Uplink channel estimation for multi-user OFDM-based systems

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***Abstract*— In this paper we propose a simple, yet flexible and efficient, channel estimator for the uplink in broadband Orthogonal Frequency Division Multiplexing (OFDM) systems. The processing is performed in the time-domain, by extracting the channel's impulse response (CIR) for each user from a joint training signal. In this OFDM system, the pilot sequence we advocate, where all users share the same pilot sub-carriers, consists of one OFDM-symbol endowed with time-shifted properties per user, which isolates each user's CIR and is robust against multi-user interference. The feasibility of our approach is substantiated by system simulation results obtained using BRAN-A broadband mobile wireless channel model.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is the choice for a variety of wideband applications due to its robustness against frequency selective channels [1], common in mobile personal communications. In addition, in a multi-user scenario, there exists the so-called multi-user diversity [2]: the probability that, at least, one user experiences good channel propagation conditions increases with the number of users. If there are many users, it is highly probable that not all of them will experience poor channel propagation and at least one of them will have a good channel, and thus nowadays such multi-user systems are rapidly growing to improve system efficiency. Most systems require accurate channel estimation either for demodulation/decoding or resource allocation. Usually channel estimation methods are pilot-based, i.e. the channel information is extracted from known transmitted symbols that, although decreasing system efficiency since no data information is conveyed, they provide better performance than blind methods. In centralized networks, this problem is particularly important in the uplink, where multiple users must send pilots efficiently to the access point, so that it can perform the multi-user channel estimation accurately. In order to do so, in [3] overlapped pilots are proposed for channel estimation where different terminals utilize the same pilot sub-carriers avoiding the decrease in efficiency as the number of users increases. However, the performance results are not very favourable. In this paper, we present an uplink channel estimation method specifically adapted for wideband OFDM-based transmission systems, where several users share the same pilot positions with-in the frame, with minimal interference among them, but attaining better performance. The system must have dedicated subcarriers for the transmission of those pilots. Perfect synchronization is assumed at the Central Point and the channel response is considered constant during each OFDM symbol.

This paper is organized as follows. Section II briefly presents the system model for multi-user OFDM-based systems. The multi-user channel estimation algorithm for the uplink scenario based on a time-domain procedure is developed in Section III, and also, pilot sequences design with special shifting properties is presented. Simulation results and discussions are provided in Section IV. Finally, conclusions are drawn in Section V.

II. System model for multi-user OFDM in the uplink

Let us consider OFDM modulation over N_c sub-carriers for transmission over a multipath fading channel. In the single-user case, the transmitted signal in frequency-domain is described by $\mathbf{s} = \mathbf{d} + \mathbf{p}$, where $\mathbf{s}$ is the column N_c-vector whose components $S[k]$, $k = 0, \ldots, N_c - 1$, are complex symbols transmitted at k-th sub-carrier. These complex symbols will belong to an elementary two-dimensional constellation (M-ary PSK or QAM). This complex symbol may be either a pilot $P[k]$ or data $D[k]$. The column N_c-vector $\mathbf{d}$ collects the data symbols, in which its k-th element $D[k]$ is data symbol

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at k-th sub-carrier. Similarly, the column N_c-vector $\mathbf{p}$ collects the pilot symbols, in which its k-th element $P[k]$ is pilot value at k-th sub-carrier. Both vectors contain values at disjoint sub-carriers, and therefore, $S[k] = D[k] + P[k]$. Define the IDFT (Inverse Discrete Fourier Transform) $N_c \times N_c$ matrix,

$$\mathbf{Q} \triangleq \frac{1}{N_c}\left(e^{j2\pi kn/N}\right)_{k,n=0,0}^{N_c-1,N_c-1} \tag{1}$$

then the transmitted time-domain signal vector $\widetilde{\mathbf{s}}$, whose components are $s[n], n = 0, \ldots, N_c-1$, can be expressed with the IDFT transform pair as

$$\widetilde{\mathbf{s}} = \mathbf{Qs} = \mathbf{Q}(\mathbf{d}+\mathbf{p}) = \mathbf{Qd} + \mathbf{Qp} = \widetilde{\mathbf{d}} + \widetilde{\mathbf{p}} \tag{2}$$

where the time-domain transmitted data column N_c-vector $\widetilde{\mathbf{d}}$ collects $d[n]$ components while pilot column N_c-vector $\widetilde{\mathbf{p}}$ collects $p[n]$ components, yielding $s[n] = d[n] + p[n]$. The received signal in time-domain $r[n]$ will be given by $r[n] = s[n] * h[n] + w[n]$, where $h[n]$ is the Channel's Impulse Response (CIR) and $w[n]$ is the Additive White Gaussian Noise (AWGN). The channel's frequency response can be obtained via DFT transform and then $\widetilde{\mathbf{h}} = \mathbf{Qh}$, where $\widetilde{\mathbf{h}}$ and $\mathbf{h}$ are time-domain and frequency-domain channel column N_c-vectors, respectively. Each component of the frequency-domain vector $H[k], k = 0, \ldots, N_c - 1$ is the channel's frequency response at k-th sub-carrier.

For the multi-user system in an uplink scenario, U users simultaneously access the Central Point. All users perform OFDM modulation over N_c sub-carriers. In this case, the pilot values in frequency domain per u-th user, $u = 0, \ldots, U-1$, will be denoted as $P_u[k]$, while the pilot sequence in time domain per u-th user is denoted as $p_u[n]$. The time-domain received signal in this multi-user scenario is then given by

$$r[n] = \sum_{u=0}^{U-1} r_u[n] + w[n] = \sum_{u=0}^{U-1} p_u[n] * h_u[n] + w[n] \tag{3}$$

where $r_u[n]$ is the received signal from u-th user and $h_u[n]$ is the channel impulse response for that user.

III. Channel estimation method

For the single-user case, Minn *et al.* [4] proposed the following pilot information to carry out their Most Significant Taps channel estimation method. Let the pilot symbols $P[k]$ be given by

$$P[k] = \sum_{m=0}^{N_c/N_f-1} \delta[k - mN_f], \; 0 \le k \le N_c - 1 \tag{4}$$

where N_f is the spacing between pilot sub-carriers in frequency and N_c is the total number of sub-carriers. If $N_t = \frac{N_c}{N_f}$ is integer, then the time-domain pilot signal is,

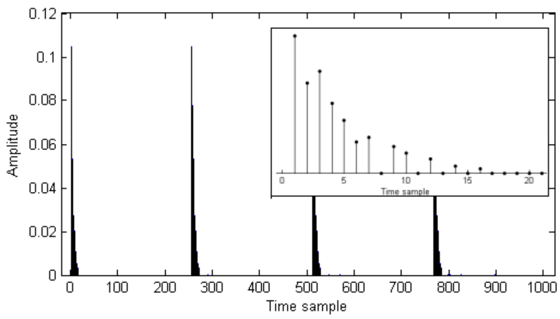

Fig. 1. Received signal, for the single-user case, when only pilot sub-carriers are sent, without loading data in the remaining ones.

$$\begin{aligned} p[n] &= \frac{1}{N_c}\sum_{k=0}^{N_c-1}\sum_{m=0}^{N_t-1}\delta[k-mN_f].\exp\left(j2\pi\frac{kn}{N_c}\right) \\ &= \frac{1}{N_c}\left[e^{j2\pi\frac{0n}{N_c}} + e^{j2\pi\frac{N_f n}{N_c}} + \ldots + e^{j2\pi\frac{\left(N_c-N_f\right)n}{N_c}}\right] \\ &= \begin{cases} \frac{1}{N_f}, \; n = \ell\frac{N_c}{N_f} = \ell N_t, \; \ell \in \mathbb{N}_0 \\ 0, \; others \end{cases} \\ &= \frac{1}{N_f}\sum_{m=0}^{N_f-1}\delta[n-mN_t] \end{aligned} \tag{5}$$

Equation (5) puts in evidence that the pilot vector in time domain appears as N_f discrete samples uniformly separated N_t samples. Transmitting this pilot signal through a wireless multipath channel with a maximum delay spread shorter than $\Delta t N_t$, where Δt is the sampling time, will result in a received vector $r[n]$ containing N_f scaled replicas of the channel's impulse response $h[n]$, independently corrupted by AWGN noise $w[n]$.

$$r[n] = p[n]*h[n]+w[n] = \frac{1}{N_f}\sum_{m=0}^{N_f-1} h[n-mN_t]+w[n] \tag{6}$$

For the purpose of illustration, Fig. 1 shows an example of a received pilot vector transmitted through an 18 path BRAN-A [5] wireless channel with a maximum delay spread of $390ns$, a pilot distance $N_f = 4$ and a sampling time $\Delta t = 25ns$. The zoomed figure in the top right corner of Fig. 1 details the first received replica of the CIR.

The channel estimator changes the received vector by keeping L samples (Most Significant Taps) of each replica of the channel impulse response, starting at positions $mN_t + 1, m = 0, \ldots, N_f - 1$ and zeroing the remaining symbol samples,

$$r'[n] = \begin{cases} r[n], \; n = mN_t + i; \; i = 0, .., L-1 \\ 0, \; others \end{cases} \tag{7}$$

A new signal $\bar{r}[n]$ is defined, made up of the averaging of the N_f replicas of the previous vector $r'[n]$, padded with zeros up to N_c,

$$\bar{r}[n] = \begin{cases} \frac{1}{N_f} \sum_{m=0}^{N_f-1} r[mN_t + n], \; n = 1, ..., L \\ 0, \; n = L+1, ..., N_c \end{cases} \quad (8)$$

The estimate of the channel's frequency response at k-th sub-carrier $\hat{H}[k]$ is the result of the DFT of the signal $\bar{r}[n]$,

$$\hat{H}[k] = DFT_{N_c}\{\bar{r}[n]\} \quad (9)$$

where we define this DFT $N_c \times N_c$ matrix as $\mathbf{Q}' \triangleq \left(e^{-j2\pi kn/N}\right)_{k,n=0,0}^{N_c-1,N_c-1}$. Most practical multipath channels will have its energy concentrated in a few significant taps, selecting the most significant ones of each replica and neglecting the remaining ones will result in a performance improvement [4] in comparison with DFT-based channel estimation methods [6]. The transmission of randomized data along with pilots will result in some performance degradation of the channel estimation as the data part of the kept samples of the received symbols will act as noise in the estimation process [4].

To deal with several users on the uplink, we propose to use a similar set-up, with all users using the same uncoded pilot sub-carriers in the same symbols that completely add up at the receiver, in the frequency domain. To be able to get estimates of each channel, each user will use different values for pilots. We propose a multi-user pilot sequence $P_u[k]$ in the frequency domain given by,

$$P_u[k] = \sum_{m=0}^{N_t-1} \delta[k - mN_f] \,. \exp\left(-j\frac{2\pi}{U}um\right) \quad (10)$$

As an example, for the case of $U = 4$ users, the frequency-domain pilot vectors will be,

$$\begin{cases} \mathbf{p}_0 = [+1, 0 \ldots + 1, 0 \ldots + 1, 0 \ldots + 1, 0 \ldots + 1 \ldots] \\ \mathbf{p}_1 = [+1, 0 \ldots - i, 0 \ldots - 1, 0 \ldots + i, 0 \ldots + 1 \ldots] \\ \mathbf{p}_2 = [+1, 0 \ldots - 1, 0 \ldots + 1, 0 \ldots - 1, 0 \ldots + 1 \ldots] \\ \mathbf{p}_3 = [+1, \underbrace{0 \ldots}_{N_f-1}, +i, 0 \ldots - 1, 0 \ldots + i, 0 \ldots + 1 \ldots] \end{cases} \quad (11)$$

The use of these different pilot values for each user results in the shifting of the replicas of the received symbols for u-th user by $u\frac{N_t}{U}$ samples in the time-domain, thus allowing the Central Point to estimate the channel for each terminal, at the cost of some added interference from data of other terminals. The time domain pilot signal per u-th user $p_u[n]$ is,

$$\begin{aligned} p_u[n] &= \frac{1}{N_c} \sum_{k=0}^{N_c-1} \sum_{m=0}^{N_t-1} \delta[k - mN_f] \,.e^{-j2\pi\frac{u}{U}m} e^{-j\frac{2\pi}{N_c}kn} \\ &= \frac{1 + ... + e^{-j2\pi\frac{u}{U}(N_t-1)} e^{-j\frac{2\pi}{N_c}(N_c-N_f)n}}{N_c} \\ &= \begin{cases} \frac{1}{N_f}, \; n = \frac{u}{U}N_t + mN_t, \; m = 0, ..., N_f - 1 \\ 0, \; others \end{cases} \\ &= \frac{1}{N_f} \sum_{m=0}^{N_f-1} \delta\left[n - \frac{u}{U}N_t - mN_t\right], \; \frac{N_t}{U} \text{ integer} \end{aligned} \quad (12)$$

or equivalently,

$$\begin{cases} p_o[n] = \frac{1}{N_f} \sum_{m=0}^{N_f-1} \delta[n - mN_t] \\ p_1[n] = \frac{1}{N_f} \sum_{m=0}^{N_f-1} \delta\left[n - \frac{1}{U}N_t - mN_t\right] \\ \vdots \\ p_{U-1}[n] = \frac{1}{N_f} \sum_{m=0}^{N_f-1} \delta\left[n - \frac{U-1}{U}N_t - mN_t\right] \end{cases} \quad (13)$$

Considering the transmission of symbols from U users with independent channels and perfectly synchronized, the received signal, recalling (3), will be,

$$\begin{aligned} r[n] &= \sum_{u=0}^{U-1} p_u[n] * h_u[n] + w[n] \\ &= \sum_{u=0}^{U-1} \frac{1}{N_f} \sum_{m=0}^{N_f-1} \delta\left[n - \frac{u}{U}N_t - mN_t\right] * h_u[n] + w[n] \\ &= \frac{\sum_{m=0}^{N_f-1} h_0[n - mN_t]}{N_f} + \frac{\sum_{m=0}^{N_f-1} h_1\left[n - \frac{1}{U}N_t - mN_t\right]}{N_f} \\ &+ \ldots \frac{\sum_{m=0}^{N_f-1} h_{U-1}\left[n - \frac{U-1}{U}N_t - mN_t\right]}{N_f} + w[n] \end{aligned} \quad (14)$$

Figure 2 shows an example where one OFDM-symbol carrying only pilots (data sub-carriers are not loaded in this example) from each user is transmitted through independent BRAN-A channels and arrives synchronized at the Central Point.

The Central Point channel estimator changes the vector of the received samples by keeping L samples of each replica starting at positions $mN_t\frac{1}{U} + 1, m = 0, \ldots, N_fU - 1$ and zeroing the remaining vector samples. We define signal $\bar{r}_u[n]$ per u-th user, each made up of the averaging of the N_f replicas pertaining to the desired user, padded with zeros up to N_c,

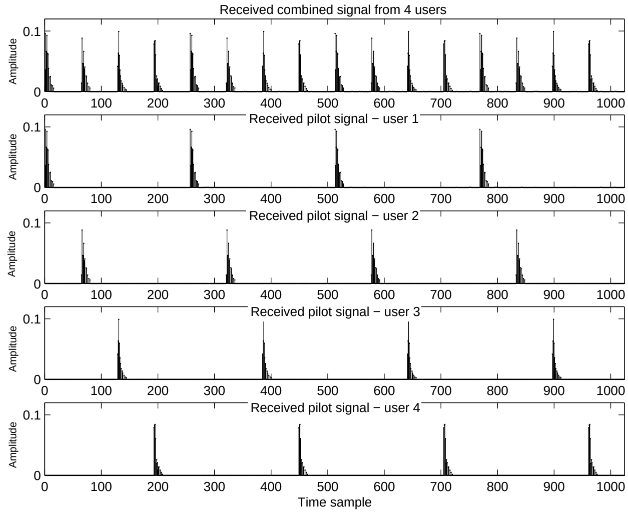

Fig. 2. Received signal for the multi-user case ($U = 4$), when only pilot sub-carriers are sent (data sub-carriers are not loaded).

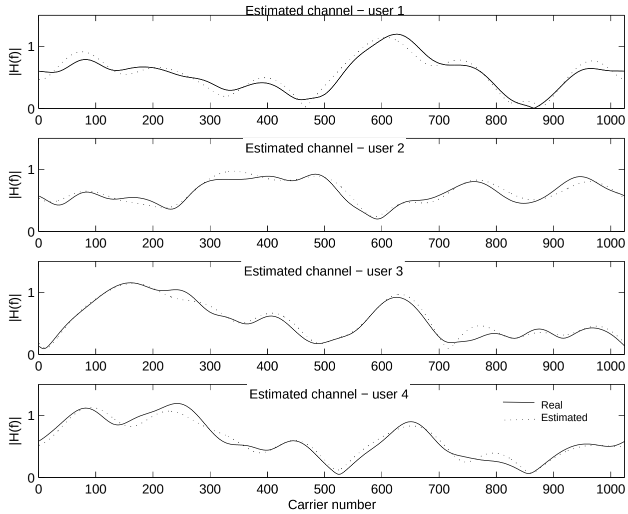

Fig. 3. Comparison between real and estimated channel's frequency response for the four users. The solid line refers to the real channel while dashed line represents the estimated channel.

$$\bar{r}_u[n] = \begin{cases} \dfrac{1}{N_f} \displaystyle\sum_{m=0}^{N_f-1} r\left[mN_t + N_t{}^{u}\!/\!_{U} + n\right], \; n = 1, ..., L \\ 0, \; n = L+1, ..., N_c \end{cases} \tag{15}$$

The estimates of the channel's frequency response at k-th sub-carrier for u-th user $\hat{H}_u[k]$ is the result of the DFT of the signal $\bar{r}_u[n]$, yielding $\hat{H}_u[k] = DFT_{N_c}\{\bar{r}_u[n]\}$. Observing Fig. 2 it is clear that the maximum number of elements that can use the same pilot carriers is limited and depends on the pilot distance N_f and the channel's maximum delay spread $\tau_{u,max}$, $U \leq \frac{N_c \Delta t}{N_f \tau_{u,max}}$. The overall system uses U times less pilot carriers, improving the performance in terms of system's power efficiency.

In the presence of a time-variant channel, the channel estimation in the time direction may be performed using some sort of simple interpolator (quadratic, cubic, ...) to keep the overall complexity low. In this case, a 2-D diamond shaped pilot pattern should be used [7] to achieve the best performance with the same complexity.

IV. Performance evaluation

A simulation scenario with $U = 4$ users was implemented, where all users share the same pilot sub-carriers. Since this channel estimation method performs frequency-domain estimation, the measurements presented in this section are only with respect to the symbols that carry both pilots and data. Channel estimation in the time direction, that would involve estimating the channel's frequency response of the symbols carrying only data, is not considered. The system uses $N_c = 1024$ QPSK modulated sub-carriers, with a pilot separation $N_f = 4$ carriers and a sampling time $\Delta t = 25$ ns. Fig. 3 shows the comparison between real and estimated channel's frequency response, when the four users transmit symbols through independent BRAN-A channels. The receiver's signal to noise ratio was set to $SNR = 10$ dB and $L = 10$ samples from each replica were kept.

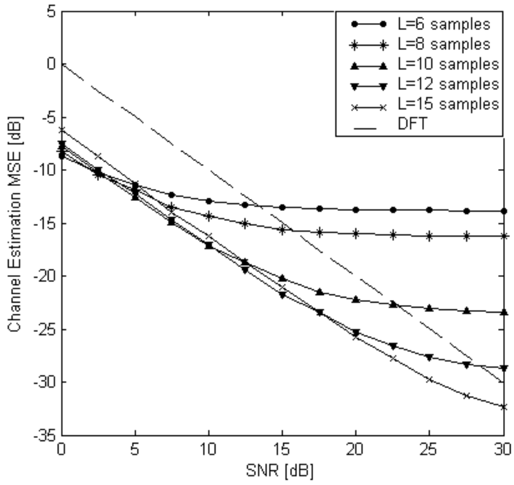

Fig. 4. MSE performance as a function of L. The dashed line refers to DFT-based channel estimation method for one user.

To assess the performance of the proposed method, several simulations were performed for different SNR values. Fig. 4 presents several performance curves of channel estimation Mean Squared Error (MSE), averaged over the four users, for different values of L, when the SNR at the input of the receiver ranges from 0 to 30 dB. Each user transmitted symbols (carrying pilots and data) through independent BRAN-A channels. For comparison purposes, the frequency domain DFT channel estimation method [6] was also implemented for a single user employing the same pilot structure. The attained channel estimation MSE of the DFT method is also included in Fig. 4.

The utilization of the proposed method implies a trade-off between the amount of distortion introduced by removing energy from significant channel taps and the amount of noise inputed to the estimation process. An MSE noise floor will be present due to removing some of the channel's energy and it will lower as the number of samples L is increased, at the price of a worse performance for low values of SNR. In the region were the channel distortion is not the dominant factor on the performance (low SNR values), this method outperforms DFT channel estimation considerably, with the added advantage of using U less sub-carriers to transmit pilots.

The dependence of the presented method on the pilot distance N_f is an important topic to address to complete its evaluation. Fig. 5 depicts several curves of the attained channel estimation MSE for different values of SNR, with different values of N_f. As reference, the DFT channel estimation performance is also included in the figure. The performance of the reference method is mainly independent on the pilot distance N_f as along as the Nyquist Criterion is fulfilled.

By observing (14), the received signal $r_u[n]$ of user u can be written as:

$$r_u[n] = \frac{1}{N_f}\sum_{m=0}^{N_f-1} \delta\left[n - \frac{u}{U}N_t - mN_t\right] * h_u[n]$$
$$= \frac{1}{N_f}\sum_{m=0}^{N_f-1} h_u\left[n - \frac{u}{U}N_t - mN_t\right] \quad (16)$$

and signal power S_{r_u} is,

$$S_{r_u} = E\left\{|r_u[n]|^2\right\}$$
$$= E\left\{\left|\frac{1}{N_f}\sum_{m=0}^{N_f-1} h_u\left[n - \frac{u}{U}N_t - mN_t\right]\right|^2\right\} = \frac{1}{N_f}S_{H_u} \quad (17)$$

where S_{H_u} is the channel power for user u. The power of each of the U vectors $\bar{r}_u[n]$ defined in (15) is

$$S_{\bar{r}_u} = \frac{1}{N_f}S_{r_u} = \frac{1}{N_f^2}S_{H_u} \quad (18)$$

Considering that the noise samples that make up $w[n]$ in (15) are independent and identically distributed with a variance σ_N^2, then the noise term in (15), after averaging the N_f replicas will be $\sigma_{N,avg}^2 = \frac{\sigma_N^2}{N_f}$ resulting in a signal to noise ratio before the DFT of

$$SNR_{ChEst} = \frac{S_{\bar{r}_u}}{\sigma_{N,avg}^2} = \frac{1}{N_f}\frac{S_{H_u}}{\sigma_N^2} \quad (19)$$

Equation (19) justifies the performance curves present in Fig. 5. In the region where noise is the dominant factor on the performance (low SNR values), decreasing the pilot distance will improve the channel estimation. This effect grows smaller as the received SNR increases and the channel distortion becomes increasingly more important.

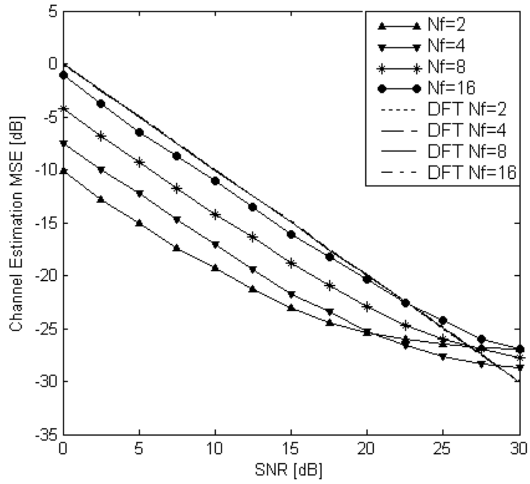

Fig. 5. Performance dependence on the pilot distance N_f.

V. Conclusions

We have proposed a simple channel estimator for multi-user OFDM-based systems in the uplink scenario. It works in the time-domain, by averaging each user's CIR replicas, which appear separated due to the use of the presented pilot sequences with time-shifting properties. Our estimator can be easily used in either acquisition (preamble-based) or tracking (pilot-tones based) mode, and its structure remains the same for any type of training pattern in the two-dimensional time-frequency space. We have also provided the pilot sequences that allows identification of the multi-user channel by ensuring special time-shifting properties. The feasibility of our approach was substantiated by computer simulation results obtained for BRAN-A broadband wireless channel models.

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