Effect of Carrier Frequency Offset on Channel Capacity in Multi User OFDM-FDMA Systems

Martin Stemick and Hermann Rohling

***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) is a very robust transmission procedure in multipath and frequency selective radio channels. A Frequency Division Multiple Access (FDMA) resource allocation technique offers the opportunity of a detailed link adaptation scheme. The combination of these transmission- and multiple access techniques in OFDM-FDMA is an ideal and very strong candidate for the downlink of future 4G mobile communication systems.**

This technical combination offers high cell capacities by exploiting the inherent multi user diversity effect of the system. To apply OFDM-FDMA in the uplink, the time and carrier synchronization accuracy becomes very important. Non-ideal synchronization of the user signals to the carrier frequency of the base station leads to intercarrier interferences (ICI). In this paper, an analytical model for the ICI consideration in the uplink of a multi user OFDM-FDMA based system is derived. The impact of the carrier frequency offset (CFO) on the performance of a cellular multi user system with respect to different subcarrier allocation schemes is analyzed.

***Index Terms*—OFDM-FDMA, Uplink, Synchronization, ICI.**

I. Introduction

The fourth generation (4G) of mobile communication systems will have to offer hundreds of Mbit/s of data rate both in the downlink and in the uplink direction in order to meet the demands and requirements of future multimedia applications. This can be fulfilled if the utilized communication system has a high degree of flexibility and adaptivity. OFDM based transmission systems in combination with multiple access schemes were found to be very promising to provide broadband communication schemes and high system flexibility simultaneously [1], [2]. Among all other multiple access schemes the OFDM-FDMA approach permits a very high degree of adaptivity in frequency selective radio channels combined with a moderate technical system complexity [2]. However, in order to provide high data rates not only in the downlink but also in the uplink, a number of additional technical challenges has to be met. One of these challenges is the synchronization of all mobile terminals (MT) to the base station (BS) inside a single cell. An OFDM-FDMA based system divides the available bandwidth into a set of orthogonal, but spectrally overlapping subcarriers. Such a system will be especially sensitive to carrier frequency offsets (CFO). In the uplink, every MT allocates an exclusive set of subcarriers, which are superimposed with all other user signals at the BS to an overall OFDM symbol.

If every MT provides an individual carrier frequency offset, strong intercarrier interferences (ICI) are observed at the BS [3], [4]. Therefore, a precise synchronization of all MTs to the BS is vital for such a radio system.

This carrier synchronization scheme can e.g. be done in such a way that in a setup phase the BS measures the CFO for each MT individually. The measured CFO are then fed back to the MTs, in order to adjust their local oscillators. The actual measurement of the CFO can be done with dedicated signals or blindly, cf. [5], [6], [7] and references therein. However, such a procedure needs large a measurement overhead and its performance will strongly depend on measurement accuracies. Therefore, in most systems a certain amount of CFO has to be tolerated.

The objective of this paper is therefore to investigate the general impact of CFO to the performance of multi user OFDM-FDMA based systems. The resulting ICI does not only depend on the amount of CFO introduced from different users, but also on the employed subcarrier allocation scheme.

In Section II, the considered system model is introduced, followed by an analytical model for the CFO depending ICI in Section III. As will be seen, the effect of ICI can be described by a Gaussian noise signal added to the wanted signals of all users [7], [8]. In Section IV, the dependency between the utilized subcarrier allocation scheme and the level of ICI noise for different users is shown. Section V introduces multi user scenarios to evaluate the impact of CFO onto the channel capacity. Section VI finally gives simulation results on how much CFO decreases the performance of the introduced OFDM-FDMA based multi user scenarios.

II. System Model

The uplink of a mobile communication system based on the OFDM-FDMA transmission and allocation technique is considered in this paper. The system consists of a single cell with a central BS and N_u mobile terminals (MT). The overall bandwidth B is divided into N subcarriers. Each MT allocates $N_c = N / N_u$ subcarriers exclusively to fulfill its transmission demands. In a first step it is assumed that all MT have the same distance to the BS in order to avoid any

Manuscript received March 26, 2007. This work was supported by the German Research Council (DFG) under Grant RO 761/14-1.

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shadowing and path loss effects inside the used signal model. The considered system model is depicted in Fig. 1. The assumed radio channel model is AWGN, to evaluate solely the effects of ICI. It is assumed that there exists some means of synchronization inside the overall system in such a way that the MT are able to perform a coarse synchronization based on OFDM symbol level analysis. Therefore, a perfect positioning of the FFT window for all MT and therefore zero interference between consecutive OFDM symbols (ISI) can be assumed. Also, sampling offsets between MT and BS are assumed to be negligible.

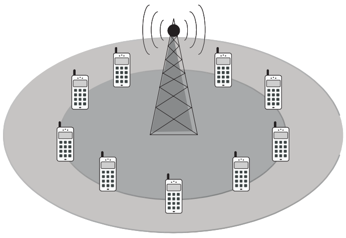

Fig. 1. Considered system model

III. Analytical Model for CFO

In OFDM based systems the transmitted time signal $x_{l,n}$ of a user l can be described by the following equation:

$$x_{l,n} = \frac{1}{N}\sum_{k=0}^{N-1} X_{l,k} e^{j2\pi\frac{kn}{N}} \tag{1}$$

Where $X_{l,k}$ is the complex valued modulation symbol of user l on subcarrier k. In the following, k is the subcarrier index on the transmitter (MT) side. To model the multiple access procedure based on the OFDM-FDMA transmission technique the modulation symbols $X_{l,k}$ of a user l show non-zero values on the allocated subcarriers and zero values elsewhere. The transmit time signals $x_{l,n}$ of every MT will pass through individual multipath radio channels and show individual CFO. All user signals are then combined and superimposed at the BS to an overall receive signal. The overall received time signal y_n is

$$y_n = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N_u-1} X_{l,k} Z_{l,k}(n) e^{j2\pi\frac{kn}{N}} + \eta_n^{AWG} \tag{2}$$

with η_n^{AWG} as the white Gaussian noise component at the receiver and $Z_{l,k}(n)$ as the combined channel fading and CFO influence to the signal of user l. At the receiver side perfect positioning of the FFT window and no signal sampling offset is assumed, so $Z_{l,k}(n)$ can be expressed as

$$Z_{l,k}(n) = H_{l,k} e^{j2\pi\frac{n\cdot\delta f_l}{N}} \tag{3}$$

where $H_{l,k}$ represents the channel transfer factor for user l on subcarrier k and δf_l describes the CFO of user l normalized to the subcarrier spacing Δf.

In the following, we will consider the interference situation in the receiver (BS) for a specific user ς on a subcarrier v assigned to this user, where v is the subcarrier index in the receiver. After calculating the FFT in the receiver and combining (2) and (3), the received symbol Y_v on subcarrier v can be described by

$$Y_v = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\sum_{l=0}^{N_u-1} X_{l,k} H_{l,k} e^{j2\pi\frac{n\cdot\delta f_l}{N}} \cdot e^{j2\pi\frac{n(k-v)}{N}} + N_v^{AWG} \tag{4}$$

Equation (4) can be split up into the wanted user modulation symbol $X_{\varsigma,v}$ of user ς who exclusively transmits on subcarrier v and ICI noise N_v^{ICI} on v from the remaining subcarriers $k \in \{[0, N-1] \setminus v\}$, see (5).

$$Y_v = \alpha_{\varsigma,v} H_{\varsigma,v} X_{\varsigma,v} + N_v^{ICI} + N_v^{AWG} \tag{5}$$

The ICI noise N_v^{ICI} can be split into the individual ICI contributions $N_{l,v}^{ICI}$ from each user l (including user ς) inside the cell. An analytical expression for $N_{l,v}^{ICI}$ is given in (6). Besides the channel influence $H_{\varsigma,v}$ there is an additional loss $\alpha_{\varsigma,v}$ on modulation symbol $X_{\varsigma,v}$ due to the individual CFO of user ς on subcarrier v, see (7).

$$N_{l,v}^{ICI} = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{\substack{k=0\\k\neq v}}^{N-1} X_{l,k} H_{l,k} e^{j2\pi\frac{n(\delta f_l + k - v)}{N}} \tag{6}$$

$$\alpha_{\varsigma,v} = \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{n\cdot\delta f_\varsigma}{N}} \tag{7}$$

The modulation symbols are assumed to be statistically independent random variables from a predefined modulation alphabet, i.e. $E\left\{X_{l,k} X_{i,j}^*\right\} = 0$ for $l \neq i \vee k \neq j$. Additionally, the transmit power for the modulation symbols is normalized such that $E\left\{\left|X_{l,k}\right|^2\right\} = 1$ holds. Therefore according to the central limit theorem, the ICI noise $N_{l,v}^{ICI}$ can be modeled as a zero-mean Gaussian distributed noise signal with variance $\sigma_{ICI}^2(l,v) = E\left\{\left|N_{l,v}^{ICI}\right|^2\right\}$, cf. [7], [8]. The analytical expression is as follows:

$$\begin{aligned}\sigma_{ICI}^2(l,v) &= \sum_{\substack{k=0\\k\neq v}}^{N-1}\sum_{n=0}^{N-1}\sum_{d=0}^{N-1} \frac{1}{N^2}\left|H_{l,k}\right|^2 e^{j2\pi\frac{(n-d)}{N}[\delta f_l + k - v]} \\ &\approx \sum_{\substack{k=0\\k\neq v}}^{N-1}\left|H_{l,k}\right|^2 \frac{\sin^2\left(\pi\left(\delta f_l + k - v\right)\right)}{\left(\pi\left(\delta f_l + k - v\right)\right)^2} \quad \text{for } N \gg 1\end{aligned} \tag{8}$$

Further, from (7) the power loss $L_{\varsigma,v}$ due to CFO of user ς allocated to subcarrier v can be calculated, see (9).

$$\begin{aligned}L_{\varsigma,v} &= E\left\{\left|\alpha_{\varsigma,v}\right|^2\right\} = \left|\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi n\frac{\delta f_\varsigma}{N}}\right|^2 \\ &= \frac{1}{N} + \sum_{d=1}^{N-1}\frac{2(N-d)}{N^2}\cos\left(\frac{2\pi}{N} d \cdot \delta f_\varsigma\right)\end{aligned} \tag{9}$$

To give an impression what happens if multiple users transmit to the BS with certain CFOs δf_l, a simple example with two users is depicted in Fig. 2. This example shows the reception of two users at the BS, where the integer sampling points of the subcarriers are shown on the x-axis. At the BS, user 0 shows a frequency shift of δf_0. This leads to a loss of signal power on the sampling points of user 0 (cf. (9)), which is marked with a square on one exemplary subcarrier. On the same subcarrier, self-interference of user 0 can be observed, which is marked with triangels. Additionally, user 0 introduces ICI to the signals of user 1 (cf. (8)), which is highlighted with circular markers for one exemplary sampling point. All this can be summarized for the general case using (5), (8) and (9), which yields

$$SNR_v = \frac{L_{\varsigma,v}\left|H_{\varsigma,v}\right|^2}{\sum_{l=0}^{N_u-1}\sigma_{ICI}^2(l,v)+\sigma_{AWG}^2(v)} \quad (10)$$

Equation (10) describes the overall SNR on subcarrier v. This SNR depends on the CFO and channel situation of user ς who is allocated to v and also on disturbances by ICI from all users. Additionally, the Gaussian noise power at the receiver is represented by $\sigma_{AWG}^2(v)=E\left\{\left|N_v^{AWG}\right|^2\right\}$. The variance of the overall interference noise on subcarrier v is therefore

$$\sigma_{ICI}^2(v)=\sum_{l=0}^{N_u-1}\sigma_{ICI}^2(l,v)\,. \quad (11)$$

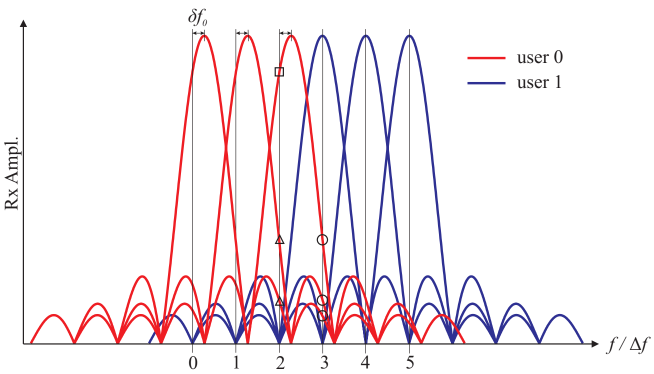

Fig. 2. Example for two users at BS, where user 0 exhibits CFO

IV. Evaluation of ICI

After providing the analytical background in the previous paragraph, now the quantitative impact of CFO-induced ICI can be shown for general OFDM-FDMA systems. One important factor in this context is the subcarrier allocation scheme that is used in the uplink. To increase the performance of OFDM-FDMA, adaptive subcarrier allocation schemes which exploit the multiuser diversity inside the system can be used [9], [10]. The two allocation schemes considered here are either blockwise- or interleaved allocation. A blockwise allocation means that a single user allocates only blocks of adjacent subcarriers, see Fig. 3. The blocksize can vary between one and N_c according to the system requirements. In the interleaved scheme, each user assigns a set of equidistantly spaced subcarriers. The subcarrier sets are positioned in an interleaved way over the system bandwidth, which is depicted in Fig. 4. This is a special case of a blockwise allocation with blocksize = 1, where the subcarriers of an individual user have the maximum mutual distance.

In the following, the level of interference power $\sigma_{ICI}^2(v)$ is evaluated for these different allocation schemes for the simple case of two users inside the system.

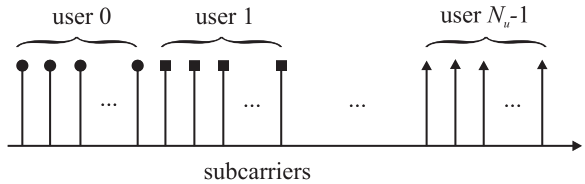

Fig. 3. Blockwise subcarrier allocation

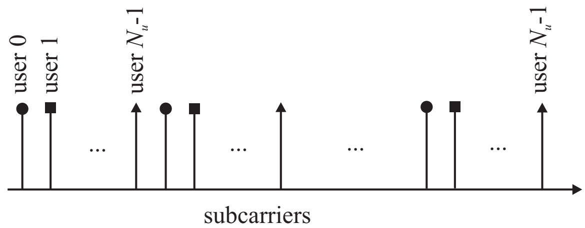

Fig. 4. Interleaved subcarrier allocation

Two examples are considered. In both examples, a total bandwidth of 256 subcarriers is assumed which is equally shared between the users. At the BS a CFO of $\delta f_0 = 0.1$ is observed for user 0, while user 1 is perfectly synchronized. Then the level of $\sigma_{ICI}^2(v)$ is calculated (cf. (8),(11)) on each subcarrier. The dB-values of $\sigma_{ICI}^2(v)$ in Fig. 5 and Fig. 6 are related to the transmit power $E\left\{\left|X_{l,v}\right|^2\right\}=1$. In the first example, blockwise allocation with a blocksize of 128 is considered, so only two blocks are present in the system, see Fig. 5.

Because user 0 is not well synchronized, some ICI is observed on all subcarriers. This ICI is constantly high for user 0, because his allocated subcarriers interfere with each other. Only at the border to user 1 the ICI decreases, because there is no interference from user 1 to user 0 due to his perfect synchronization. All ICI, which will be observed by user 1 stems from user 0 and is steadily decreasing for subcarriers which are far away from the interfering carriers. Most ICI to user 1 is well below -30dB.

A different situation can be seen in Fig. 6. There the blocksize is decreased to 16 and the blocks are allocated in an interleaved manner. The situation for user 1 worsened compared to Fig. 5, because now every block of user 1 experiences heavy interference from both sides.

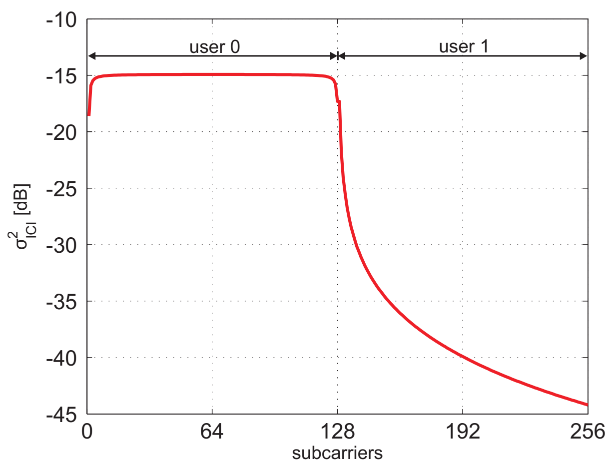

Fig. 5. ICI for two users with blockwise allocation; 128 subcarriers for each user, blocksize 128, user 0 with $\delta f_0 = 0.1$, user 1 with $\delta f_1 = 0$.

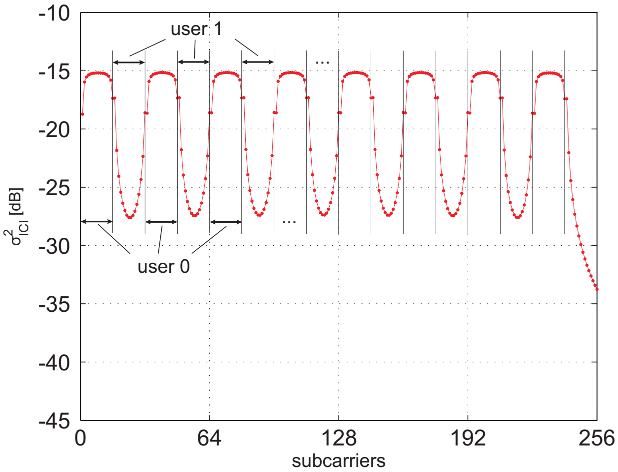

Fig. 6. ICI for two users with blockwise allocation, 128 subcarriers for each user, blocksize 16, user 0 with $\delta f_0 = 0.1$, user 1 with $\delta f_1 = 0$

As a result the interference level of user 1 is always higher than -30 dB. On the other hand, user 0 profits from the changed subcarrier allocation. At the border-subcarriers of his allocated blocks, he still observes a low interference level. But now, the number of blocks has increased, and therefore the number of border-subcarriers also has increased, so that the overall performance of user 0 is improved.

If the blocksize in Fig. 6 is further reduced to one, a purely interleaved allocation scheme is obtained. Then the interference situation for the two users is conversed: Now user 0 sees the lowest interference, since the immediate neighbors of his subcarriers are the perfectly synchronized carriers of user 1. This effect can be shown by a simple analysis: In an interleaved scheme with two users and blocksize 1, the subcarrier allocation pattern is very regular. Therefore, if the subcarriers at the borders of the bandwidth are neglected, the interference situation is almost the same for all subcarriers of a specific user. For further simplification we assume, that the strongest interference to a subcarrier stems from its two neighboring subcarriers. Using (8), and the above prerequisites, the interference power $\sigma^2_{ICI0}(v)$ for all subcarriers of user 0 can be calculated as follows:

$$\sigma^2_{ICI0}(v) = \sin^2\left(\pi\left(\delta f_1 + 1\right)\right)\left[\frac{2\left(\delta f_1^2 + 1\right)}{\left(\pi\left(\delta f_1^2 - 1\right)\right)^2}\right] \qquad (12)$$

To calculate the interference $\sigma^2_{ICI1}(v)$ for user 1, the CFO in (12) has to be set to δf_0. Due to our assumptions, (12) is independent of *v*. Since in our example $\delta f_1 = 0$ holds, equation (12) yields $\sigma^2_{ICI0}(v) = 0$. This means, user 0 sees no interference, almost as assumed above. Of course user 0 will experience ICI from its own more far away subcarriers, but these have minor influence and are neglected here.
On the other hand, every subcarrier of user 1 now observes heavy interference from user 0. This can be seen by substituting δf_1 with δf_0 in (12). Since $\delta f_0 = 0.1$ holds, $\sigma^2_{ICI1}(v)$ will take a value of -17dB. So the situation from Fig. 5 and Fig. 6, where user 0 saw the highest and user 1 the lowest interference is completely conversed.

V. CFO and System Capacity

After discussing the influence of the subcarrier allocation scheme on the interference level for individual users, it is now of interest to evaluate which effect the CFO will have on the overall performance of a multi user system.
The system model introduced in section II will be considered in the following. Inside the system, 16 users will equally share a bandwidth of 256 subcarriers. The channel between the MTs and the BS will be AWGN with CFO-induced ICI as an additional disturbance. Interleaved and blockwise allocation will be employed. As a performance criterion, the theoretical channel capacity C_l of each user *l* will be measured, based on

$$C_l = \sum_{\substack{\forall\, v \text{ of} \\ \text{user } l}} \log_2\left(1 + SNR_v\right) \quad \left[{}^{\text{bits}}/_{\text{OFDM-Sym.}}\right]. \qquad (13)$$

The AWGN-power is assumed to be $\sigma^2_{AWG}(v) = 0.1$.

To get significant average results for every user, the allocation of all users to the subcarrier blocks is done in a random manner.

In the simulations, two scenarios were considered: In the first scenario, all MTs except one are perfectly synchronized to the BS. The capacity of the badly synchronized user is evaluated for various allocation schemes and CFOs. In this way, the self inflicted interference of a user can be evaluated in dependency on the allocation scheme.

The second scenario assumes that all users except one have an absolute CFO of δf. The capacity of the single perfectly synchronized user is also evaluated for various allocation schemes and CFOs. This scenario shows the performance loss due to interference inflicted from other users.

VI. Results

In the following we will present some quantitative results for the scenarios introduced in the previous section. Fig. 7 shows the results for the first scenario where we observe the capacity of a non-ideally synched user. In general, the performance of this user decreases with increasing CFO. W.l.o.g., we will assume the observed non-synchronized user to be user 0.
The worst performance figures are observed for a blockwise allocation with blocksize = 16. This means in the considered system model that every user gets only one block of subcarriers. The bad performance, shown for user 0 stems from his strong self-interference due to the accumulation of all of his subcarriers in a single block (cf. also Fig. 5, user 0). When the blocksize decreases, the performance of user 0 improves, because also his self-interference decreases. For the interleaved allocation, there is almost no performance loss, because the subcarriers of user 0 have maximum distance to each other and therefore observe only low interference.

Fig. 8 shows the results for the second scenario, where only one user in the system is perfectly synchronized and all others experience a uniform CFO to the BS. We observe the capacity of the synchronized user (w.l.o.g.: user 0). User 0 only observes interference noise from other users, no self interference. Therefore, allocation schemes with large block-sizes are advantageous because then interference is only observed at the borders of one block. In Fig. 8, the triangle marked curve refers to the allocation with the largest blocksize and therefore shows the lowest capacity loss for user 0. According to our considerations, the performance loss grows worse when the blocksize decreases.

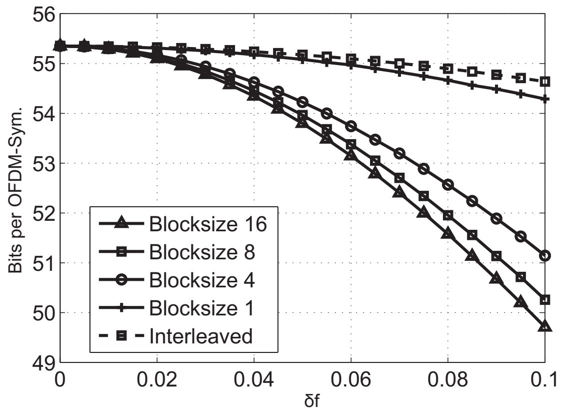

Fig. 7. Capacity of the badly synchronized user for various allocation schemes. All other users in the system are perfectly synchronized.

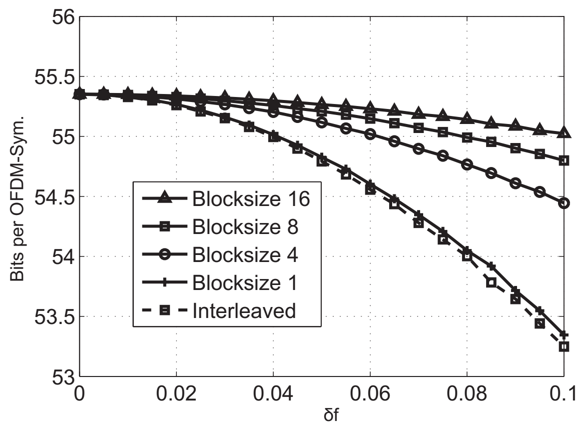

Fig. 8. Capacity of the perfectly synchronized user for various allocation schemes. All other users in the system observe an absolute CFO of δf .

These two examples depicted in Fig. 7 and Fig. 8 show not only the dependency of system performance on CFO and subcarrier allocation scheme. It also becomes obvious that in the first example a small blocksize is of advantage while in the second example it is just the other way round. This is because in example 1 only self-inflicted ICI is observed while in example 2 only ICI from adjacent users is observed. The choice of an ICI-optimized subcarrier allocation scheme should therefore also depend on the overall interference situation inside the communication system. For given CFO, the system designer can choose how the ICI is distributed between the users by choosing a suitable subcarrier allocation scheme. Of course, not the absolute amount of ICI inside the system can be influenced, but its distribution.

VII. Conclusion

In this paper, the influence of CFO-induced ICI on a multi user OFDM-FDMA system was evaluated. It was shown that the performance loss of individual users through ICI depends strongly on the subcarrier allocation scheme. Considering the overall system performance, one has to be aware of the fact, that changing the allocation scheme structure only varies the distribution of ICI inside the system, but has no effect on the overall ICI power.

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