Joint Modeling of the Multipath Radio Channel and User-Access Method

Zsolt A. Polgar, Vasile Bota, Mihaly Varga, Atilio Gameiro

Abstract—**The paper proposes two methods for the joint modeling of the mobile multipath Rayleigh-faded radio channels and of two OFDMA-based multi-user access techniques appropriate for high bit rate mobile transmissions. These mathematical models are used to compute the probability density functions (p.d.f.) and the state probabilities of the SINR at the mobile receiving end and to build Markov-chain models that describe jointly the channel and the multi-user access technique. It also discusses the effects of the user speed of the carrier load upon the p.d.f. of the received SINR and upon the state and state-transition probabilities for the access methods considered.**

Key words: **OFDMA multi-user access, multipath propagation, Rayleigh fading, total SINR p.d.f., Markov chain**

I. INTRODUCTION

An important problem related to the theoretical performance evaluation of a mobile transmission system is the mathematical characterization of the multipath Rayleigh-faded radio channel [1] [2] and the multi-user access technique employed [2]. The characterization of a channel affected by frequency and time selectivity requires the derivation of the channel state (instantaneous SINR) p.d.f. Pure analytical solutions of the stated problem are given only for the particular case characterized by a uniform power delay profile and the same Rayleigh fading parameter on each path, [3]. The entire transmission chain could also be described by Markov chains [4], which is particularly useful for packet-based transmissions. Besides the p.d.f. of the received SINR, it also describes its variation, for small time intervals, due to users' mobility.

The paper analyzes a joint modeling of the channel and multi-user access technique in an OFDMA-based scheme, for two allocation methods of the user frequency-band, i.e. the allocation based on Frequency Hopping (FH) [5] and on Best-Frequency Position (BFP), [6], [7]. Two mathematical modeling techniques are analyzed for each allocation method, i.e. the modeling based on the p.d.f. of the channel's SINR and on the channel SINR-state Markov chain, respectively.

The paper is structured as follows: section II presents briefly the BFP-OFDMA and FH-OFDMA access techniques. Section III presents the two proposed methods to compute the p.d.f. of the received SINR, which consider jointly the channel parameters and the user-chunk BPF or FH allocation method. Section IV deals with the Markov-chain modeling of the channel SINR states, by considering jointly its parameters and the allocation method. Section V summarizes the main conclusions.

II. OFDMA ACCESS TECHNIQUE PROPOSED FOR MOBILE WIRELESS SYSTEMS

The OFDMA type multi-user access is based on a frequency-time signal pattern that can be adjusted to different propagation scenarios and which is able to cope with the frequency-selectivity generated by the multipath propagation and the time-variability owned to the user motion. Such a signal pattern, called bin or chunk, is proposed by [7] and would be used in this paper. Within these chunks, non-coded or coded QAM modulations are used adaptively [6], [8], based on channel SINR measurements and prediction performed by the MS, using a number of scattered pilot symbols [7]. The chunk-allocation to different users in the downlink connection employs either the BFP or the FH methods. The BFP method assigns to each user the chunk that would provide the best SINR in the frequency-band of that chunk, according to his channel prediction, [6] and [7]. The chunk-allocation is performed by a scheduler block in the BS that uses the predicted channel map sent by each user on the uplink. The FH method allocates the chunks, in the frequency domain, according to a pseudorandom sequence. The OFDMA multi-user access technique based on the chunk pattern and the proposed structures and parameters of these chunks are presented in fig. 1 and table I for the FDD and TDD duplex transmissions, [7].

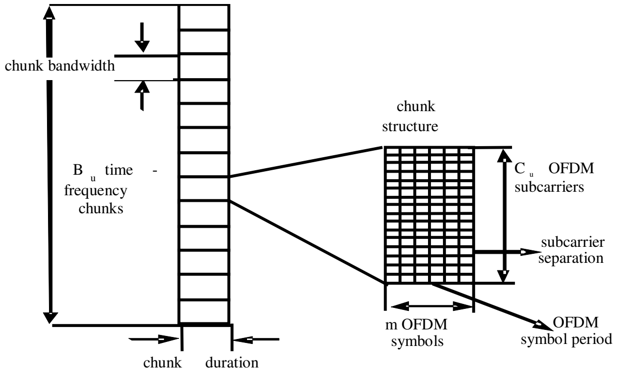

Fig. 1 OFDMA multi-user access principle and chunk structure.

III. COMPUTATION OF RECEIVED SINR P.D.F. IN THE CONSIDERED OFDMA MULTI-USER ACCESS SCHEME

Considering a multipath channel with N Rayleigh paths, out of

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TABLE I

PARAMETERS OF THE MULTI-USER ACCESS SCHEMES

Parameter	FDD duplexing mode: 2×20MHz	TDD duplexing mode: 1×100MHz
Centre frequency	5GHz DL / 4.2GHz UL	5GHz
FFT bandwidth	20MHz	100MHz
No. of subcarriers	512	2048
Subcarrier spacing	39062Hz	48828Hz
Symbol length	25.6µs	20.48µs
Cyclic prefix length	3.2µs	1.28µs
Total symbol length	28.8µs	21.76µs
No. of used subcarriers	416	1664
Signal bandwidth	16.25MHz	81.25MHz
Chunk size (subcarr×symb)	8 × 12 = 96	16 × 5

which the first one arrived has a x_1 level, and the other N-1 exhibit a_t relative attenuations and τ_t relative delays, the probability of signal's level, at the mobile receiver's input, to to equal r on frequency f_l, is expressed by, [9]:

$$\begin{aligned} P_{f_l}(r) = & \int_0^{\infty} p_1(x_1) \int_0^{\infty} p_2(x_2) \int_0^{\infty} p_3(x_3) \cdots \int_0^{\infty} p_{N-1}(x_{N-1}) \cdot \\ & \cdot Ql\left(r, x_1, x_2, x_3, \cdots x_{N-1}, \Omega l_{1:N}, \Omega l_{2:N}, \Omega l_{3:N}, \cdots \Omega l_{N-1:N}\right) \cdot \\ & \cdot dx_{N-1} \cdots dx_3 dx_2 dx_1 \end{aligned} \quad (1)$$

In (1) the function $Ql(r, x_1, x_2 \ldots, x_{N-1}, \Omega l_{1:N}, \Omega l_{2:N}, \ldots, \Omega l_{N:\,N-1})$, see (2), expresses the occurrence probability of a level x_N on the last path, N, that would make the value of the entire received signal equal r, when the signals on the other N-1 paths have levels $x_1 - x_{N-1}$; the signals of paths 2,…N are affected by the stated attenuations and relative delays, [9].

$$\begin{aligned} & Ql\left(r, x_1, x_2, x_3, \ldots, x_{N-1}, \Omega l_{1:N}, \Omega l_{2:N}, \Omega l_{3:N}, \ldots, \Omega l_{N-1:N}\right) = \\ & = \begin{cases} 0 & ;\Delta < 0 \\ p_N(\alpha l_1) & ;\Delta = 0 \quad ; \quad \Omega l_{i:j} = 2\pi \cdot f_l \cdot \left(\tau_i - \tau_j\right) \\ p_N(\alpha l_1) + p(\alpha l_2) & ;\Delta > 0 \end{cases} \end{aligned} \quad (2)$$

In (1) and (2) $p_t(\cdot)$ denotes the Rayleigh p.d.f. of the level on the propagation path t, [2]. The αl_1 and αl_2 are expressed by:

$$\begin{aligned} & \alpha l_{1,2} = -\sum_{t=1}^{N-1} x_t \cos(\Omega l_{t:N}) \pm \Delta; \\ & \Delta = \sqrt{r^2 - \left(\sum_{t=1}^{N-1} x_t \cdot \sin(\Omega l_{t:N})\right)^2} \; ; x_t = x_1 / a_t, t=2,\ldots,N \end{aligned} \quad (3)$$

The channel instantaneous SINR on frequency f_l can be obtained by dividing the received signal power level on the considered frequency, f_l, to the noise + interferences power at receiver's input. Since the noise power is constant for a short time interval and over a small frequency bandwidth, we may assume that the SINR p.d.f. is also expressed by (1), (2), (3).

Since the assessment of the overall performances of a transmission system employing adaptively a set S of QAM constellations requires the joint consideration of the channel characteristic and multi-user access method, the probability w_k of the channel to be in state S_k, i.e. the total received SINR to have a level between thresholds T_{k-1} and T_k, is obtained by integrating (1) between them with some particular aspects imposed by the method used for the user-chunk allocation, BFP (4) or FH (5).

For an OFDMA-type access the channel state is defined by the average SINR value for the chunk with index m, $SINR_m^{av}$. This average value is computed considering both the frequency and time dimension of the chunk. Due to the limited dimensions of the chunk, smaller than the channel coherence bandwidth and time, we may assume that each subcarrier of a chunk m has the same $SINR_m^{av}$.

For the BFP allocation, the probability w_{kBFP}, computed in (4), of the channel to be in state S_k is the probability that the average SINR value $SINR_m^{av}$ of chunk m lies between thresholds J_k, J_{k+1}, (the linear values corresponding to T_k, T_{k+1}) and that it has the maximum value, denoted by d in (4), out of all B_u available chunks. The w_{kBFP} equals the probability P_{km} of the $SINR_m^{av}$, denoted with d_m in (4), of chunk m to lie in this domain, multiplied with the probability Q_m that the average SINR levels of all other (B_u - 1) chunks to be smaller than the average SINR level of chunk m. This combined probability should be computed for m ranging from 1 to B_u, and accumulated on all chunks.

$$\begin{aligned} & w_{kBFP}\left(J_k < d < J_{k+1}\right) = \sum_{m=1}^{B_u} P_{km} \cdot Q_m = \\ & = \sum_{m=1}^{B_u} \left(\int_{J_k}^{J_{k+1}} P_{am}(d_m) \prod_{\substack{j=1 \\ j \neq m}}^{B_u} Q_{mj}(d_m) d(d_m) \right) \\ & P_{am}(d_m) = \frac{1}{C_u} \sum_{q=1}^{C_u} P_{f_{(mC_u+q)}}(d_m); \\ & Q_{mj}(d_m) = \int_0^{d_m} P_{aj}(y)\, dy \; ; d_m - \text{av. SINR in chunk m} \end{aligned} \quad (4)$$

where $P_{am}(d_m)$ represents an average probability of the subcarriers in chunk m (C_u distinct subcarriers in a chunk) to have the SINR level d_m and q denotes the subcarrier index in a chunk. The method described by (1)-(4) provides a good accuracy when compared to simulation results. The differences, between the state-probabilities obtained by computation and by simulations, are smaller than 1-2%, [9]. The error increases slightly with the carrier load L_c, i.e. the number of active users requiring a chunk, due to the probability of more users to have the best SINR on the same chunk (not included in the model), but this increase does not affect significantly the method's accuracy for carrier loads smaller than 50-60%.

For the FH allocation method, the probability w_{kFH} of the channel to be in state S_k, meaning that the average SINR level $SINR_m^{av}$ lies between thresholds J_k, J_{k+1} is given by:

$$\begin{aligned} & w_{kFH}\left(J_k < d < J_{k+1}\right) = \sum_{m=1}^{B_u} P_m \cdot \left(\int_{J_k}^{J_{k+1}} P_{am}(d_m) d(d_m) \right) \\ & P_{am}(d_m) = \frac{1}{C_u} \sum_{q=1}^{C_u} P_{f_{(mC_u+q)}}(d_m); P_m = \frac{1}{B_u} \quad ; \quad N_u = B_u \cdot C_u \\ & \Downarrow \\ & w_{kFH}\left(J_k < d < J_{k+1}\right) = \frac{1}{N_u} \cdot \sum_{l=1}^{N_u} \left(\int_{J_k}^{J_{k+1}} P_{f_l}(d_m) d(d_m) \right) \end{aligned} \quad (5)$$

where P_m is the probability of a user to get a certain chunk, probability that is independent of the selected chunk and N_u denotes the total number of payload subcarriers.

The error between computed and simulated results is smaller than 0.2%, [9], in the case of a multipath radio channel characterized by 4 different propagation paths defined in [6].

The approach described above, which should be performed for every SINR value when evaluating by computation the performances of the transmission scheme, requires a large amount of computation. Besides, for the BFP allocation, it should also be performed for every cell-carrier load (L_c). Therefore, a second approximate method to compute the p.d.f. of the total received signal level (and the received SINR), for any other average $SINR_a$ of the first arrived path was developed. It consists of three steps:

a. compute, using the analytical method described above, simulate or measure the probabilities of the receiver's SINR to lie between an imposed set of thresholds T_k, for a given $SINR_0$ of the first arrived path; note that these probabilities are affected by the allocation method employed, BFP or FH, and by the cell carrier load, $L_{c,\ for}$ the BFP allocation.

b. find an interpolation function f(x) that approximates the distribution of the SINR on the channel, fulfilling the conditions imposed by step a; x denotes the linear values of SINR.

c. translate and scale f(x) around the desired value of the SINR of the first arrived wave, $SINR_a$.

This method requires a smaller amount of computation, since it uses the analytical method for only one value of $SINR_0$ (for a multipath configuration and cell-carrier load, L_c). As an example, we analyze the WP5 Macro channel model (18 paths), [7], for an average $SINR_0$=16dB of the first arrived path [6]; the SINR values were split into S domains, corresponding to the regions of optimality of S = 11 QAM modulations [10], separated by SINR thresholds T_k, of table II. Assuming that the user chunk is BFP-allocated, the probabilities w_k of the total SINR to lie within each domain k (or state S_k), obtained as indicated in point a. above, are shown in table II, for L_c= 2%, 50%, 75% and 100% cell-carrier load.

TABLE II

SNR PROBABILITY DISTRIBUTION FOR SNR_0 = 16 dB OBTAINED BY SIMULATION; BFP USER CHUNK ALLOCATION

T_k [dB]	T_1=-2	T_2=8.3	T_3=13.2	T_4=16.2	T_5=20.2	T_6=23.6
L_c=2%	0	0	0	$2\cdot10^{-5}$	$1.4\cdot10^{-3}$	$3.9\cdot10^{-2}$
L_c=50%	0	0	0	$5\cdot10^{-5}$	$3.2\cdot10^{-3}$	$5.5\cdot10^{-2}$
L_c=75%	0	0	0	$8\cdot10^{-5}$	$8.5\ 10^{-3}$	$7.3\cdot10^{-2}$
L_c=100%	0	0	$3.1\cdot10^{-4}$	$6.8\cdot10^{-3}$	$2.6\cdot10^{-2}$	$8.8\cdot10^{-2}$

T_k [dB]	T_7=26.6	T_8=29.8	T_9=33	T_{10}=36.2	T_{11}=39.4
L_c=2%	$3.4\cdot10^{-1}$	$5.3\cdot10^{-1}$	$8.7\cdot10^{-2}$	$7.8\cdot10^{-4}$	0
L_c=50%	$3.5\cdot10^{-1}$	$5.0\cdot10^{-1}$	$8.8\cdot10^{-2}$	$7.9\cdot10^{-4}$	0
L_c=75%	$3.4\cdot10^{-1}$	$4.8\cdot10^{-1}$	$8.7\cdot10^{-2}$	$8.2\cdot10^{-4}$	0
L_c=100%	$3.1\cdot10^{-1}$	$4.8\cdot10^{-1}$	$8.9\cdot10^{-2}$	$1.0\cdot10^{-3}$	0

If f(x) is the function that interpolates the p.d.f. of the SINR, then this function should observe (6); condition (6.b) includes a modified upper threshold T_{11}=36.4 dB and insures the solvability of the system described below (J_k - linear values corresponding to the logarithmic values of the T_k thresholds).

$$w_k = \int_{J_k}^{J_{k+1}} f(x)dx; k = 1, \ldots, S; \text{a.} \quad f(J_{S+1}) = f(J_1) = 0; \text{b.} \qquad (6)$$

The interpolation function f(x) should also be positive and should have only one maximum across the whole range of the x-variable. To simplify its computation, the number of SINR domains is reduced by suppressing those who exhibit a w_k below an imposed value, e.g. $w_k<1\cdot10^{-3}$, and by adjusting accordingly the lowest and highest thresholds to T_{km} and T_{kM}. The f(x) was chosen to be a polynomial function of order k_M-k_m+1, (7.a). By obtaining the probabilities w_k of the SINR to lie within the k-th interval, relation (6.a) a. above and table II, the coefficients of f(x) are computed using (7.b) and (7.c).

$$\begin{aligned} f(x) &= \sum_{i=0}^{k_M-k_m+1} c_i \cdot x^i; \text{a.} \quad f(J_{k_m}) = f(J_{k_M}) = 0; \text{c.} \\ w_k &= \sum_{i=0}^{k_M-k_m+1} \frac{c_i}{i+1}(J_{k+1}^{i+1} - J_k^{i+1}); k = k_m \ldots k_{M-1}; \text{b.} \end{aligned} \qquad (7)$$

For the SNR_0=16dB case, the number of SINR domains, their thresholds and the order of f(x) are shown in table III.

TABLE III

PARAMETERS FOR THE DERIVATION OF f(x) FUNCTION

L_c	T_{km} [dB]	T_{kM} [dB]	SINR domains	f(x) order
2%, 50%, 75%	20.2	36.39	6	8
100%	16.2	36.39	7	9

The graphs f(SINR) vs. SINR obtained for $SINR_0$ = 16dB and L_c=2%, 50%, 75%, 100% are presented in fig. 2.

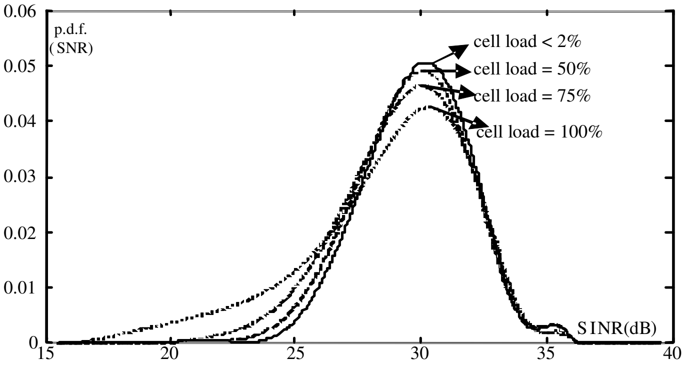

Fig. 2 Interpolation functions of the channel SINR p.d.f. for cell-carrier load L_c=2%, 50%, 75%, 100% and BFP chunk allocation.

The probabilities of the SINR to lie within each interval are obtained by integrating f(x) between the corresponding thresholds (6.a). The values obtained are shown in table IV, row 3, and are compared to the ones obtained by computer simulation, row 2, for L_c=2%. Comparisons between computed and simulated values, performed for L_c=25%, 75% and 100%, also indicate a good matching between the two sets of values.

TABLE IV

CHANNEL STATE PROBABILITIES COMPUTED WITH THE APPROXIMATE METHOD (C) AND OBTAINED BY SIMULATION (S) - BFP

T_k [dB]	T_1 -2	T_2 8.3	T_3 13.2	T_4 16.2	T_5 20.2	T_6 23.6	T_7 26.6	T_8 29.8
S. 16 dB	0	0	0	$2\cdot10^{-5}$	$1.4\cdot10^{-3}$	0.039	0.34	0.62
C.16 dB	0	0	0	$1\cdot10^{-5}$	$1.4\cdot10^{-3}$	0.039	0.34	0.62
S. 4 dB	0	0.0095	0.138	0.67	0.181	0.002	0	0
C. 4 dB	0	0.0077	0.144	0.665	0.176	0.0067	0	0
S. 1 dB	$8.7\cdot10^{-4}$	0.146	0.502	0.348	$3.5\cdot10^{-3}$	0	0	0
C. 1 dB	0.0014	0.150	0.494	0.341	$1.5\cdot10^{-2}$	0	0	0

The probabilities of the SINR to lie between the given thresholds, for different values of the average $SINR_0$, can also be computed using the function f(x). Denoting by $SINR_{ref}$ the value $SINR_0$ =16dB for which the f(x) = f_r(x) was derived, and by $SINR_a$ the $SINR_0$ for which the p.d.f. is to be computed, the interpolating function f_a (SINR) can be obtained by translating and scaling the $f_r(x)$, for the desired value of L_c, on the x-axis (in dB) which is equivalent to (8) for linear SINR.

$$f_a(x) = f_r(x \cdot c); \quad c = SNR_{ref} / SNR_a \tag{8}$$

The SINR probabilities w_k to lie between any pair of thresholds were computed by integrating the $f_a(x)$ between them. The values of w_k for SNR_a= 4dB and 1dB are presented in table IV together with the values obtained by computer simulations, for L_c = 2%. Additional comparisons performed by the authors for different values of $SINR_a$ and L_c show that the errors of the approximate method rest in the same range, smaller than 3%, as those of table IV.

The BFP allocation method ensures great state-probabilities only for a limited number of channel states, 2 maximum 3, the rest of the states being employed quite seldom. The SINR average values of these states are with 10-14 dB higher (for a channel with many propagation paths) than the SINR of the firstly arrived path, leading to the employment of larger QAM constellations, at the same BER, and to a significant throughput increase. This is because the BFP method ensures (with high probability) the chunk with the highest average SINR for each user, taking advantage of the frequency and user diversities of the multipath Rayleigh channel, in a very efficient way.

The w_{kFH} probabilities are computed taking the same steps as above, but usind the analytical expression given in (5). The w_{kFH} of the total SINR to lie inside each domain, for the same WP5 channel and average $SINR_0$ =16dB, are listed in table V and the corresponding p.d.f. is shown in fig. 3. The cell-carrier load does no longer affect the p.d.f., because the user chunks are allocated independently according to a pseudorandom sequence, as in the multi-user access scheme proposed in [5].

TABLE V

CHANNEL STATE PROBABILITIES FOR SNR_0 = 16 dB - FH

T_k [dB]	T_1=-2	T_2=8.3	T_3=13.2	T_4=16.2	T_5=20.2
L_c=x%	$5\cdot10^{-4}$	$1.1\cdot10^{-2}$	$3.7\cdot10^{-2}$	$1.47\cdot10^{-1}$	$2.38\cdot10^{-1}$
T_k [dB]	T_7=26.6	T_8=29.8	T_9=33	T_{10}=36.2	
L_c=x%	$2.21\cdot10^{-1}$	$7\cdot10^{-2}$	$4.5\cdot10^{-3}$	$2\cdot10^{-5}$	

Due to the larger number of SINR domains that have to be considered for the computation of the SINR's p.d.f., the inferior and superior thresholds were set as T_{km}=8.3dB and T_{kM}=36.2dB and the interpolation polynomial was of order 10.

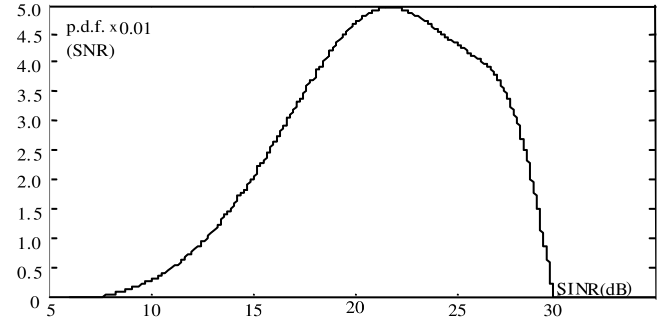

Fig. 3 Interpolation functions of the channel SINR p.d.f. - FH

For the FH allocation, the w_k probabilities delivered by the approximate method are differing with less than 0.3% from the ones obtained by computer simulations.

The number of channel states employed with non-negligible probabilities w_k is larger, 5 or 6, because the FH does not place the user on the particular chunk that ensures the best SINR. The average SINRs of these states are smaller, with even 8 dB, or greater, with up to 14 dB, than the $SINR_0$. The most employed states ensure average SINRs higher with 0-10 dB than SNR_0. This is because the FH allocation only performs an averaging over the whole available bandwidth.

IV. MARKOV CHAIN MODELING OF THE CHANNEL SINR STATES IN THE CONSIDERED OFDMA MULTI-USER ACCESS SCHEME

The p.d.f. of the total SINR derived in the previous section gives the average probabilities of the channel's SINR to rest between each pair of thresholds, and only allows the computation of average performances of a transmission scheme, over a relatively long time interval. If the transmission analysis requires the probability distributions of the studied performances, distributions that are dependent on the mobile speed, a Markov-chain modeling jointly the channel and multi-user access method should be elaborated, [4]. Note that different chains should be derived for different cell-carrier loads and mobile speeds, for BFP. The theoretical construction of such a Markov-chain is a very complex task and is currently studied in literature. This paper presents some Markov-chains obtained by computer simulations, for some significant situations of the considered OFDMA-access scheme. Computer simulations required to derive these chains with acceptable accuracy are far less time consuming than the complete evaluation, by simulations, of the performances of a transmission system. Out of the set of possible non-coded QAM modulations, this section employs adaptively in each chunk only the first 8, see table VI, according to the current value of the channel SINR, as in references [6], [8]. The complete Markov-chain assigned to the channel SINR-states has 8 possible states, one for each SINR domain, but it may be simplified by dropping out some states with very low associated probabilities, i.e. $w_k \leq 0.01$, to allow a simpler and still accurate performance evaluation.

TABLE VI

CHANNEL STATES AND SINR THRESHOLDS.

SNR state no.	1	2	3	4	5	6	7	8
SNR thr. (dB)	-∞	8.3	13.2	16.2	20.2	23.6	26.6	29.8

Tables VII.a. – VII.c show the state w_k and state-transition w_{kj}, probabilities that characterize the Markov-chains associated to the channel for the BFP allocation; these probabilities are determined for two mobile velocities and three significant values of the cell-carrier load L_c. For L_c = 2% and 50%, the state S5 can be neglected. The increase of L_c from 2% to 50% increases the probabilities of the lower states S6 and S7. Besides this effect, a further increase of L_c up to 100% leads to an greater number of states, i.e. from 3 to 4 in table VII.c, state S5 being also used.

For low user speeds, e.g. v = 4 km/h, the transition-probabilities w_{kj} between different states are very small and the transition-probabilities w_{kk} between the same states are close to 1. For high user-speeds, e.g. v=120 km/h, the w_{kj} probabilities increase, while the w_{kk} probabilities decrease significantly.

TABLE VII

STATE AND STATE-TRANSITION PROBABILITIES FOR TWO USER- SPEEDS AND DIFFERENT L_C; BFP USER-CHUNK ALLOCATION.

TAB.a BFP ALLOCATION, L_C=2%

INITIAL STATE →		6	7	8	State probab
Final state ↓	SPEED ↓				
6	120 km/h	0.0839	0.0554	0.0269	0.03775
	4 km/k	0.9713	0.0018	0.00077	
7	120 km/h	0.464	0.3972	0.1128	0.33888
	4 km/k	0.0158	0.9807	0.0095	
8	120 km/h	0.4484	0.545	0.3563	0.62305
	4 km/k	0.0127	0.0174	0.9897	

TAB.B BFP ALLOCATION, L_C=50%

INITIAL STATE →		6	7	8	State probab
Final state ↓	SPEED ↓				
6	120 km/h	0.55177	0.0375	0.02038	0.05652
	4 km/k	0.972	0.00243	0.001235	
7	120 km/h	0.22413	0.6999	0.1532	0.34746
	4 km/k	0.014	0.98148	0.00937	
8	120 km/h	0.2203	0.2603	0.82546	0.59292
	4 km/k	0.0135	0.016	0.98938	

TAB.C BFP ALLOCATION, L_C=100%

INITIAL STATE →		5	6	7	8	State probab
Final state ↓	SPEED ↓					
5	120 km/h	0.0524	0.0463	0.0362	0.0206	0.0291
	4 km/k	0.97	0.00142	0.0011	0.0006	
6	120 km/h	0.1475	0.1336	0.1128	0.0718	0.0917
	4 km/k	0.0004	0.9729	0.0035	0.0021	
7	120 km/h	0.3882	0.3786	0.3563	0.2868	0.3196
	4 km/k	0.0119	0.0116	0.9797	0.0088	
8	120 km/h	0.4117	0.4412	0.4944	0.6163	0.5565
	4 km/k	0.013	0.0136	0.0153	0.9882	

As for the Markov-chain that jointly models the channel and the FH allocation method, it would not be affected by the cell-load and mobile velocity, due to the predefined order of allocating chunks to one user. Table VIII shows the w_k and w_{kj} probabilities parameters that describe the Markov chain of the studied channel for the FH. As already seen in section III, the FH allocation method leads to more channel states that have significant occurrence probabilities w_k, e.g. 7 instead of 3 or 4, on the same channel profile. The state-transition probabilities w_{kj} and w_{kk} have comparable values, because the FH method allocates the user on a predefined frequency band, regardless its SINR value, and so the SINR takes a wider range of values.

TABLE VIII

STATE AND STATE-TRANSITION PROBABILITIES FOR FH ALLOCATION; ANY CELL-CARRIER LOADING AND USER SPEED.

Init. st→	2	3	4	5	6	7	8
Final st. ↓							
2	0.0111	0.0135	0.0119	0.0122	0.0125	0.0111	0.0077
3	0.0572	0.0440	0.0382	0.0387	0.0370	0.0361	0.0338
4	0.1639	0.1577	0.1532	0.1539	0.1510	0.1408	0.1358
5	0.2553	0.2474	0.2486	0.2436	0.2363	0.2341	0.2160
6	0.2527	0.2612	0.2720	0.2652	0.2640	0.2649	0.2583
7	0.1981	0.2174	0.2097	0.2187	0.2258	0.2279	0.2469
8	0.0614	0.0578	0.0655	0.0683	0.0725	0.0843	0.100
w_k	0.0116	0.0376	0.1490	0.2385	0.2649	0.2229	0.0745

V. CONCLUSIONS

The paper presents an analytical method to compute the p.d.f. of the SINR on the mobile Rayleigh-faded multipath channel and its probability to lie between an imposed set of thresholds, considering two user-chunk allocation methods in an OFDMA multi-user access scheme. This method ensures errors below 2%, compared to the probabilities obtained by computer simulations, but requires a significant amount of computation to evaluate the performances of adaptive QAM modulations, when employed over a large domain of SINR. To avoid this inconvenience, the paper proposes a more practical approximate method to compute the same probabilities, which uses the analytical method for only a single $SINR_0$ value and ensures about the same accuracy, but requires a significantly smaller amount of computation.

The employment of the BFP allocation leads to a smaller number of states that have significant state-probabilities; moreover, these states ensure an average SINR much greater than the SINR of the firstly arrived path, leading to higher throughputs provided in adaptive modulation schemes.

The FH allocation method leads to a larger number of channel states with significant probabilities; these states ensure smaller average SINRs, thus leading to a smaller average throughput provided by a QAM set employed adaptively.

The SINR's p.d.f. provided by the BFP allocation is slightly extended with the increase of number of users on the cell-carrier, thus decreasing the average SINR ensured, while the FH allocation is not affected by the cell-carrier loading.

The state probabilities provided by both allocation methods are not affected by the users' speeds, the channel p.d.f. being independent of this parameter, but the state-transition probabilities are depending of it.

Based on the SINR's p.d.f. and additional computer simulations, a Markov-chain modeling of the "composed" channel is proposed for the transmission system considered, for both allocation methods. The state-transition probabilities are depending of the channel variability, i.e. MS's speed, and may be used for short-term analysis of the system performances.

The two proposed joint channel-multi-user access modeling methods can be applied with some minor modifications to other OFDMA multi-user access techniques and/or other channel models.

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