Multi-user detection of nonlinearly distorted MC-CDMA symbols by microstatistic filtering

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***Abstract*— Multi-carrier code division multiple access is a powerful modulation technique that is being considered in many emerging broadband communication systems. In a downlink scenario orthogonal spreading sequences are used since they reduce multiple access interference compared to non-orthogonal. However, the nonlinear amplification of the transmitted signal destroys the orthogonality and, thus, reduces the system performance. In order to avoid performance degradation without requiring large back-offs in the transmitter amplifier, it becomes necessary to use multi-user detection techniques at the receiver side. Conventional multi-user detectors (MUD) are designed for linear environments and, as a result, might not exhibit enough performance improvement. In this paper a new MUD based on microstatistic filtering is proposed. The presented MUD uses piece-wise linear filtering in conjunction with threshold decomposition of the input signal, which introduces a nonlinear effect, to improve performance when a nonlinearity is present. Maximum performance improvement compared to conventional MUD is achieved for low spreading factors and user loads no greater than 50%.**

***Index Terms*— MC-CDMA, nonlinear distortion, multi-user detection, microstatistic filter.**

I. INTRODUCTION

Multi-carrier code division multiple access (MC-CDMA) is a modulation technique that exploits the advantages of spread spectrum (SS) and orthogonal frequency division multiplexing (OFDM). One of the major disadvantages of multi-carrier (MC) systems based on OFDM is the high sensitivity to nonlinear amplification, which requires large back-off in the transmitter amplifier and, as a consequence, inefficient use of power amplifiers. On the other hand, using low back-offs leads to signal distortion and, as a result, increased performance degradation. When SS techniques are used in conjunction with OFDM, as in MC-CDMA, the nonlinear amplification of the signal destroys the properties of the spreading sequences. Particularly in the synchronous downlink, where orthogonal spreading codes are used to reduce the multiple access interference (MAI), the nonlinearity (NL) destroys the orthogonality thus increasing MAI.

Several techniques can be found in the literature to reduce the sensitivity of MC-CDMA systems to nonlinear amplification. Most common transmitter side solutions include predistortion [1] and peak-to-average power ratio (PAPR) reduction [2]. Receiver side strategies usually combine iterative decoding and multi-user detection so that both NL compensation and MAI is taken into account [3], [4]. In that sense, a multi-user detector (MUD) that performs a joint detection, estimation and cancellation of the nonlinear distortion effects was introduced in [3] and a receiver based on an iterative block decision feedback equalizer, combined with estimation and cancellation of the nonlinear distortion effects was proposed in [4]. The major drawback of such solutions is that they significantly increase the complexity requirements at the receiver side. Complexity is mainly given by the number of iterations which, in fact, increases with the number of active users. Moreover these techniques require that predistortion is used at the transmitter side. In order to reduce the complexity requirements, [5] proposes to iteratively optimize the signal constellation of each active user, via a multi-user approach, such that the intersymbol interference at the relevant decision devices is minimized. However, [5] also requiers predistortion at the transmitter side.

In this paper, a novel microstatisctic MUD (MSF-MUD) based on complex-valued multi-channel microstatistic filters (C-M-CMF) [6], [7] for the detection of nonlinearly distorted MC-CDMA symbols in a downlink scenario is presented. The key idea is to use a nonlinear but piece-wise linear structure in order to model the nonlinear behavior of the transmitter's power amplifier with high flexibility and simplicity. Then, prior to the detection stage, compensation of the received complex valued symbols is done. The transmitter's NL is modeled by using a training sequence and following the minimum mean-squared error (MMSE) criterion. Simulation results show that the proposed detector outperforms conventional linear ones [8] when low spreading factors and user loads no greater than 50% are used. For other system configurations similar performance is obtained. This improvement is achieved at expenses of an acceptable increase of the computational complexity in comparison with that of conventional minimum mean-square error MMSE-MUD. Nevertheless, the complexity requirements of a MSF-MUD based receiver are much smaller than those of [3]–[5]. Moreover, the proposed scheme neither requires predistortion at the transmitter nor iterative decoding at the receiver.

The structure of the paper is as follows. In Section II the MSF-MUD is presented. Section III describes the system configurations that are used in the simulations. Then, in Section IV, the performance improvement capabilities of the proposed MUD are evaluated and, subsequently, in Section

V the numerical results using several system configurations are shown. Finally, in Section VI some conclusions from the presented work are drawn.

II. MICROSTATISTIC FILTERING BASED MUD

In this section the basis of microstatistic filtering and its application to MUD is presented. In order to get the particular expressions describing C-M-CMF in a more readable way, discrete-time index for the simplicity is omitted.

A. C-M-CMF structure

A block diagram of C-M-CMF is shown in Fig. 1 [6]. Here, M, $y(i)$ and $\hat{d}(k)$ are the number of the input and output channels, the i-th input complex signal and the k-th output complex signal of the C-M-CMF, respectively. As it can be observed the C-M-CMF consists of M complex-valued threshold decomposers (TD) and a set of M multi-channel Wiener filters (WF) .

The i-th TD, denoted as TD_i, performs a threshold decomposition of the signal $y(i)$ into a set of the L complex-valued signals $y(i,j)$. The outputs of each TD are uniquely determined from its input signal by

$$\begin{aligned}\mathbf{Y}(i) &= [y(i,1),\cdots,y(i,L)]^{\mathsf{T}} = \mathrm{D}_i\{Y(i)\}\,e^{j\phi(i)} = \\ &= [Y(i,1),\cdots,Y(i,L)]^{\mathsf{T}}\,e^{j\phi(i)}\end{aligned} \tag{1}$$

where $Y(i)=|y(i)|$, $\phi_i = arg\{y(i)\}$ and the superscript T signifies matrix transposition. The term $\mathrm{D}_i\{Y(i)\}$ represents the threshold decomposition of $Y(i)$ into a set of L signals as

$$Y(i,j)=\begin{cases} 0 & \text{if} \quad Y(i)<l(i,j-1) \\ Y(i)-l(i,j-1) & \text{if} \quad l(i,j-1)\le Y(i)<l(i,j) \\ l(i,j)-l(i,j-1) & \text{if} \quad l(i,j)\le Y(i) \end{cases} \tag{2}$$

for $1\le j\le L$. The parameters $l(i,j)$ constituting the vector $\mathbf{L}_i=[l(i,1),\cdots,l(i,L)]^{\mathsf{T}}$ are the positive real-valued threshold levels of TD_i, which satisfy $0=l(i,0)<l(i,1)<\ldots<l(i,L)=\infty$ [6].

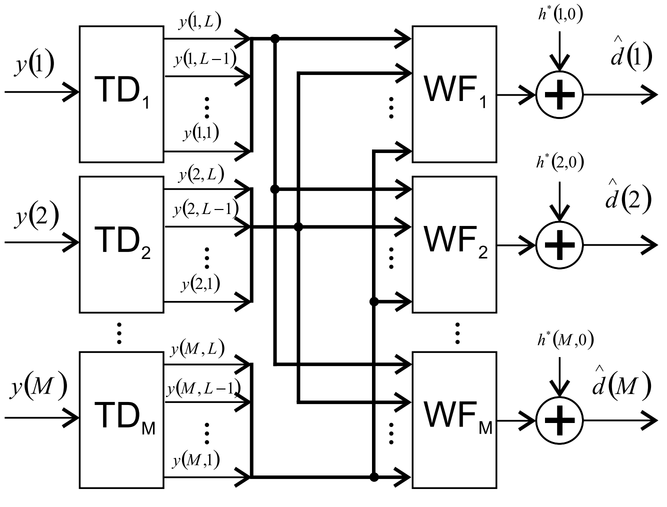

Fig. 1. Block scheme of the C-M-CMF.

According to Fig. 1, the output signals obtained after threshold decomposition are fed into the set of M multi-channel WFs. Let WF_k, $k=1,2,\ldots,M$ denote the k-th WF, then the k-th output of C-M-CMF is computed as

$$\hat{d}(k)=\mathbf{H}^{\mathsf{H}}(k)\mathbf{Y} \tag{3}$$

where $\mathbf{H}(k)=\left[h(k,0),\ \mathbf{H}'^{\mathsf{T}}(k)\right]^{\mathsf{T}}$, $\mathbf{Y}=\left[1,\ \mathbf{Y}'^{\mathsf{T}}\right]^{\mathsf{T}}$ and the superscript H denotes Hermitian transposition. $\mathbf{Y}'$ and $\mathbf{H}'(k)$ are two vectors of the same length that comprise the output signals from all TDs and the coefficients of k-th WF, respectively. The constant term $h(k,0)$ is applied to the C-M-CMF structure in order to obtain an unbiased estimation of desired signal at the output of WF_k.

Before being able to use the microstatistic filter one has to determine the threshold levels and the coefficients of the WFs. This is done by transmitting a special training sequence and by following the MMSE criterion. Let us define the set of threshold levels from all TDs as $\mathbf{L}=\left[\mathbf{L}^{\mathsf{T}}(1)\cdots\mathbf{L}^{\mathsf{T}}(L)\right]^{\mathsf{T}}$, then according to the MMSE criterion the optimum values for $\mathbf{L}$ and $\mathbf{H}(k)$ are obtained as the solution that minimizes the cost functions

$$\mathrm{MSE}(\mathbf{H}(k),\mathbf{L}) = E\left[e(k)e(k)^*\right], \tag{4}$$

where $e(k)=d(k)-\hat{d}(k)$, the asterisk denotes complex conjugation and $d(k)$ is the desired signal at the output of WF_k. The solution of this optimization problem can be obtained by means of an iterative process, where each iteration consists of three basic steps [6]. In the first step, the vector $\mathbf{L}$ is estimated. This can be done by using either a scanning method, a genetic algorithm based method or a cumulative distribution function based method [9]. In the second step, the optimum coefficients of the M WFs are computed based on the estimation of the vector $\mathbf{L}$ as

$$\mathbf{H}^{opt}(k)=\mathbf{R}^{-1}\mathbf{P}(k) \tag{5}$$

where $\mathbf{R}=E\left[\mathbf{Y}\mathbf{Y}^{\mathsf{H}}\right]$ is the autocorrelation matrix of $\mathbf{Y}$ and $\mathbf{P}(k)=E\left[d(k)\mathbf{Y}\right]$ is the cross-correlation vector of $d(k)$ and $\mathbf{Y}$. Both, $\mathbf{R}$ and $\mathbf{P}(k)$ are estimated by using a training sequence. During the last step, the cost functions in (4) are evaluated to determine whether another iteration is required or not. The iterative process is stopped either when the cost functions are minimized or the error is acceptable from an application point of view. In such case the resulting values for $\mathbf{L}$ and $\mathbf{H}^{opt}(k)$ are declared as the optimum parameters of the microstatistic filter. The reader is referred to [6] and [9] for further details regarding the design procedure of C-M-CMF.

Even though the above described algorithm is able to determine the optimum parameters of the C-M-CMF, the associated complexity is too large for a practical implementation. Thus, a suboptimum solution is used to significantly reduce both the required complexity while at the same scarcely reduce the performance. This suboptimum solution relies on the fact that a suboptimum threshold level can be found independently of the system configuration. Thus, the training algorithm will only have to determine the suboptimum coefficients of the WF, that is, only run steps 2 and 3 from the above described iterative process.

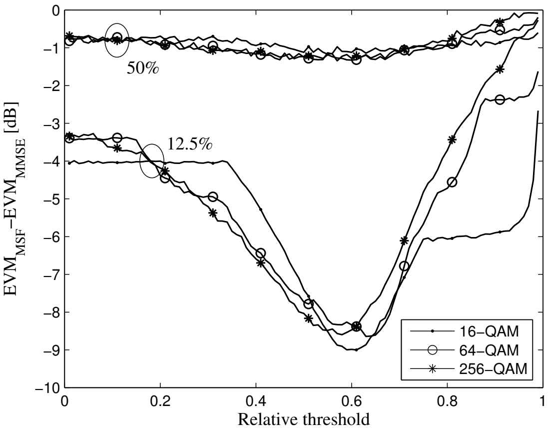

Fig. 2. EVM reduction with respect to MMSE-MUD as a function of the threshold level. 16, 64 and 256-QAM, length-32 Hadamard codes with 12.5% and 50% user load, and Saleh NL operating at IBO=20dB is used.

B. MSF-MUD structure

The structure of a MSF-MUD is very similar to that of a conventional linear MMSE-MUD [8]. Since microstatistic filters represent a sub-set of piece-wise linear filters [10], it follows that MMSE-MUD is a special case of MSF-MUD. In fact, MSF-MUD can be obtained from MMSE-MUD by just substituting the set of multi-channel linear filters in MMSE-MUD, located between the bank of matched filters (BMF) and decision devices, by the C-M-CMF described in II-A. As a result, it seems reasonable to assume that MSF-MUD has to provide better or at least the same performance properties than a corresponding MMSE-MUD. This is achieved at the expense of higher computational complexity requirements at the receiver. The complexity of MSF-MUD is greater than that of MMSE-MUD in $(L-1)MN$ complex-valued multiplications and accumulations (MAC), where N is the memory length of WF_k. Further details concerning MSF-MUD can be found in [9].

III. SYSTEM CONFIGURATIONS

In this section we describe the system configurations that will be used later in both Section IV and Section V. We setup a conventional MC-CDMA system, operating at different user loads, with length-16 and length-32 Hadamard spreading codes and 16-QAM, 64-QAM and 256-QAM baseband modulation schemes. As we already discussed, a downlink scenario is considered, thus, perfect user synchronization is assumed. In order to minimize the performance degradation introduced by the NL the spreading sequences are chosen so that average PAPR is minimized. Moreover, according to [11] an oversampling rate of 4 is used to avoid aliasing the out of band distortion into the data bearing tones.

Two major types of power amplifiers are typically used in communication systems, traveling wave tube amplifiers (TWTA) and solid-state power amplifiers (SSPA) [11]. Comparatively, TWTA introduce more significant distortion than SSPA, therefore, since our intention is to provide stringent conditions to evaluate algorithms, TWTA will be considered.

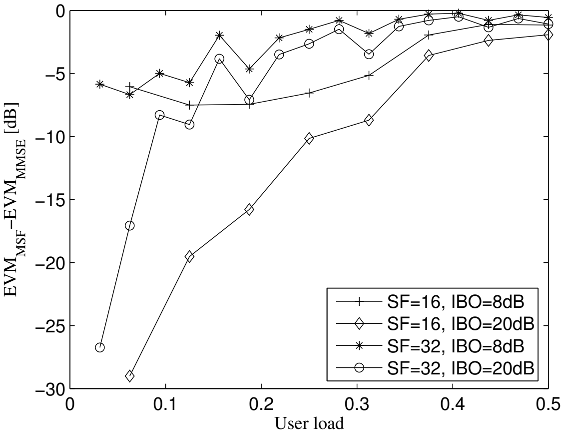

Fig. 3. EVM reduction with respect to MMSE-MUD as a function of the user load. Hadamard codes of length 16 and 32, 16-QAM baseband modulation and Saleh NL operating at IBO=8dB and IBO=20dB is used.

In this paper we use the widely accepted Saleh baseband model for TWTA [12] defined by the following amplitude-to-amplitude modulation (AM/AM) and amplitude-to-phase modulation (AM/PM) characteristics [11],

$$\mathrm{G}(u_x)=\frac{\kappa_G\cdot u_x}{1+\chi_G\cdot u_x^2},\ \Phi(u_x)=\frac{\kappa_\Phi\cdot u_x^2}{1+\chi_\Phi u_x^2} \qquad (6)$$

In the expressions above we chose $\kappa_G=2$, $\chi_G=\chi_\Phi=1$ and $\kappa_\Phi=\pi/3$. The operating point of the NL is defined by the so called input back-off (IBO) which corresponds to the ratio between the saturated and average input power,

$$\mathrm{IBO}_{dB}=10\log_{10}\left(P_{max}/\overline{P_x}\right) \qquad (7)$$

At the receiver side, a conventional MMSE-MUD and the proposed MSF-MUD are used. In order to minimize the complexity requirements of MSF-MUD just one threshold level is used for decomposition, i.e. L=2. Moreover, since an AWGN channel is used, the memory length of the multi-channel filters is set to N=1. Note that, this means increasing the complexity requirements, with respect to MMSE-MUD, in just one MAC per user.

IV. MSF-MUD PERFORMANCE EVALUATION

In this section we evaluate the performance improvement capability of the proposed MSF-MUD when a NL is present. Performance evaluation is computed by means of error vector magnitude (EVM) of the constellation at the output of the MUD. Let R_n and I_n denote the constellation point at MUD output and the ideally received constellation point, respectively, EVM is computed as

$$\mathrm{EVM}=10\log_{10}\left(E[|R_n-I_n|^2]/P_{ref}\right) \qquad (8)$$

where P_{ref} is the power of the outmost ideal constellation point. Let us recall from Section III that just one threshold level is used for decomposition. As we already discussed in Section II-A, in order to achieve maximum performance it is required to use the optimum threshold level. This is evaluated in Fig. 2 where EVM reduction of the proposed MSF-MUD with respect to MMSE-MUD as a function of the threshold level is shown. As it can be seen, maximum performance improvement is obtained when a relative threshold level around 0.6 is used. It is important to state that extensive simulations have been done to determine the threshold levels that provide best performance for each system configuration even though, for the sake of simplicity, only some simulation results are shown. In fact, after evaluating the extensive simulation results we realized that, when Hadamard codes are used, maximum performance is generally obtained if the threshold level is set to 0.6, independently of the back-off, the baseband modulation, the SF and the user load. For the remaining cases, the performance degradation with respect to the optimum relative threshold level was so small that could be neglected. Thus, as we already discussed in Section II-A, by setting the threshold level to 0.6 one significantly reduces the complexity associated to the training process while scarcely reduces the performance.

Another important aspect to notice in Fig. 2 is that, independently of the relative threshold level, the performance of the proposed MUD is always better than or, at least, equal to that of MMSE-MUD. This is further analyzed in Fig. 3 where EVM reduction using different user loads and system configurations is evaluated. Here, 16-QAM and a threshold level of 0.6 is used. Although simulation results show that the performance of the proposed MUD is never worse than that of conventional MMSE-MUD, it can be seen that a high performance improvement is only obtained for low user load and low spreading factors.

V. NUMERICAL RESULTS

In this section we evaluate the bit-error rate (BER) characteristics of the proposed MSF-MUD using different system configurations. We also compute BER for conventional MMSE-MUD so that the performance of both detectors can be compared. According to the conclusions from Section IV the MSF-MUD is configured to use a relative threshold level equal to 0.6 independently of the system configuration.

Fig. 4 to Fig. 6 show the BER characteristics of a nonlinearly distorted MC-CDMA system using length-16 Hadamard spreading codes with 12.5%, 31.25% and 50% user loads, respectively. It can be seen that for both low SFs and low to moderate user loads (see Fig. 4 and Fig. 5) a high performance improvement, in terms of BER, is obtained by using MSF-MUD compared to MMSE-MUD. Let us first evaluate the back-off requirements to achieve BER performances close to those obtained in linear AWGN channels. For MMSE-MUD and 64-QAM/256-QAM, input back offs of 20dB/24dB are required to achieve BER performances close to linear AWGN ones, while 16dB/20dB of IBO is enough if MSF-MUD is used. Thus, a reduction of 4dB in the amplifier's IBO requirements is achieved. On the other hand, if we focus on the signal-to-noise ratio requirements to achieve a target BER of 10^{-6} we can notice that for 64-QAM mapping and IBO=16dB, the required Eb/No for MSF-MUD is around 6dB to 7dB lower than that required for MMSE-MUD. Let us now consider the half-loaded system in Fig. 6, as we discussed in Section IV, here some performance improvement is still reached although it is not as important as for 12.5% and 31.25% user loads.

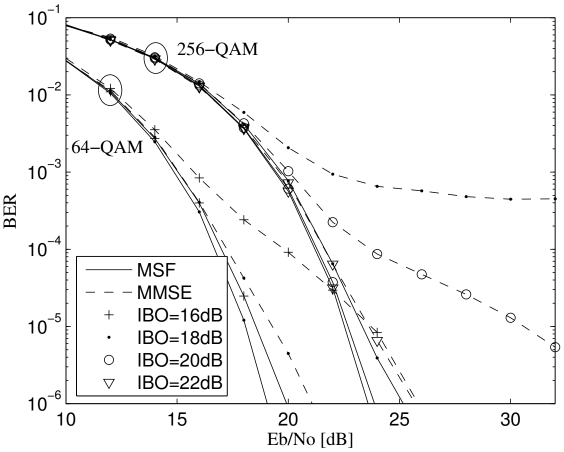

Fig. 5. BER performance of a nonlinearly distorted MC-CDMA system using MSF-MUD and MMSE-MUD. 31.25% user load and SF=16 is used.

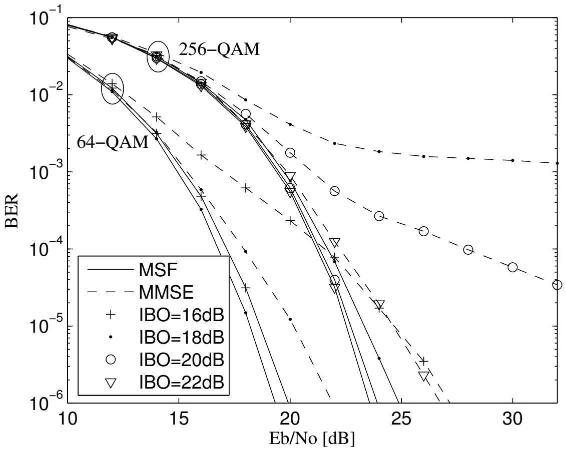

Fig. 4. BER performance of a nonlinearly distorted MC-CDMA system using MSF-MUD and MMSE-MUD. 12.5% user load and SF=16 is used.

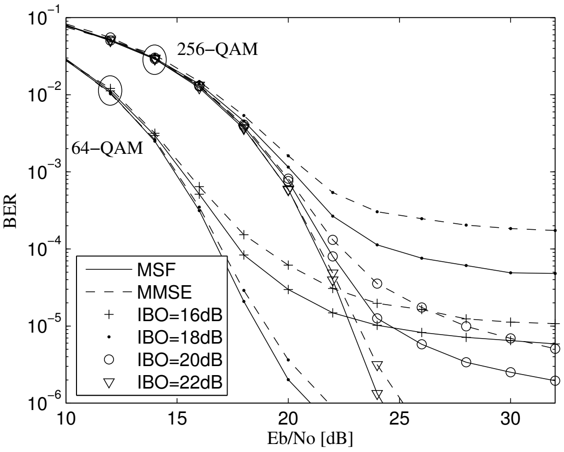

Fig. 6. BER performance of a nonlinearly distorted MC-CDMA system using MSF-MUD and MMSE-MUD. 50% user load and SF=16 is used.

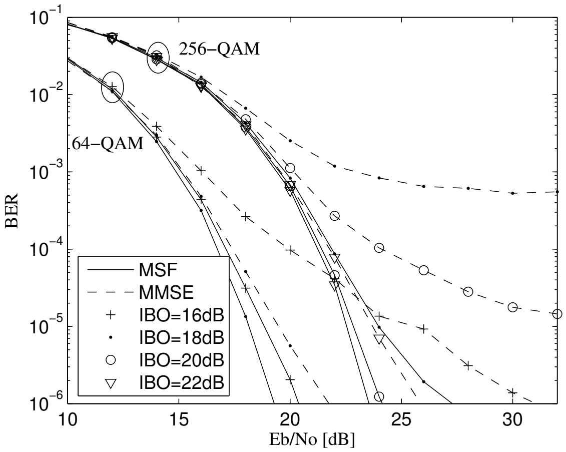

Fig. 7. BER performance of a nonlinearly distorted MC-CDMA system using MSF-MUD and MMSE-MUD. 12.5% user load and SF=32 is used.

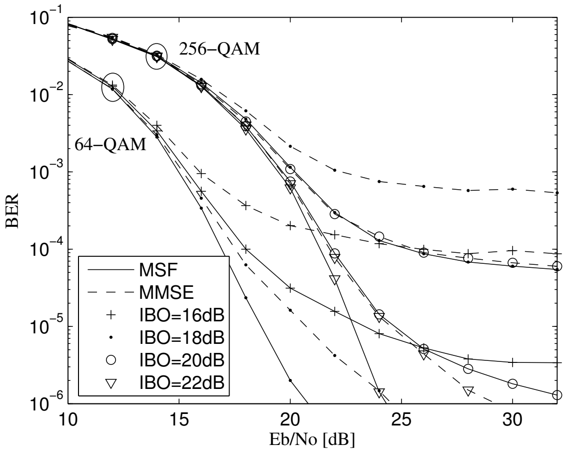

Fig. 8. BER performance of a nonlinearly distorted MC-CDMA system using MSF-MUD and MMSE-MUD. 31.25% user load and SF=32 is used.

Fig. 7 and Fig. 8 show the BER characteristics of a nonlinearly distorted MC-CDMA system using length-32 Hadamard spreading codes with 12.5% and 31.25% user loads, respectively. The BER performance for 50% user load is not shown since it is very similar to that in Fig. 6, where a SF of 16 is used. As it can be observed from Fig. 7 and Fig. 8, when a SF of 32 and low to moderate user loads are used a high performance improvement with respect to MMSE-MUD is still obtained.

From the presented simulation results we conclude that as we already pointed in Section II, MSF-MUD provides a great performance improvement compared to MMSE-MUD in low user loads scenarios with short SFs.

VI. CONCLUSIONS

In this paper a MSF-MUD for nonlinearly distorted MC-CDMA signals has been presented. MSF-MUD can be understood as a MMSE-MUD where the multi-channel linear filter located between the BMF and the decision devices is substituted by a multi-channel microstatistic filter, which consists of a complex-valued TDs and WFs. From a practical point of view two parameters must be specified before using the proposed detector, the threshold levels used for decomposition and the memory length of the WF. After evaluating extensive simulation results we concluded that for most configurations one threshold level is enough, moreover the optimum relative threshold level can be determined independently of the back-off, the baseband modulation, the SF and the user load. Thus, significantly reducing the complexity of the training process.

In this paper we also compared the performance of the proposed MUD with that of a conventional MMSE-MUD when a NL is present. Simulation results suggested that the performance of MSF-MUD is always better than or, at least, equal to that of MMSE-MUD. However, large performance improvement is achieved for low SFs and user load no greater than 50%. Although this improvement is obtained at expenses of increasing the receiver computational complexity, it has been shown that a minimum increase of the complexity requirements with respect to MMSE-MUD, just one MAC per user, is enough to achieve noticeable performance improvement in nonlinear AWGN channels.

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