Robust Transmission over Fast Fading Channels on the Basis of OFDM-MFSK

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***Abstract*—We present an OFDM-based transmission scheme which is suitable for robust transmission in fast fading environments, where a reliable channel estimate is impossible or very difficult to obtain. Our scheme is based on the combination of noncoherently detected MFSK (M-ary frequency shift keying) and OFDM (orthogonal frequency division multiplexing). Noncoherent detection of OFDM-MFSK allows an arbitrary phase choice for all subcarriers in the transmitter. One possibility to exploit this degree of freedom is to choose the subcarrier phases such, that the PAPR (peak-to-average power ratio) is reduced. A second possibility is to use the subcarrier phases to transmit additional data. This can be done by differentially modulating the subcarriers that are occupied by the OFDM-MFSK scheme. Both possibilities do not affect the robustness of the underlying noncoherently detected OFDM-MFSK modulation.**

I. Introduction

Wireless communication with high speed trains is a typical scenario where fast fading channels occur. Future trains are expected to travel at speeds up to $v = 600\,\text{km/h}$. Obtaining a reliable channel estimate is very difficult in such an environment. At the same time at least some of the data contains security relevant control data which requires a very robust transmission scheme. A simple way to solve this problem is to use a modulation scheme which can be detected noncoherently and therefore does not need channel estimation at all. For multipath channels, OFDM (orthogonal frequency division multiplexing) [1] is a scheme which can avoid intersymbol interference. In this paper we analyze a system where noncoherently detected M-ary frequency shift keying (MFSK) is combined with OFDM, resulting in a very simple receiver which does not need equalization and channel estimation. The noncoherent detection of OFDM-MFSK allows an arbitrary phase choice for all subcarriers in the transmitter. The drawback of MFSK, also in combination with OFDM, is its low spectral efficiency [2]. To mitigate this, we describe a hybrid transmission method which uses the phases of the occupied subcarriers to transmit additional data by combining OFDM-MFSK and DPSK (differential phase shift keying). A second possibility is to choose the subcarrier phases such, that the PAPR (peak-to-average power ratio) of the transmit signal is reduced. Both methods do not affect the noncoherent detection of the underlying OFDM-MFSK.

II. System Model

We use a discrete-time baseband representation of an N-tone OFDM transmission, where the time domain sample vectors $\mathbf{s}$ are obtained by transforming the transmit symbols $\mathbf{x}$ of size $N \times 1$ to the time domain, using a normalized IFFT and adding a cyclic prefix of N_g samples. After a parallel to serial conversion we obtain the discrete-time transmit signal $s(i\Delta t)$ which is convolved with the channel impulse response $h(i\Delta t)$ and affected by AWGN:

$$g(i\Delta t) = s(i\Delta t) * h(i\Delta t) + n(i\Delta t). \tag{1}$$

In the receiver, the cyclic prefix is removed from the received signal $g(i\Delta t)$ and after serial to parallel conversion and transformation to the frequency domain via FFT, the received OFDM symbols $\mathbf{y}$ can be detected independently.

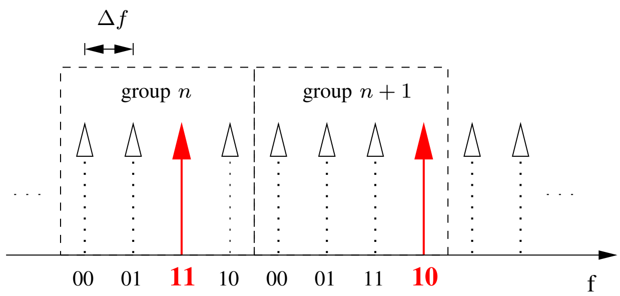

Fig. 1. Principle of OFDM-4FSK modulation: Two bits are assigned to each subcarrier using Gray coding. One out of each group of four subcarriers is taken for transmission, indicated by the solid arrow.

A. OFDM-MFSK

MFSK is a well known technique for robust transmission. It can be combined with OFDM by dividing the subcarriers of an OFDM symbol into groups of M and applying MFSK to each of these groups. This modulation scheme allows noncoherent detection which is particularly interesting for fast fading environments because no channel estimation is needed, which would require a large effort under such conditions.

Fig. 1 shows the principle of this modulation using the example of OFDM-4FSK. The subcarriers with a spacing of Δf are grouped into blocks of four. One carrier of each group is selected for transmission whereas on the other subcarriers of the group no energy is transmitted.

It is well known that for increasing M this orthogonal modulation scheme becomes more power efficient and approaches the Shannon bound [3]. Obviously it is possible to transmit $\log_2 M$ bits per M subcarriers which means that on the other hand the spectral efficiency approaches zero when M is increased. This low spectral efficiency is the major drawback of modulation schemes based on MFSK. A good compromise is the use of OFDM-4FSK because it has the same spectral efficiency as OFDM-2FSK of $\eta = 0.5\,\mathrm{bit}/(\mathrm{s}\cdot\mathrm{Hz})$ while being more robust against noise.

If we transmit over frequency selective channels caused by multipath propagation, some subcarriers can completely fade out and cause an error floor in the bit error curve. To combat this problem, channel coding in conjunction with interleaving is used. The best performance is obtained by using a soft decision detector in order to provide the decoder with a degree of reliability for each bit. A suitable approximate log-likelihood metric L_j for the j-th bit of a code symbol in a transmission with orthogonal modulation can be calculated from the components of the received vector y_n [4]

$$L_j = \max_{n \in S_j^1} |y_n|^2 - \max_{n \in S_j^0} |y_n|^2. \tag{2}$$

S_j^1 denotes the subset of all component indices where the code symbols have a '1' at the j-th digit of the bit mapping. Accordingly there is a '0' at the j-th digit for S_j^0.

Using noncoherent detection for OFDM-MFSK, the phase of the carriers is arbitrary. This degree of freedom can be used in several ways. One possibility is to reduce the PAPR which will be addressed in Section IV. A second possibility is to use the phase to transmit additional data, thus improving the spectral efficiency of the modulation scheme. Such a scheme will be presented in the following section.

III. Hybrid OFDM-MFSK with Differentially Encoded Phases

In this scheme, which was first presented in [5], the subcarrier phases of the OFDM-MFSK symbols are differentially encoded using DPSK. The differential encoding allows noncoherent detection without the need for channel estimation.

Basically the differential encoding of the OFDM-MFSK symbol phases can be done in two different ways. The DPSK modulation can be implemented in frequency direction from occupied subcarrier to occupied subcarrier within one OFDM symbol or in time direction from OFDM symbol to OFDM symbol within one group of subcarriers. For fast fading channels it is advantageous to perform the encoding in frequency direction, however the coherence bandwidth has to be quite large as the occupied subcarriers that are used for DPSK can be far apart in frequency direction. The principle of this hybrid modulation scheme is shown in Fig. 2 where the example of OFDM-4FSK in conjunction with 2DPSK is taken. For this example, one out of a group of four subcarriers is occupied according to the 4FSK bits and the phase (indicated by the arrows) of the occupied subcarriers is differentially modulated between neighboring subcarriers according to the 2DPSK bits. In the receiver the MFSK symbols are detected first, so that the occupied subcarriers are known. Then, assuming correct detection of the MFSK symbols, the differentially encoded DPSK symbols can be detected. With this hybrid scheme,

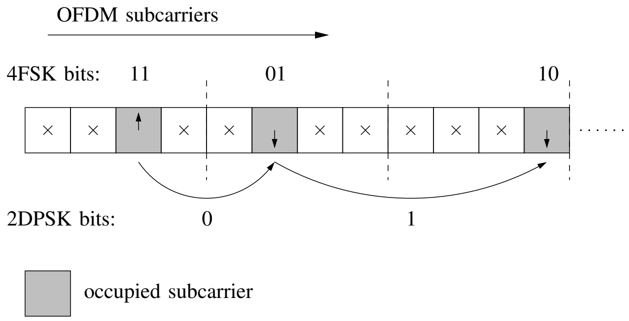

Fig. 2. OFDM-4FSK-2DPSK modulation scheme.

the spectral efficiency can be significantly increased, without influencing the noncoherent detection of the MFSK component at all. Therefore for fixed SNR the bit error probability for the MFSK bits stays the same as for the non hybrid OFDM-MFSK. The error probability of a DPSK bit however, depends on the error probability of the MFSK component. After some calculations it turns out, that for an AWGN channel the probability for a correct DPSK bit using the hybrid modulation is given by

$$\begin{aligned} P'_{c\mathrm{DPSK}} = {} & (1 - P_{b\mathrm{DPSK}})\,(1 - P_{s\mathrm{MFSK}})^2 \\ & + P_{s\mathrm{MFSK}} \left(1 - \frac{P_{s\mathrm{MFSK}}}{2}\right) \end{aligned} \tag{3}$$

where $P_{s\mathrm{MFSK}}$ denotes the probability of an MFSK symbol error and $P_{b\mathrm{DPSK}}$ denotes the probability of a DPSK bit error assuming the symbol decision for both involved MFSK symbols was correct. If we use 4FSK, the symbol error probability for the 4FSK symbols is $P_{s4\mathrm{FSK}} = \frac{3}{2} P_{b4\mathrm{FSK}}$. Neglecting powers of bit error probabilities, which is applicable for high SNR, this yields

$$P'_{b\mathrm{DPSK}} = \frac{3}{2} P_{b4\mathrm{FSK}} + P_{b\mathrm{DPSK}} \tag{4}$$

for the probability that a DPSK bit is in error using the hybrid OFDM-4FSK-DPSK scheme.

To improve the bit error performance, channel coding including interleaving can be applied to the hybrid scheme as well. To maintain the characteristic that the robustness of the noncoherent OFDM-MFSK transmission is not influenced by the additional DPSK modulation, separate encoding of the MFSK and DPSK bit streams is reasonable. In the receiver the MFSK symbols are detected and decoded first. Then the received bits are encoded again to obtain knowledge about the occupied subcarriers $\mathbf{c}_{\mathrm{occ}}$. After this, the DPSK symbols can be detected and decoded as well. A block diagram of this hybrid transmission scheme can be seen in Fig. 3.

A. Simulation Results

In this section we present simulation results for both OFDM-MFSK and the hybrid modulation scheme, where OFDM-MFSK and DPSK are combined. For all simulations a two-sided noise power spectral density of $N_0/2$ is assumed, which means that white Gaussian noise with $\sigma^2 = N_0/2$

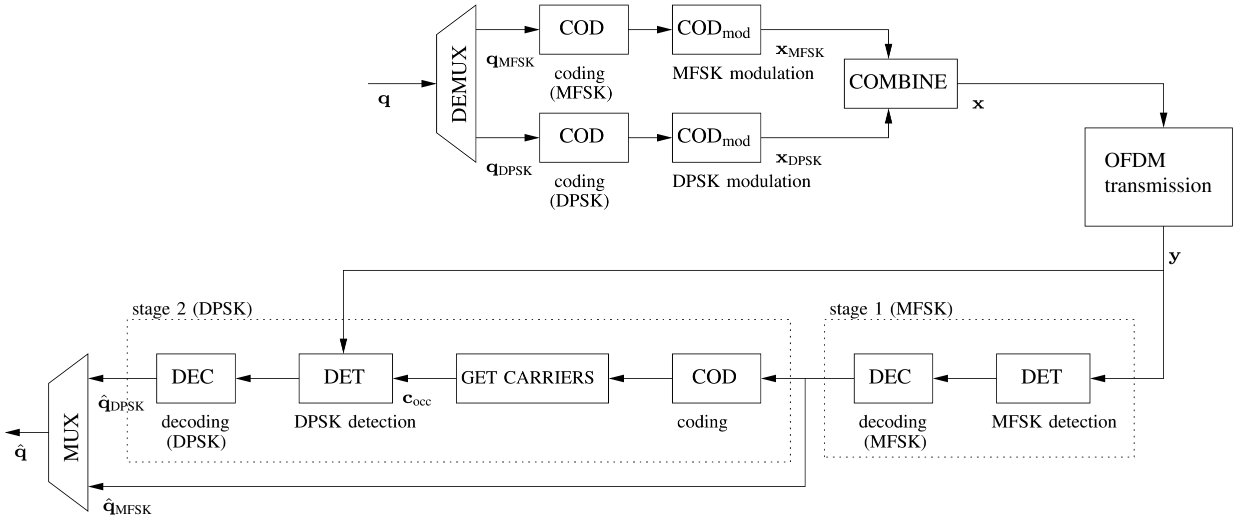

Fig. 3. Block diagram of the coded hybrid transmission scheme.

for the real and the imaginary component was added. As mentioned before, for OFDM-MFSK there is a trade off between increasing power efficiency and decreasing spectral efficiency for increasing M. Therefore $M = 4$ seems to be a good trade off and we will therefore concentrate on OFDM-4FSK in the following. For coded transmission, a rate 1/2 standard convolutional code is used for encoding both the MFSK and DPSK bits separately. In the receiver a soft input Viterbi decoder determines the received bits.

1) AWGN channel: Fig. 4 shows the BER vs. E_b/N_0 for coded transmission over the AWGN channel for noncoherently detected OFDM-4FSK and for the hybrid transmission scheme based on OFDM-4FSK. The BER for 4FSK-2DPSK and 4FSK-4DPSK is the total BER for both 4FSK and DPSK bits. The large gain in E_b/N_0 for the hybrid schemes is due to the fact, that more bits per symbol can be transmitted, therefore the energy per bit is reduced. For 4FSK-4DPSK the same spectral efficiency of $0.5\,\text{bit}/(\text{s}\cdot\text{Hz})$ (without loss due to cyclic prefix) for coding rate 1/2 can be achieved as for BPSK. For comparison the curve for BPSK is also added as a dashed line, but it has to be kept in mind, that BPSK has to be detected coherently and therefore needs channel knowledge. It has to be mentioned that in the case of AWGN only, the BER for the transmission with the above mentioned parameters using the hybrid schemes is dominated mostly by errors in the 4FSK transmission. The spectral efficiency could therefore be further optimized by adapting the modulation and coding methods for the DPSK transmission. However, as will become clear later, the DPSK transmission component is more sensitive against frequency selectivity or fast time variance.

2) Worst Case Channel Model: Because we are interested in a robust transmission scheme we first have to define a channel model which includes the disturbances of interest. As an example we take a scenario where a high speed train transmits and receives signals from a fixed base station. A worst case in this scenario would be, if in addition to a line

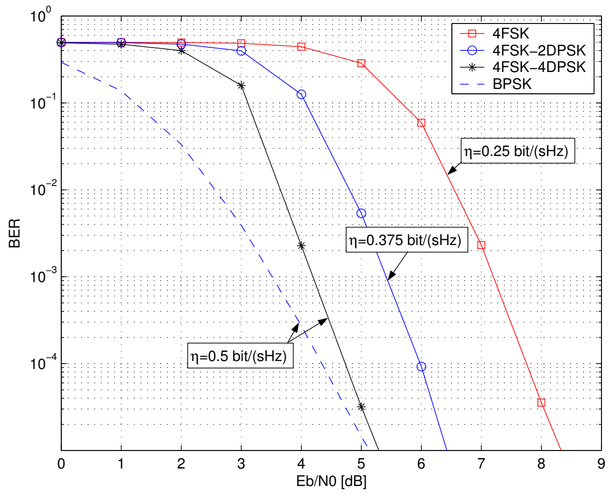

Fig. 4. BER vs. $\frac{E_b}{N_0}$ for OFDM-4FSK and hybrid OFDM-4FSK-DPSK for an AWGN channel and a rate 1/2 convolutional code ([133,171]); $N_f = 256$, $N_g = 64$, η without loss due to cyclic prefix.

of sight (LOS) path, a second path which is reflected behind the mobile station with low attenuation arrives at the receiver, leading to a maximum Doppler spread of $2f_d = 2f_c\frac{v}{c}$ in the received signal. Here v denotes the velocity of the mobile station and f_c is the carrier frequency of the OFDM system, assuming f_c is much greater than the OFDM bandwidth. Such a scenario is shown in Fig. 5. The parameters used for all simulations with the time variant 2-path channel are listed in Table I. Furthermore it was assumed that the reflected path is not attenuated compared to the LOS path. Simulation results have shown, that besides an overall degradation due to the frequency selective fading, noncoherently detected OFDM-4FSK is very robust against large Doppler spreads, occurring in fast fading environments. Even for $v = 600\,\text{km/h}$ the degradation in $\frac{E_b}{N_0}$ is less than 0.5 dB at a BER of 10^{-5}. The results for OFDM-4FSK are plotted in Fig. 6 for a velocity of

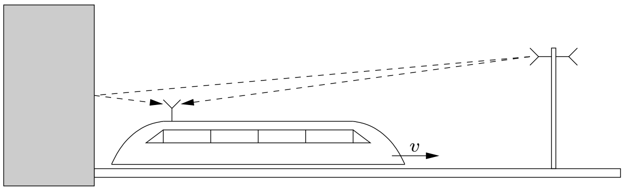

Fig. 5. Model for a two path channel in a high speed scenario, reflection at a bridge or tunnel entrance.

carrier frequency	$f_c = 38\,\mathrm{GHz}$
FFT length	$N_f = 256$
no. of subcarriers used	$N_{f\mathrm{used}} = 160$
subcarrier separation	$\Delta f = 312.5\,\mathrm{kHz}$
cyclic extension	$T_g = N_g \Delta t = 0.8\,\mu\mathrm{s}$
symbol duration	$T_s = (N_g + N_f)\,\Delta t = 4\,\mu\mathrm{s}$

TABLE I
PARAMETERS FOR THE HIGH VELOCITY SCENARIO

$v = 600\,\mathrm{km/h}$ and a path delay of $t_d = 0.075\,\mu\mathrm{s}$. If the path delay is increased up to the total guard time the performance is not degraded. This robustness of OFDM-4FSK of course also holds for the 4FSK bits in the hybrid transmission scheme, however, the DPSK bits are much more sensitive against frequency selectivity, especially if the modulation is done in frequency direction. The coherence bandwidth is inversely proportional to the path delay. From Fig. 6 we can see that even for a path delay of only $t_d = 0.075\,\mu\mathrm{s}$ the frequency selectivity still causes a severe performance degradation in the DPSK component. For a smaller delay of $t_d = 0.03\,\mu\mathrm{s}$ this degradation becomes less. Note that the simulations in Fig. 6 were done with a velocity of $v = 600\,\mathrm{km/h}$ for the mobile station and DPSK modulation in frequency direction, proving that also the hybrid scheme is very robust against fast time variance. If the DPSK modulation is done in time direction, the BER is much more sensitive against fast time variance, making

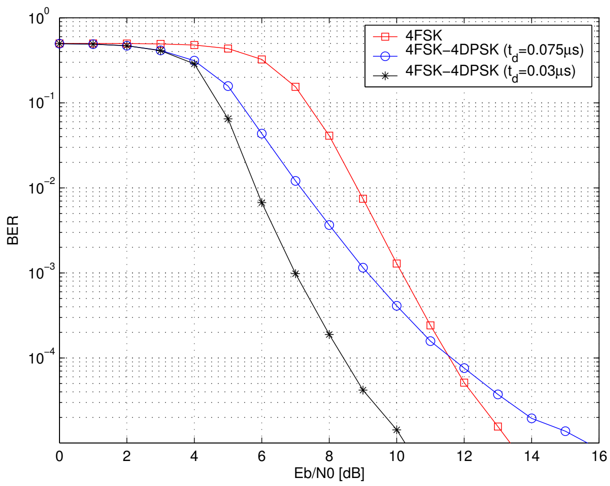

Fig. 6. Total BER vs. $\frac{E_b}{N_0}$ at the receiver for OFDM-4FSK-4DPSK with with a rate 1/2 convolutional code ([133,171]) for the two path channel; $v = 600\,\mathrm{km/h}$, $N_f = 256$, $N_g = 64$.

a transmission over the DPSK component at $v = 600\,\mathrm{km/h}$ impossible. The sensitivity against frequency selectivity is lowered but still quite high, because different subcarriers are occupied in consecutive symbols and differentially modulated in time direction.

IV. PAPR Reduction

Instead of using the subcarrier phases for transmitting additional data, it is possible to use them for PAPR reduction. It is a well known problem, that multicarrier transmission schemes like OFDM suffer from a large PAPR [1]. The reason for this is, that for certain phase selections of the subcarriers, the superposition of the orthogonal subcarriers leads to very large peaks in the amplitude. So the arbitrary phases of noncoherently detected OFDM-MFSK should be chosen such, that the PAPR is as low as possible. Several methods to achieve this were presented in [6] and will be summarized here.

The PAPR of an OFDM symbol is defined as the square of the maximum amplitude divided by the mean power. If

$$\|s\|_\infty = \max_t |s(t)| \tag{5}$$

is the maximum amplitude and

$$\|s\|_2^2 = \frac{1}{T_s}\int_0^{T_s} |s(t)|^2 dt \tag{6}$$

is the mean power of an OFDM symbol, then the PAPR is defined as [1]

$$\mathrm{PAPR} = \frac{\|s\|_\infty^2}{\|s\|_2^2}. \tag{7}$$

The largest PAPR is obtained if all subcarrier phases are the same and therefore add up coherently. A straightforward method to avoid this is to assign random phases φ_n to all subcarriers. Simulations have shown that allowing only discrete phases $\varphi_n \in \{0, \pi\}$ leads to a lower PAPR than allowing continuous phases $\varphi_n \in [0, 2\pi)$ (cf. Fig. 7). Furthermore, there are many PAPR reduction techniques for QAM and MPSK schemes [7] that can also be applied to OFDM-MFSK. Due to the noncoherent detection it is not necessary for OFDM-MFSK to transmit side information about the phase selection in the transmitter.

A very simple method is *selected mapping* [8] where several replicas of each OFDM symbol with random subcarrier phases are generated and the one with the lowest PAPR is transmitted. The larger the number of generated symbols the lower the achievable PAPR. However, for each candidate symbol the time domain signal has to be calculated by an IFFT which leads to a trade off between complexity and performance.

A second method is to use a *time-frequency domain swapping algorithm* [9] where the PAPR of each OFDM symbol is iteratively reduced by clipping the signal in the time domain and reconstructing the subcarrier amplitudes in the frequency domain. By adapting the clipping level (CL) relative to the maximum amplitude the performance and complexity can be influenced. This algorithm yields the largest PAPR reduction. However, the complexity is very high because for each OFDM symbol several hundred FFTs are needed.

In [6] a *sequential algorithm* was presented which systematically changes the subcarrier phases to reduce the PAPR. After an initial random phase selection $\varphi_n \in \{0, \pi\}$ the phases of the subcarriers are flipped sequentially. In each step the time domain signal is calculated which means that one IFFT has to be performed for each occupied subcarrier. Only if the PAPR has been reduced, the flipped phase is saved, otherwise the original phase is kept. It is possible to reduce the complexity of this algorithm by exploiting the linearity of the DFT. If we know the value $s(i_{\text{peak}})$ and the sample index i_{peak} of the peak in the time domain, we can estimate if it is possible that a phase flip of the current subcarrier can reduce the PAPR. The contribution of the current subcarrier to the signal peak can be written as

$$s_n(i_{\text{peak}}) = \frac{1}{\sqrt{N}} x_n \cdot e^{j2\pi n \Delta f i_{\text{peak}}}, \tag{8}$$

where n is the number of the current subcarrier and N the FFT-size. x_n denotes the element on subcarrier n before the phase flip. Only if the angle between $s(i_{\text{peak}})$ and $s_n(i_{\text{peak}})$ is smaller than 90° the amplitude of the peak can be reduced. In the other case the phase flip would increase the amplitude of the peak and we can go to the next occupied subcarrier without calculating the time domain signal. By this the number of IFFTs can be reduced by about 30%. The second reduction in complexity can be made in the calculation of the time domain signal. Because only the phase of one subcarrier is flipped we can calculate the time domain signal after the phase flip from the previous time domain signal by

$$s(i)' = s(i) - \frac{2}{\sqrt{N}} x_n \cdot e^{j2\pi n \Delta f i}, \quad i = 0, \ldots, N-1. \tag{9}$$

The complexity of (9) is $O(N)$ whereas the complexity of a complete IFFT is $O(N \log_2 N)$.

All of the presented PAPR reduction algorithms have the problem that the phases have to be selected for each OFDM symbol individually. As this has to be done in real time for a practical system, the complexity of the algorithms is very important.

Fig. 7 compares the cumulative density function (CDF) of the PAPR for the different algorithms using OFDM-4FSK modulation. Assigning random phases to all occupied subcarriers does not increase the complexity but the PAPR is quite high. By using selected mapping the PAPR can be reduced but one IFFT has to be performed for each candidate symbol. In our system for 65 candidate symbols the complexity of selected mapping is the same as for the sequential algorithm. As we can see, the sequential algorithm outperforms selected mapping and the complexity of the sequential algorithm can be further reduced. The best performance it terms of PAPR reduction is obtained by the swapping algorithm but its complexity is unacceptably large.

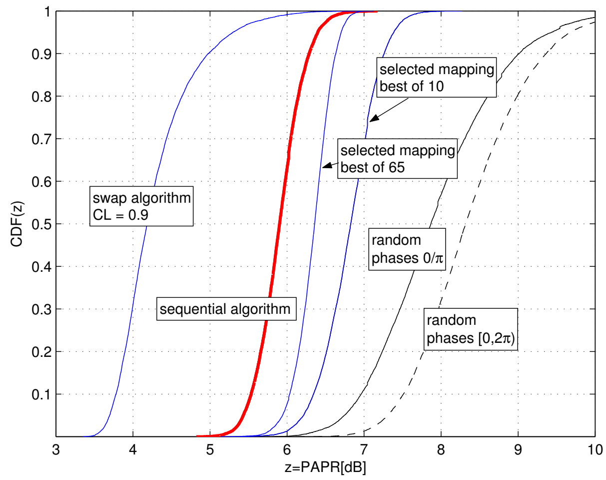

Fig. 7. CDF of the PAPR using OFDM-4FSK modulation for different algorithms; $N_{f\text{used}}$ = 256, 8 times oversampling

V. Summary and Conclusion

In this paper a robust modulation scheme was described and analyzed which is based on the combination of OFDM and MFSK. Noncoherent detection of OFDM-MFSK is possible, making the scheme very robust against fast fading channels and rendering channel estimation unnecessary. To increase the spectral efficiency it is possible to differentially modulate the phases of occupied subcarriers. This additional phase modulation does not affect the noncoherent OFDM-MFSK transmission but offers additional data rate for moderate channel conditions. Alternatively the subcarrier phases can be used to reduce the PAPR of the transmit signal. We gave an overview over several PAPR reduction algorithms. A sequential algorithm has been found to offer a good tradeoff between complexity and performance.

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References

[1] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications.* Artech House Publishers, 2000.

[2] J. Lindner, *Informationsübertragung.* Springer-Verlag, Berlin, 2005.

[3] J. G. Proakis, *Digital Communications.* McGraw-Hill, New York, 3rd Edition, 1995.

[4] A. J. Viterbi, *CDMA - Principles of Spread Spectrum Communication.* Addison-Wesley, Reading 1995.

[5] M. Wetz, I. Periša, W. G. Teich, J. Lindner, "OFDM-MFSK with Differentially Encoded Phases for Robust Transmission over Fast Fading Channels," 11th International OFDM Workshop, Hamburg, Germany, August 2006.

[6] M. Wetz, W. G. Teich, J. Lindner, "PAPR Reduction Methods for Noncoherent OFDM-MFSK," 3rd COST 289 Workshop, Aveiro, Portugal, July 2006.

[7] S. H. Han and J. H. Lee, "An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission," *IEEE Wireless Communications*, vol. 12, pp. 56-65, April 2005.

[8] R. W. Bäuml, R. F. H. Fischer, J. B. Huber, "Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selected Mapping," *IEEE Electronics Letters*, vol. 32, pp 2056-2057, October 1996.

[9] E. van der Ouderaa, J. Schoukens, J. Renneboog, "Peak Factor Minimization Using a Time-Frequency Domain Swapping Algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 37, pp 145-147, March 1988.