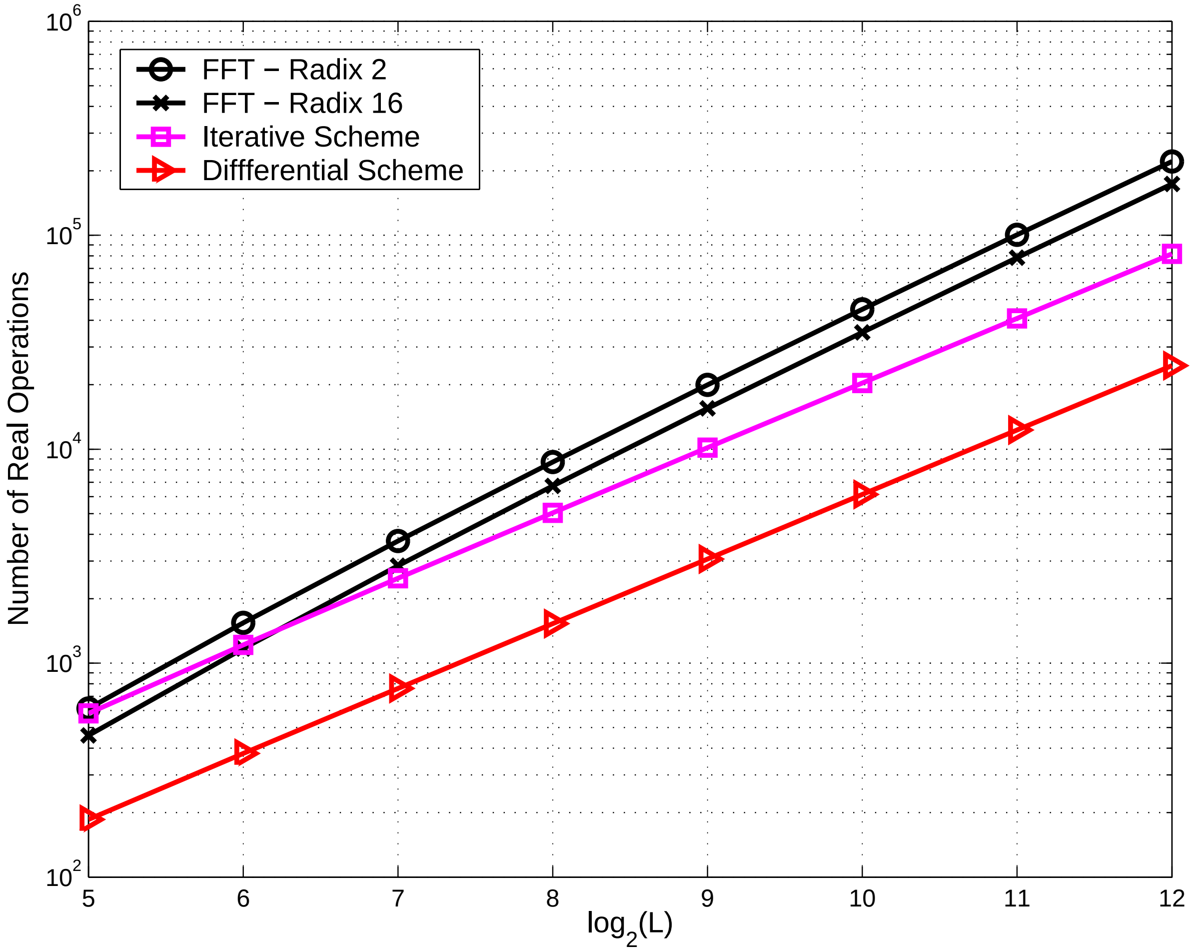
- Sequence length $L = 256$, False alarm probability $P_{FA} = 10^{-3}$, $N = 10$ for the M&M-estimator

Detection Probabilities with Combining



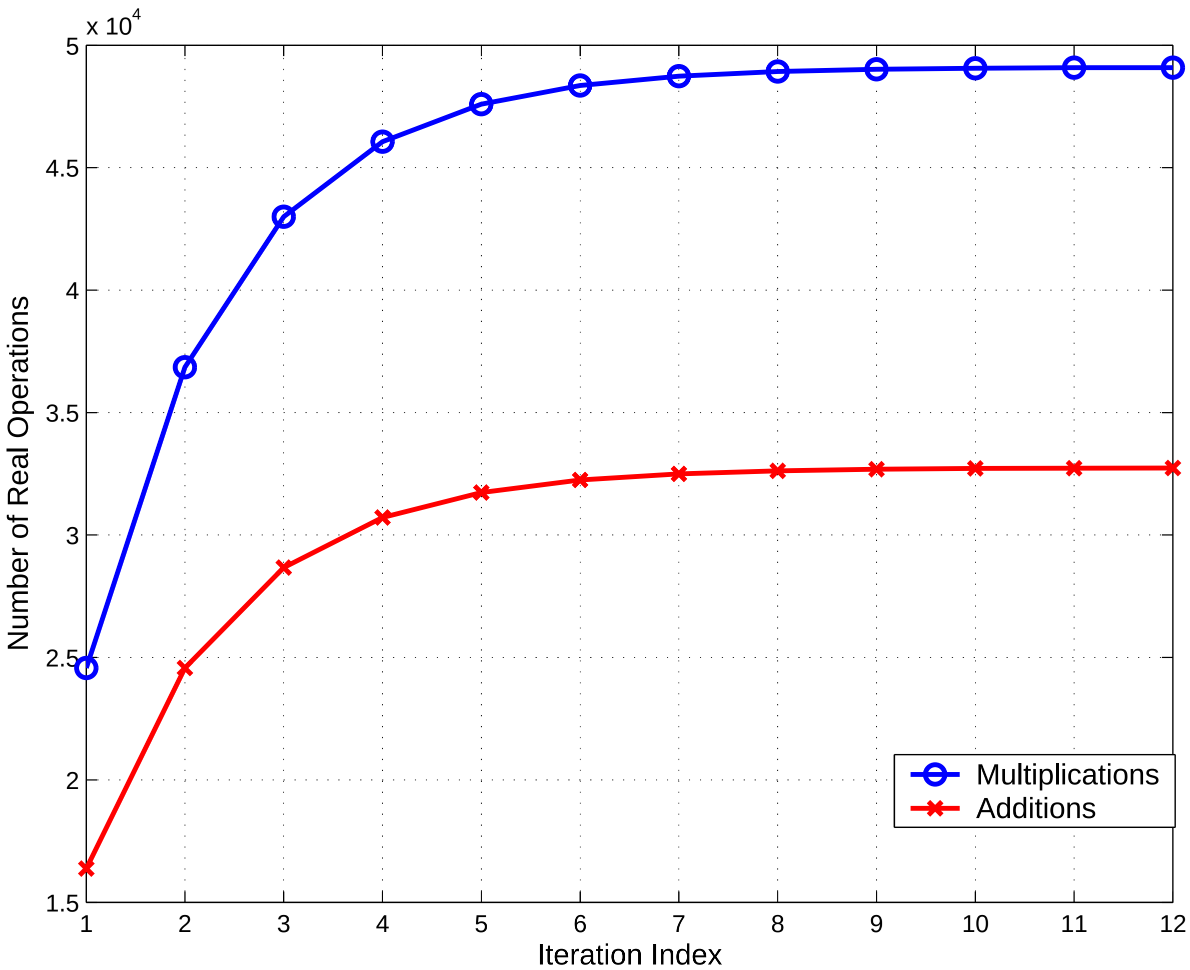
- Sequence length $L = 256$, False alarm probability $P_{FA} = 10^{-3}$, Combining factor $M = 4$, $N = 10$ for the M&M-estimator

Computational Complexity of Different Schemes



- Computational complexity for different sequence lengths L
- Non-Padded FFT without max-search

Computational Complexity of Iterative Scheme by Iteration



- Computational complexity for sequence length $L = 4096$
- Initial iterations account for most of the complexity

Summary and Conclusions

- Frequency estimation schemes can be applied to acquisition problems with small modifications
- Several schemes can close the huge performance gap between FFT-based acquisition and simple differential schemes
- Schemes that operate in time-domain have an inherent flexibility to trade off complexity for performance
- Time-domain schemes do not have a frequency-dependant performance
- Further optimization of iterative scheme possible
- Exact derivation of the detection thresholds still an open problem