Channel Spacing of Random Access Wideband Channels

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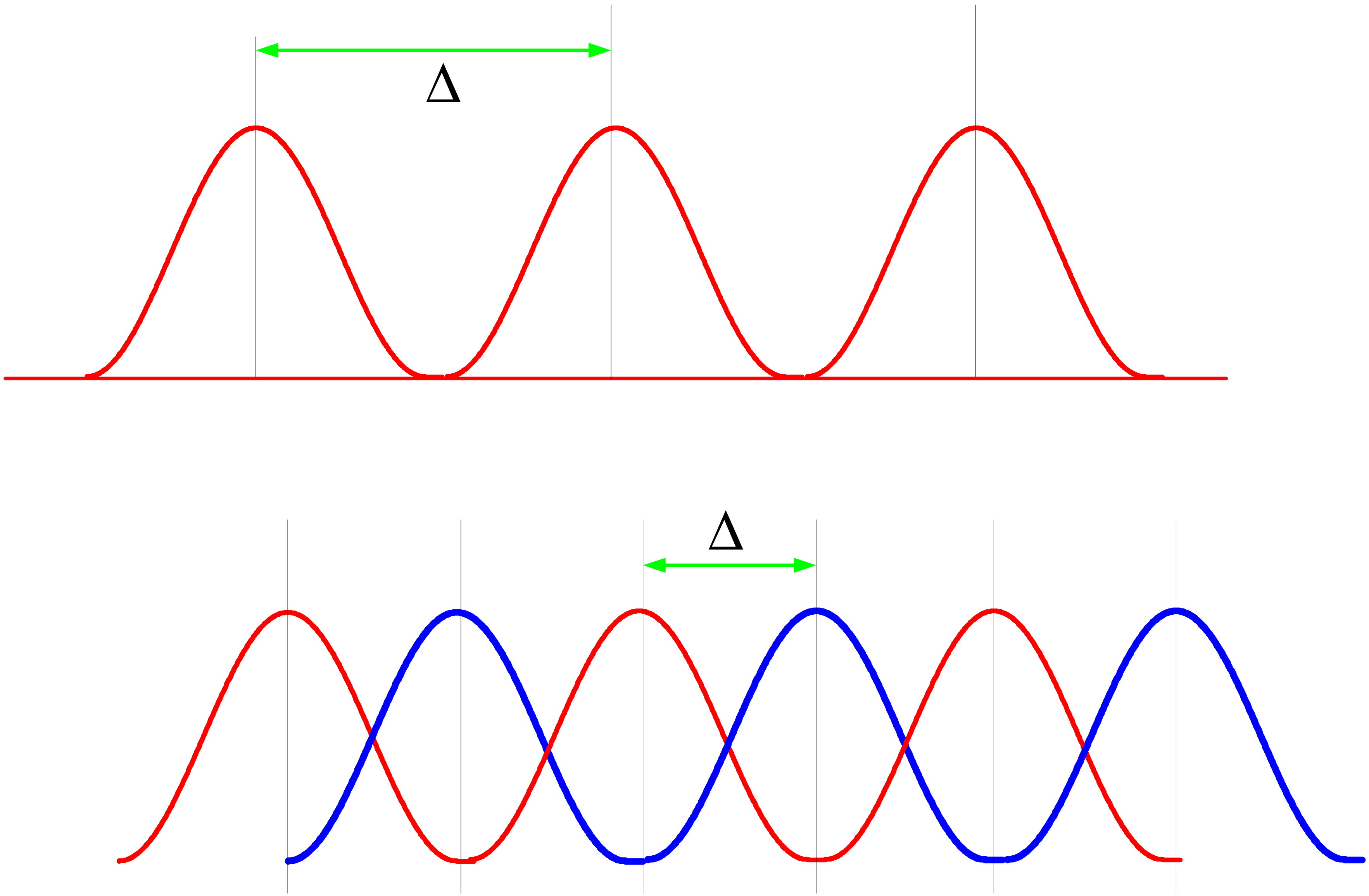
The Problem

- Given K users sharing a common wideband channel of bandwidth W

- The channel is divided into $N = \frac{W}{\Delta}$ subchannels, and each user (independently of other users) selects a subchannel randomly.

- What is the best selection of the **channel spacing Δ**???


Δ
Δ

The Problem

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- What is the best selection of the **channel spacing Δ**???
  - small $\Delta \rightarrow$ large $N \rightarrow$
    - reduce the probability that two users hop to the same subchannel
    - but increases the adjacent channel interference

The Model

- The kth signal is

$$y_k(t) = \text{Re}\left\{s_k(t)\, e^{j2\pi f_k t}\right\}$$

where $f_k \in \{F_0, F_0 + \Delta, F_0 + 2\Delta, ..., F_0 + (N-1)\Delta\}$ with equal probability

- $s_k(t)$are independent Gaussian signals, having identical power spectral density $S(f)$.

- The composite signal at the kth receiver

$$r_k(t) = x_k s_k(t)\, e^{j2\pi f_k t} + \sum_{i \neq k} x_i s_i(t)\, e^{j2\pi f_i t} + n(t)$$

where $n(t)$ is the AWGN signal having two sided power spectral density $N_0/2$

The Capacity

- Assuming NO coordination between users.

- The interference signal $\left\{\sum_{i\neq k} s_i(t)\, e^{j2\pi f_i t} + n(t)\right\}$ is conditionally Gaussian

- Then, for a given power spectral density $S(f)$, and $\{f_0, f_1, ..., f_K\}$

$$C(f_0, .., f_{K-1}) = \sum_{k=0}^{K-1} \int_{A_k} \log\left(1 + \frac{S(f - f_k)}{\sum_{n=0, n\neq k}^{K-1} S(f - f_n) + N_0/2}\right) df \quad \text{[bps]}$$

where A_k is the set of frequencies $S(f - f_k) \neq 0$.

Average Capacity

$$C = \sum_{k=0}^{K-1} \int_A \frac{1}{N^K} \sum_{f_0} \sum_{f_1} .. \sum_{f_{K-1}} \log\left(1 + \frac{S(f)}{\sum_{n=0,n\neq k}^{K-1} S(f+f_k-f_n)+N_0}\right)$$

Average Capacity

$$C = \sum_{k=0}^{K-1} \int_A \frac{1}{N^K} \sum_{f_0} \sum_{f_1} .. \sum_{f_{K-1}} \log \left(1 + \frac{S(f)}{\sum_{n=0,n\neq k}^{K-1} S(f+f_k-f_n)+N_0} \right)$$

Main contribution:

$$C = \sum_{k=0}^{K-1} \int_A \int_0^\infty \frac{1-e^{-zS(f)}}{z} e^{-zN_0/2} \left[1 - \frac{\Delta}{W} \sum_n \left(1 - e^{-zS(f+n\Delta)} \right) \right]^{K-1} dz.$$

Spectral Efficiency

- The spectral efficiency [bits/sec/Hz] of the shared spectrum is

$$
\begin{aligned}
\eta &= \lim_{\substack{K,W\to\infty \\ K/W=\lambda}} \frac{C}{W} \\
&= \lambda \int_A \int_0^\infty \left[1 - e^{-zS(f)}\right] e^{-\lambda\Delta \sum_n \left(1 - e^{-zS(f-n\Delta)}\right)} e^{-z\frac{N_0}{2}} \frac{dz}{z}
\end{aligned}
$$

Rayleigh Faded Channels

- $x_0, x_1, ..., x_{K-1}$ are independent exponential random variables

$$\mathsf{E}\left[\log\left(1+\frac{x_k S(f)}{\sum_{n=0,n\neq k}^{K-1} x_n\ S(f+f_k-f_n)+N_0/2}\right)\right]$$

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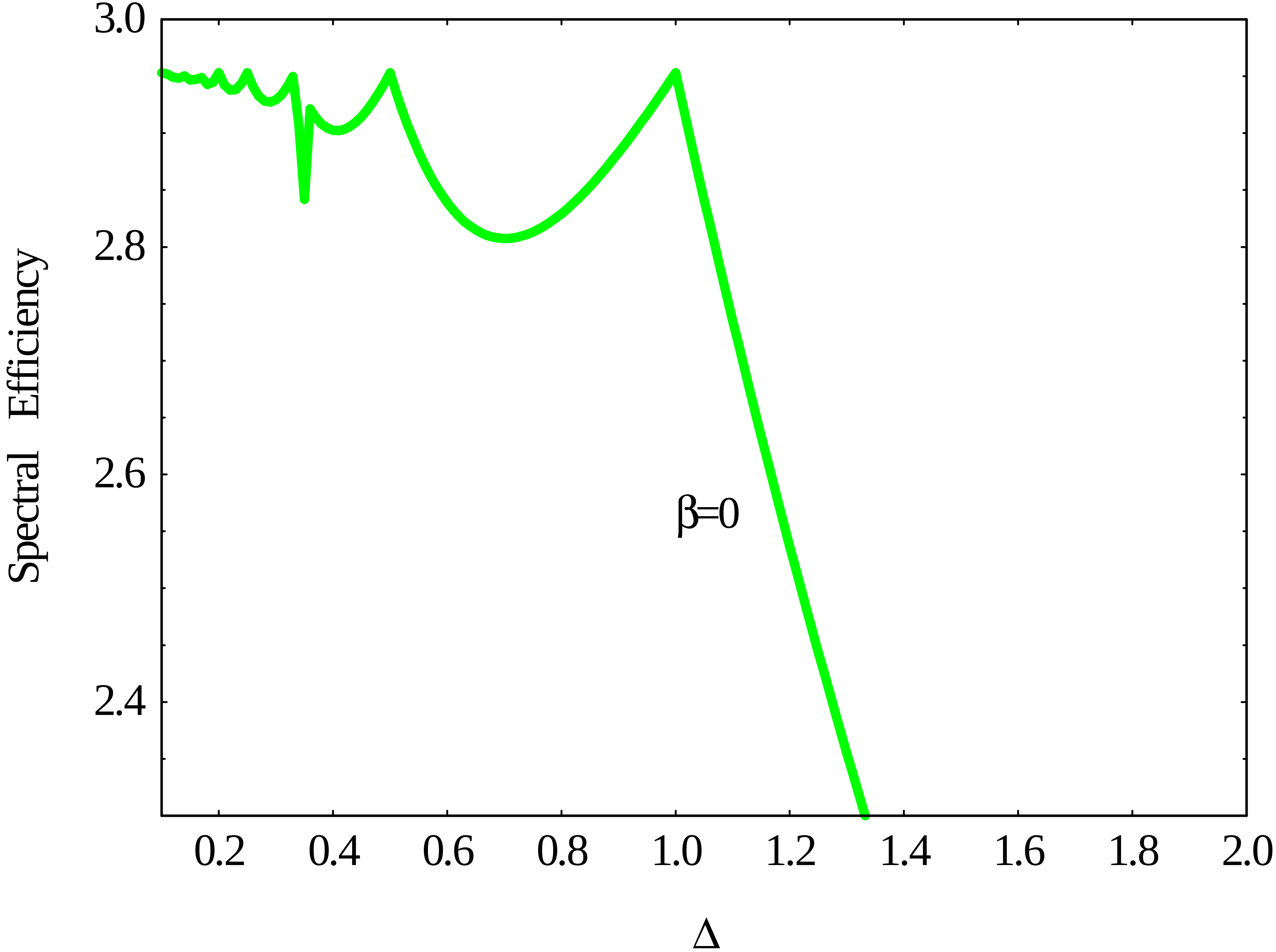
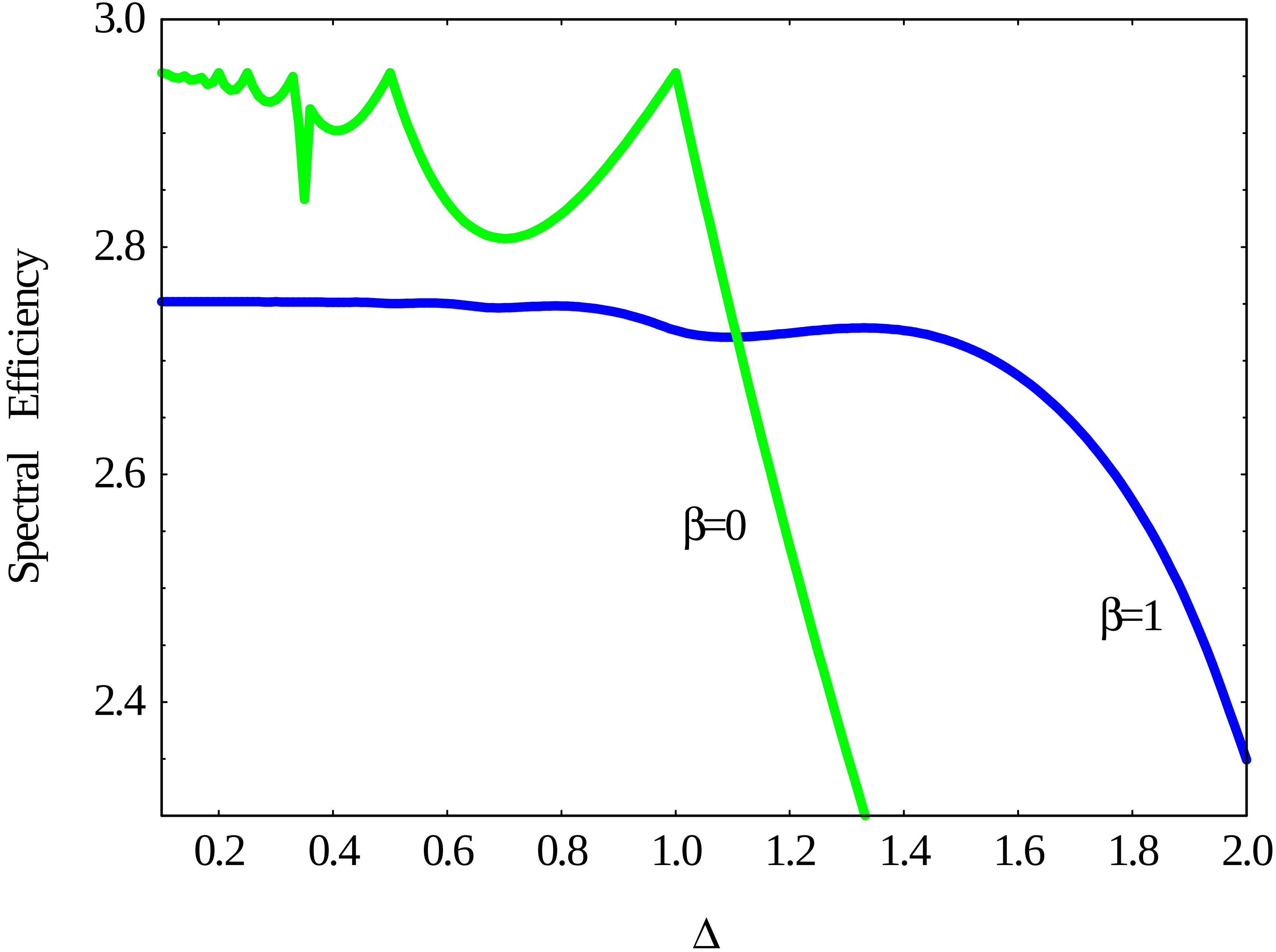
$$\mathsf{E}\left[\log\left(1+\frac{x_k S(f)}{\sum_{n=0,n\neq k}^{K-1} x_n \; S(f+f_k-f_n)+N_0/2}\right)\right]$$

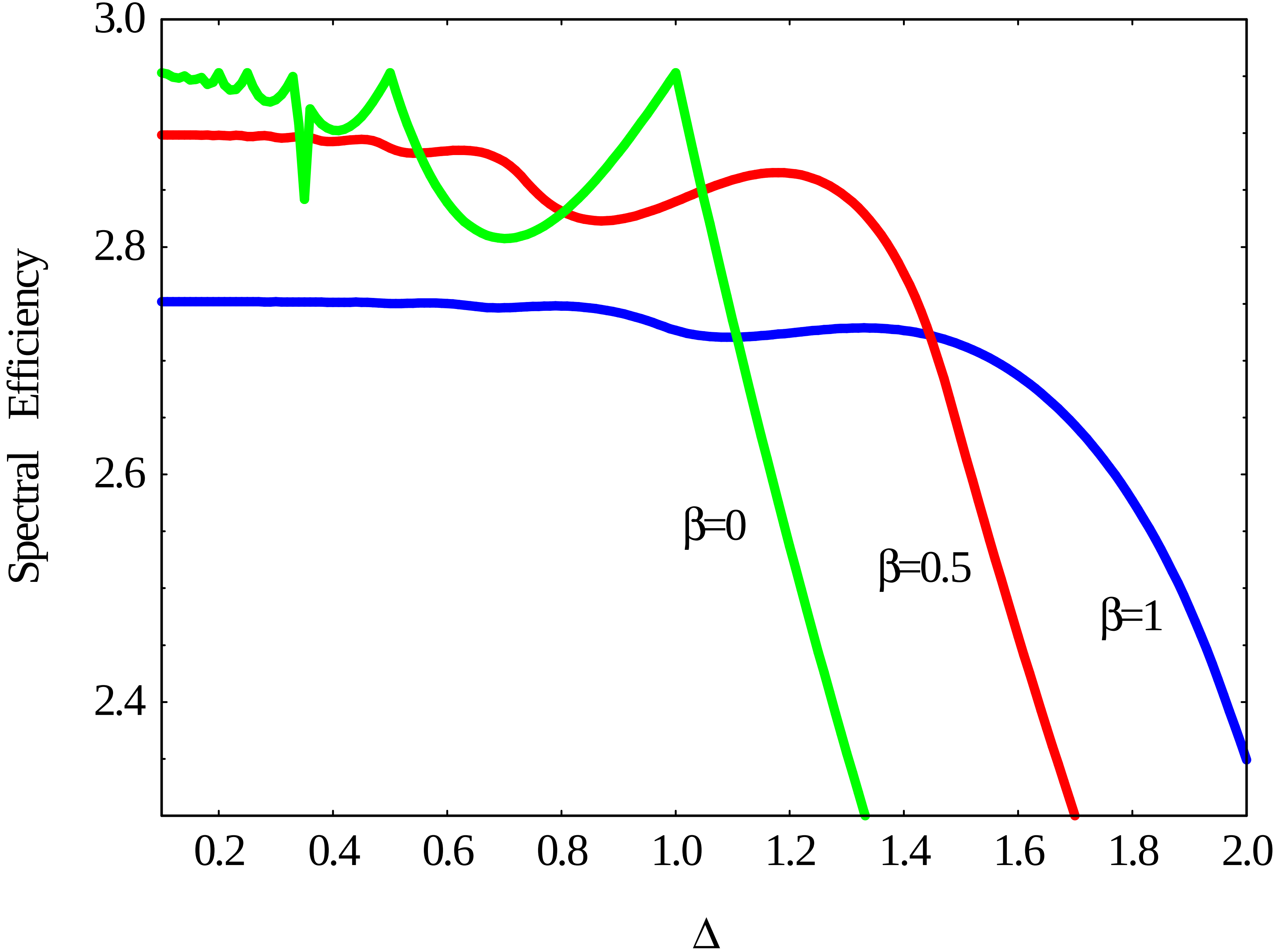
$$= \int_0^{\infty}\left(1-\frac{1}{1+zS(f)}\right)\left[1-\frac{\Delta}{W}\sum_n\left(1-\frac{1}{1+zS(f+n\Delta)}\right)\right]^{K-1} e^{-z\frac{N_0}{2}}\frac{dz}{z}.$$

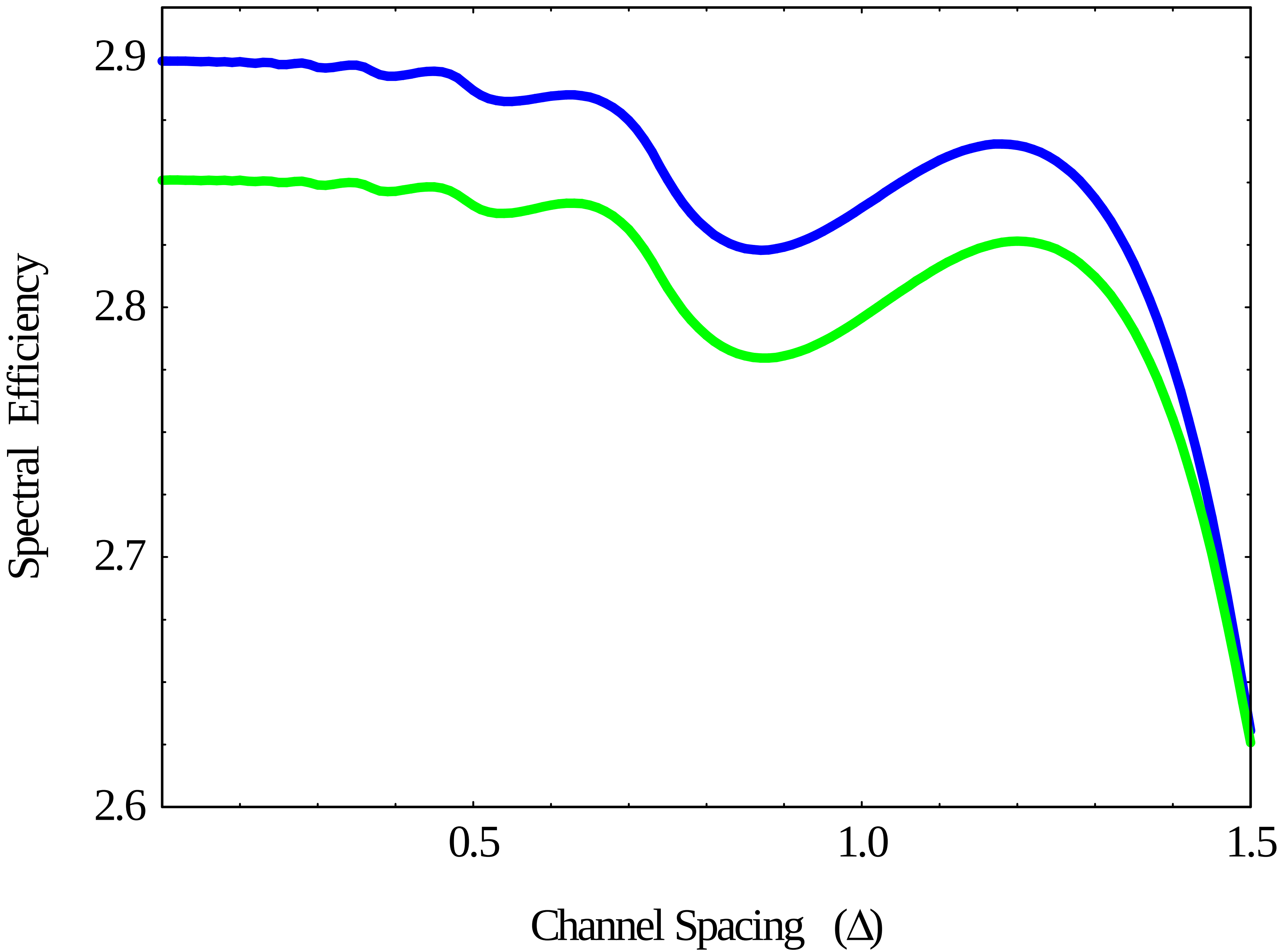


3.0
2.8
2.6
2.4
Spectral Efficiency
β=0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Δ

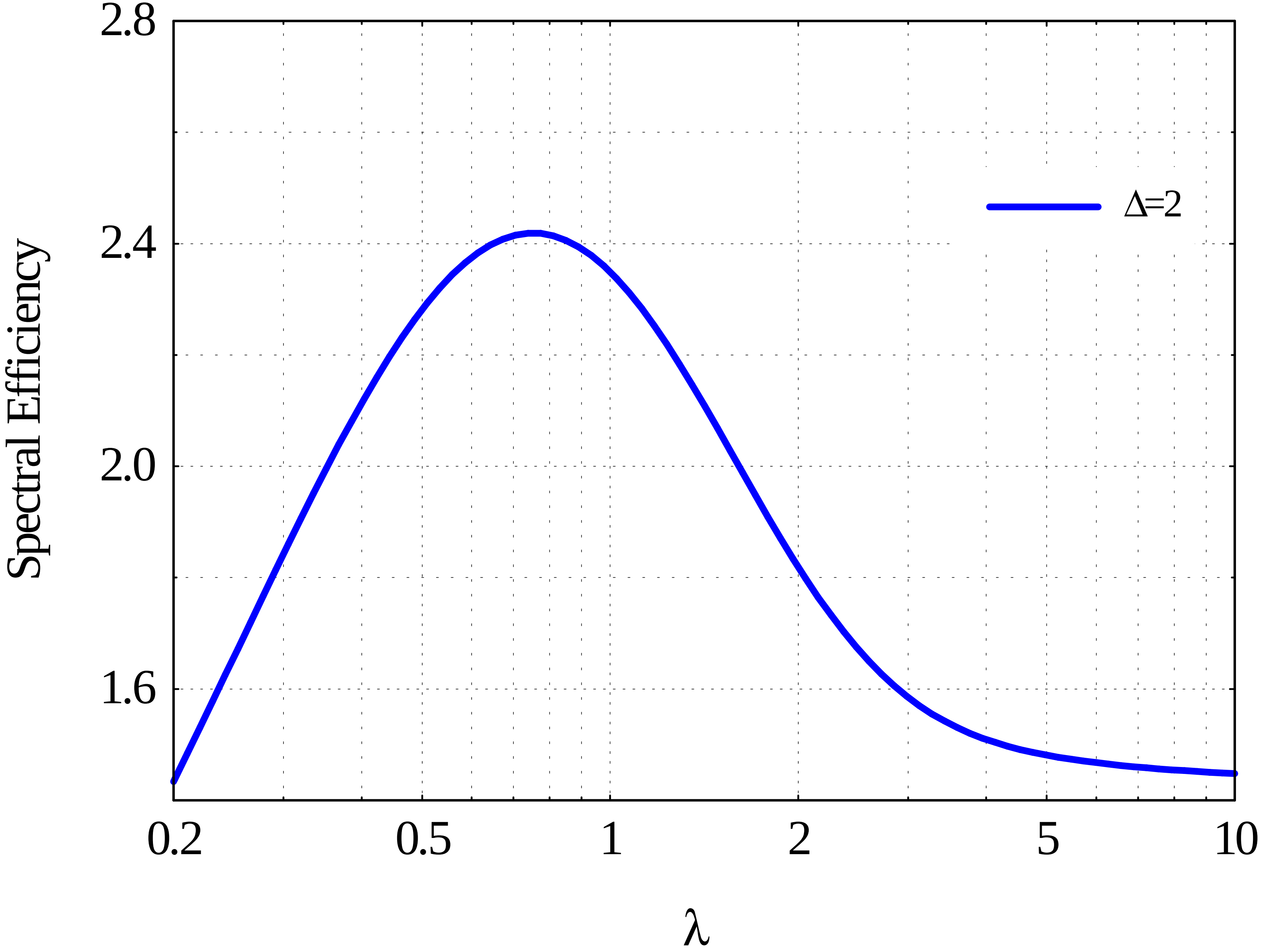


3.0
2.8
2.6
2.4
Spectral Efficiency
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Δ
β=0
β=1

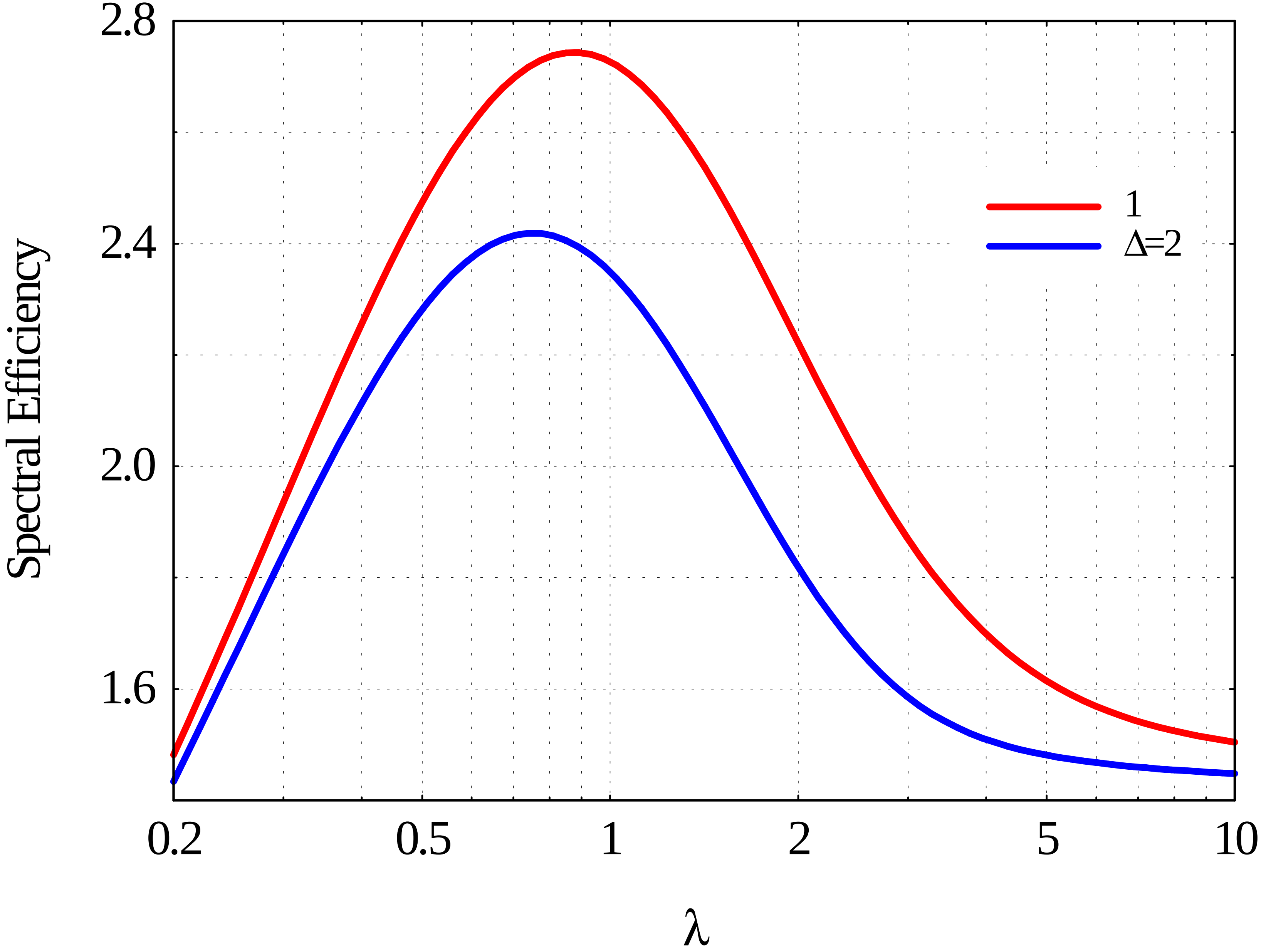

3.0
2.8
2.6
2.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Spectral Efficiency
Δ
β=0
β=0.5
β=1



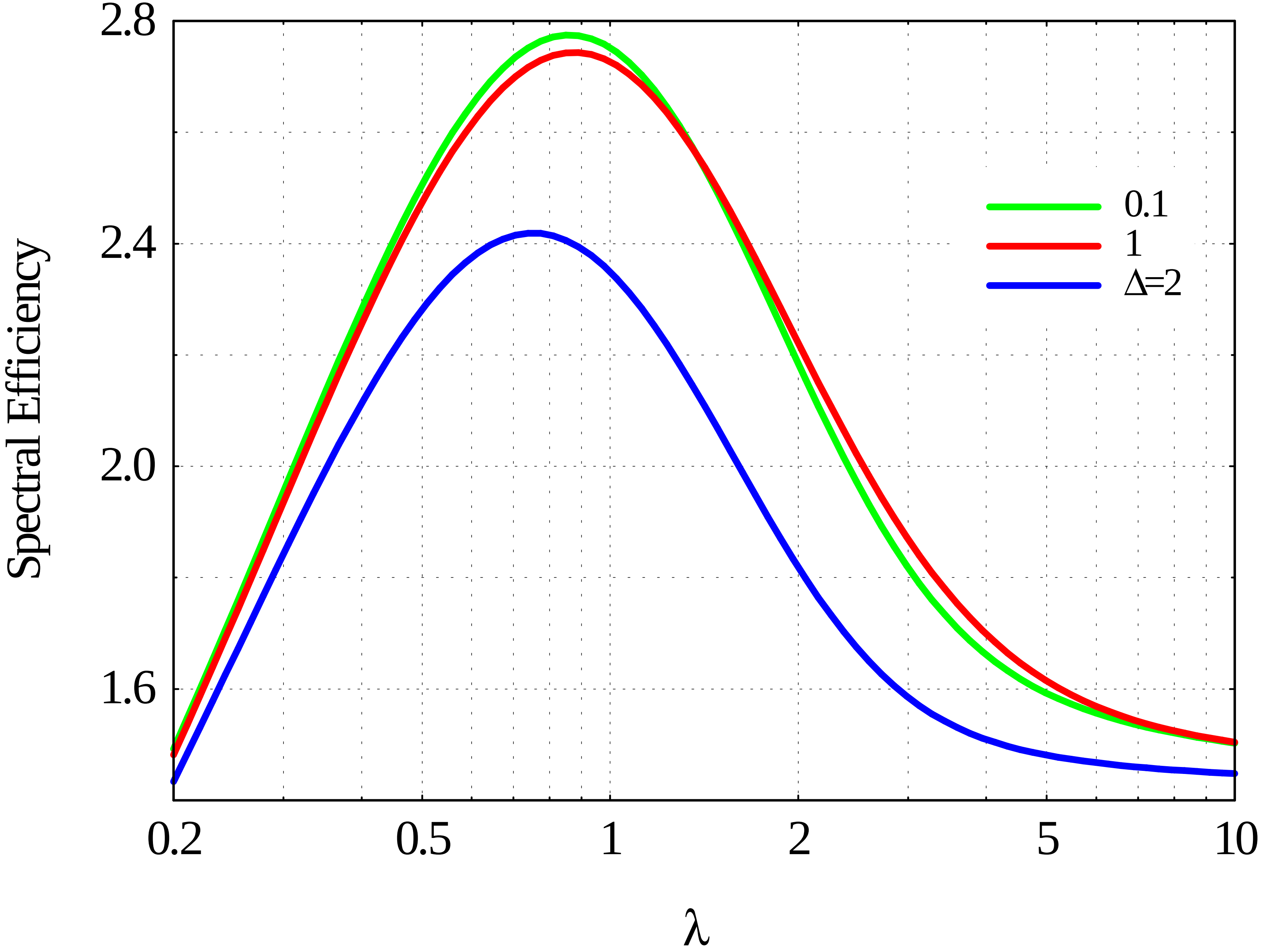
2.9
2.8
2.7
2.6
0.5
1.0
1.5
Spectral Efficiency
Channel Spacing (Δ)



2.8
2.4
2.0
1.6
Spectral Efficiency
Δ=2
0.2
0.5
1
2
5
10
λ



2.8
2.4
2.0
1.6
Spectral Efficiency
0.2
0.5
1
2
5
10
λ
1
Δ=2



Spectral Efficiency
2.8
2.4
2.0
1.6
0.2
0.5
1
2
5
10
λ
0.1
1
Δ=2

Summary

- Sum ergodic capacity is used to measure the spectral efficiency of a random access shared media

- Spectral efficiency depends on
  - pulse shape
  - channel spacing
  - density of users

- **Do we need to channelize the shared spectrum??**