Evaluation of performance improvement capabilities of PAPR-reducing methods

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***Abstract*— One of the major drawbacks of multicarrier modulation is the large envelope fluctuations which either require an inefficient use of high power amplifiers or decrease the system performance. Peak-to-average power ratio (PAPR) is a very well known measure of the envelope fluctuations and has become the cost function used to evaluate and design multicarrier systems. Several PAPR-reducing techniques have been proposed with the aim to alleviate back-off specifications or increase the system performance. Besides the fact that these techniques have varying PAPR-reduction capabilities, power, bandwidth and complexity requirements, it is interesting to notice that the performance of a system employing these techniques has not been fully analyzed. In this paper we, first, develop a theoretical framework for both PAPR and the distortion introduced by a nonlinearity, and then simulate an OFDM system employing several well known PAPR-reducing techniques from the literature. By means of the theoretical analysis and the simulation results we will show the relation between PAPR and the performance of OFDM systems when a clipping device is present and we will evaluate the real performance improvement capabilities of the PAPR-reducing methods. The agreement between the theoretical and the simulation results demonstrate the validity of the analysis.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a powerful modulation technique being used in many new and emerging broadband communication systems. The main advantage of this technique is its robustness against time dispersion in multipath fading channels. The downside is the large amplitude variations of the OFDM signal, which requires large back-off in the transmitter amplifier and, as a consequence, inefficient use of high power amplifiers (HPA).

In order to reduce the distortion caused by a HPA without setting it to large back-offs, several techniques have been introduced that limit the peak of the envelope of the signal [1], a problem that is usually referred to as peak-to-average power ratio (PAPR) reduction. Besides the fact that these techniques have varying PAPR-reduction capabilities, power, bandwidth and complexity requirements, it is important to notice that the performance of a system employing these techniques has not been fully analyzed. PAPR is a very well known measure of the envelope fluctuations of a multicarrier (MC) signal and has become the figure of merit used in the literature to define the goodness of a method. As a result, the problem of reducing the envelope fluctuations with the aim to increase the system performance has turned to reducing PAPR.

In this paper we, first, present a quantitative study of PAPR and the distortion introduced by a nonlinearity, and then simulate an OFDM system employing several well known PAPR-reducing techniques from the literature. By means of the theoretical analysis and the simulations results we will show that the performance of an OFDM system when a nonlinear amplifiers is present is not clearly related to its PAPR. Moreover, it will be shown that altough spectral spreading is reduced when applying PAPR-reduction, a bit error rate (BER) performance improvement is not always achieved.

The structure of the paper is as follows. In Section II we present the system model and develop a theoretical framework of both PAPR and the distortion introduced by a nonlinearity. In Section III we discuss some important considerations about PAPR-reduction. Subsequently, in Section IV, the performance improvement capabilities of some well-known PAPR-reduction techniques are evaluated. Finally, in Section V some conclusions from the presented analysis are drawn.

II. Theoretical analysis of the PAPR and system performance

An OFDM signal consists of N data symbol transmitted over N distinct subchannels. Consider one complex baseband OFDM symbol $s(t)$ defined over the time interval $t \in [0, NT]$,

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi kt/NT}, \tag{1}$$

where S_k is the complex baseband modulated symbol and N is the number of subcarriers. If the OFDM signal of (1) is sampled at $t = nT$, the complex samples can be described as

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j\frac{2\pi kn}{N}}, \qquad n = 0, 1, \ldots, N-1. \tag{2}$$

To study the statistics of the OFDM signal we note that s_n is a sum of independent and identically distributed random variables. According to the central limit theorem if the number of subcarriers is large enough, the signal can be approximated as a complex Gaussian distributed random variable. Therefore the envelope of the OFDM signal (2) follows a Rayleigh distribution as

$$f_X(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}}, \tag{3}$$

with

$$E[X] = \sigma\frac{\sqrt{\pi}}{2} \quad \text{and} \quad \text{var}[X] = \sigma^2(1-\frac{\pi}{4}), \qquad (4)$$

where the variance of the real and imaginary parts of the signal is assumed to be $\sigma^2/2$.

A. Peak-to-Average Power Ratio

PAPR is a common measure of the envelope fluctuations of an OFDM signal. Let $\mathbf{s}^{(m)}$ be the m-th OFDM symbol generated by (2). The PAPR of $\mathbf{s}^{(m)}$ is defined as the ratio between its peak power and the average power of all OFDM symbols

$$\text{PAPR}_m = \frac{\|\mathbf{s}^{(m)}\|_\infty^2}{E[\|\mathbf{s}^{(m)}\|^2]/N}, \qquad (5)$$

where the expectation is taken over all OFDM symbols. Assuming that the samples are mutually independent, which is true for non-oversampled signals, the complementary cumulative density function (CCDF) of the PAPR of an N-subcarrier OFDM signal, that is, the probability that the PAPR exceeds the threshold γ_0, is

$$\Pr(\gamma > \gamma_0) = 1-(1-e^{-\gamma_0})^N. \qquad (6)$$

B. High Power Amplifiers

Two major type of power amplifiers are used in communication systems, traveling wave tube amplifiers (TWTA) and solid state power amplifiers (SSPA). In order to reduce the performance degradation introduced by the nonlinear HPA digital predistortion (PD) is often used at the transmitter side [2]. The idea of PD is to modify the input signal of the HPA so that the output is as close as possible to the linearly amplified original signal. However, since the output power is limited, linearization can only be achieved up to the saturation point. In this paper we use a soft limiter (SL) nonlinearity since it models a situation where predistortion is used to linearize the HPA. Let $x_n = |x_n|e^{j\theta_n}$ be the signal at the input of a SL with A being the maximum output amplitude allowed, then the output signal becomes

$$y_n = \begin{cases} x_n & \text{if } |x_n| \le A \\ Ae^{j\theta_n} & \text{otherwise} \end{cases} \qquad (7)$$

The operating point of the nonlinearity is defined by the input back-off (IBO) that corresponds to the ratio between the saturated and average input powers.

$$\text{IBO}_{dB} = 10\log_{10}(\frac{P_{max}}{\overline{P}_x}) \qquad (8)$$

where P_{max} is maximum output power that in this case has been normalized so that $P_{max} \equiv P_{max,y} = P_{max,x} = A^2$.

C. In-Band and Out-of-Band Distortion

In this section we will analyze the effect of nonlinearities in an OFDM system. From the Bussgang theorem and by extending that to complex Gaussian processes (e.g. OFDM), the output y_n of a memoryless nonlinearity with a Gaussian input x_n can be written as the sum of a scaled input replica and an uncorrelated distortion term as [3]

$$y_n = \alpha x_n + d_n, \quad \text{where } \alpha = \frac{E[y_n x_n{}^*]}{E[|x_n|^2]}. \qquad (9)$$

The term α introduces a uniform attenuation and rotation to the data bearing tones that can be easily compensated at the receiver by introducing a correcting factor $\alpha^*/|\alpha|^2$. Hence, when α is compensated the performance degradation is just caused by the distortion term

$$d_n = \tilde{s}_n - \alpha s_n. \qquad (10)$$

Let us define

$$\mathbf{D} = [\mathbf{D}^{(in)}\ \mathbf{D}^{(out)}] \qquad (11)$$

where $\mathbf{D} \in \mathbb{C}^{L\cdot N}$ is the frequency domain representation of $\mathbf{d} \in \mathbb{C}^{L\cdot N}$ and $\mathbf{D}^{(in)} \in \mathbb{C}^N$ and $\mathbf{D}^{(out)} \in \mathbb{C}^{(L-1)\cdot N}$ represent the in-band and the out-of-band distortion respectively. $\mathbf{D}^{(in)}$ is the part of distortion that increases the bit error rate at the receiver, while $\mathbf{D}^{(out)}$ is, directly, the out-of-band radiation. Figure 1 shows the distortion term, computed using (10), that has been introduced to an OFDM system by a clipping nonlinearity. As it can be seen $\mathbf{D}^{(out)}$ matches perfectly the shape of the out-of-band radiation and $\mathbf{D}^{(in)}$ introduces an in-band noise.

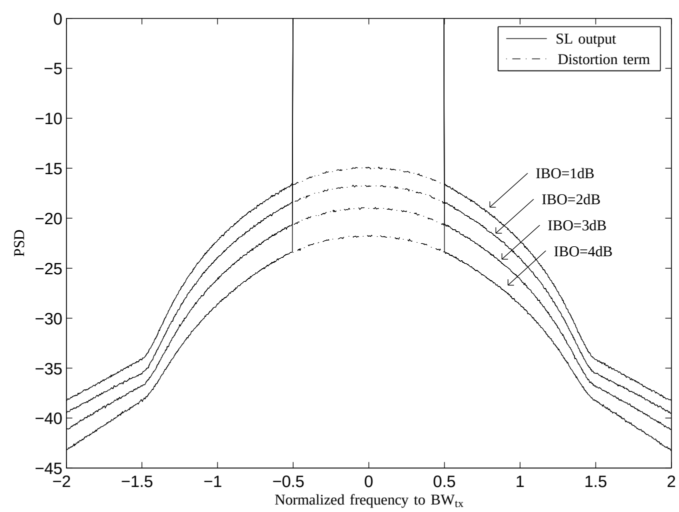

Fig. 1. Distortion introduced by a SL to an OFDM system.

From Figure 1 one can see that the spectrum of the distortion term depends on the working point of the nonlinearity, while it is independent of the number of subcarriers and the mapping scheme. Moreover, by further analyzing the distortion term we realized that its distribution just depends on the working point of the nonlinearity, while being independent of the number of subcarriers and the mapping. As a result we conclude that the probability of error when a nonlinearity is present is independent of the number of subcarriers in the system while its dependence on the mapping scheme is only due to the the different sensitivity of the mappings to a noise level.

III. Some considerations on PAPR

In this section we discuss some important considerations on the PAPR measure and the problem of PAPR-reduction.

Let us first consider PAPR as a cost function used to evaluate and design MC systems when nonlinear amplifiers are present. In order to improve the system performance, we note that PAPR should predict the amount of distortion introduced by the nonlinear amplifier. From (6) we see that PAPR depends on the number of subcarriers in the OFDM system. However, by analyzing the effect of a nonlinearity over an OFDM system we showed that its performance degradation is independent of the number of subcarriers. The spectral outgrowth is determined by the out-of-band distortion term while the BER degradation is caused by the in-band distortion and the uniform attenuation and rotation introduced by the term α. In Section II-C we showed that both distortion terms just depend on the back-off of the nonlinearity and in [3], α is also shown to depend only on the back-off. As a result, we note that although PAPR depends on the number of subcarriers in the OFDM signal, the performance degradation of an OFDM system when a nonlinear amplifier is present, does not. Thus, the effect of a nonlinearity on an OFDM signal is not clearly related to its PAPR.

Another important aspect is to determine which are the performance improvement capabilities of PAPR-reducing methods. Let us assume that we feed a given nonlinear amplifier with either a conventional OFDM signal, $s_{n/L}$, or a PAPR-reduced OFDM signal, $\bar{s}_{n/L}$. Then, the effective energy per bit of the signal at the input of the nonlinearity can be expressed as

$$E_b^{(eff)} = \frac{E_o}{K}\eta_p \tag{12}$$

where E_o is the average energy of the signal at the input of the nonlinearity, K is the number of bits per symbol and η_p is the power efficiency of the PAPR-reducing technique. In general $\eta_p < 1$, and as a result the BER performance of the peak-reduced signal at the input of the nonlinearity will be worse than the BER performance of the original signal. On the other hand, since $\bar{s}_{n/L}$ suffers from less envelope fluctuations than $s_{n/L}$, one can assume that the power of the distortion term (in-band and out-band) introduced by the nonlinearity to $\bar{s}_{n/L}$ is lower than that of the conventional OFDM signal $s_{n/L}$.

Let us now consider the signals at the output of the nonlinearity. From the previous statements we conclude that when the PAPR of an OFDM system is reduced, the spectral outgrowth due to the nonlinear distortion will be reduced compared to that of a conventional OFDM system. However, the BER performance will not always improve since it depends on both the reduction of the in-band distortion and the power efficiency of the transmitted signal. Hence, there will only be a BER performance improvement when the effect of reducing the in-band distortion becomes noticeable and more important than the loss of power efficiency, which in a practical situation means that there is a trade-off between PAPR-reduction and maintaining a high power efficiency. For example if PAPR-reduction is achieved at the expenses of largely increasing the transmitted power, the BER performance of the system might be worse compared to the OFDM system with no PAPR-reduction. This is specially critical for small constellation sizes since they are less affected by the distortion term. These statements are not taken into account in the majority of PAPR-reducing methods.

IV. Performance of the PAPR-reduced signals

In this section we compute the performance improvement capabilities of several well-known PAPR-reducing methods by comparing the power spectral density (PSD) and the BER of a conventional OFDM system with those of the PAPR-reduced OFDM systems. Active constellation extension (ACE) [4], tone reservation (TR) [5] and partial transmit sequences (PTS) [6] PAPR-reduction techniques are used.

In the simulations we set up an OFDM system with both QPSK and 16-QAM baseband modulation schemes. ACE, TR and PTS techniques are implemented as described in [4], [7] and [1] respectively. In TR approximately 4.3% of the subcarriers are reserved for PAPR-reduction. Those have been properly distributed to maximize the PAPR-reduction capabilities. In PTS $V = 3$ subblocks and $W = 4$ phase factors are used. As previously described we consider a SL nonlinearity operating at IBO = 2dB, 4dB and 6dB and, in order to avoid aliasing the out-of-band distortion into the data bearing tones, an oversampling rate $L = 8$ has been used. Generally $L \geq 4$ is required in nonlinear MC system simulations.

A. Active Constellation Extension

Performance improvement capabilities of ACE are strongly related to the constellation size, in general, larger constellation sizes result in less ACE flexibility. This phenomena can be appreciated in Figure 2 where the PSD of a conventional and ACE based PAPR-reduced OFDM systems are depicted. For all back-offs a larger reduction of the out-of-band radiation is achieved by using QPSK compared to 16-QAM. On the other hand, one should notice that, since by applying ACE the transmitted power increases, then the power efficiency of the transmitted signal will decrease as the ACE flexibility increases. Figure 3 shows the BER performance improvement capabilities of ACE. When QPSK mapping is used a BER degradation occurs at IBOs of 4dB and 6dB, while a slight improvement is appreciated for $\mathrm{Eb/No} \geq 10$dB when a IBO $=$ 2dB is used. In case of using 16-QAM mapping the BER improvement is only appreciated at IBOs of 2dB and 4dB. Hence, as we expected in Section III PAPR-reduction does not always lead to a BER performance improvement.

The PSD shown in Figure 2 was computed by means of periodogram, as the average of the PSD of the $L = 8$ oversampled signal in each OFDM symbol interval. This assures that the out-of-band radiation is computed just from the distortion term. In a practical situation an spectral outgrowth due to sharp transition between consecutive OFDM symbols would ocurr. Figure 4 shows the PSD of a conventional and an ACE-based PAPR-reduced OFDM system in a rectangular window based transmission.

B. Tone Reservation

Figures 5 and 6 show the PSD and the BER of a conventional and a TR-based PAPR-reduced OFDM system obtained when a SL is used. TR performance is independent of the

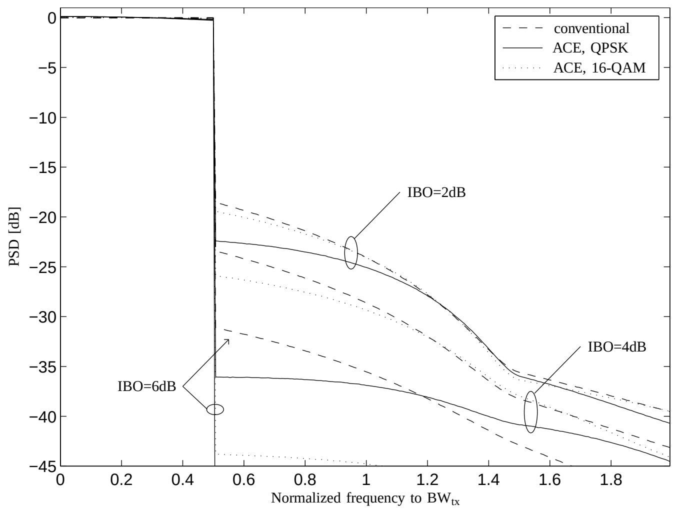

Fig. 2. PSD of a conventional and an ACE-based PAPR-reduced OFDM system obtained when a SL is used.

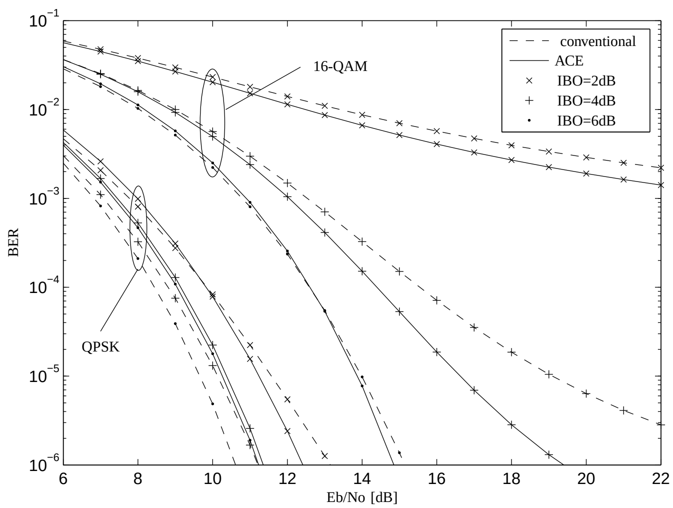

Fig. 3. BER performance of a conventional and an ACE-based PAPR-reduced OFDM system obtained when a SL is used.

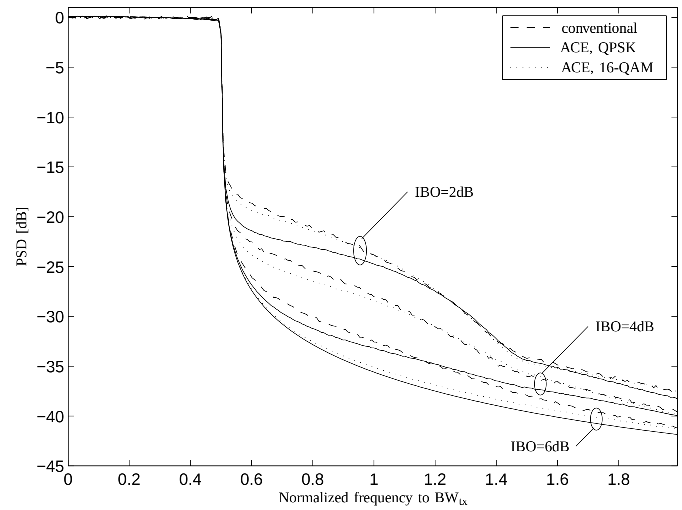

Fig. 4. PSD of a conventional and an ACE-based PAPR-reduced OFDM system obtained when a SL is used in a rectangular window based transmission.

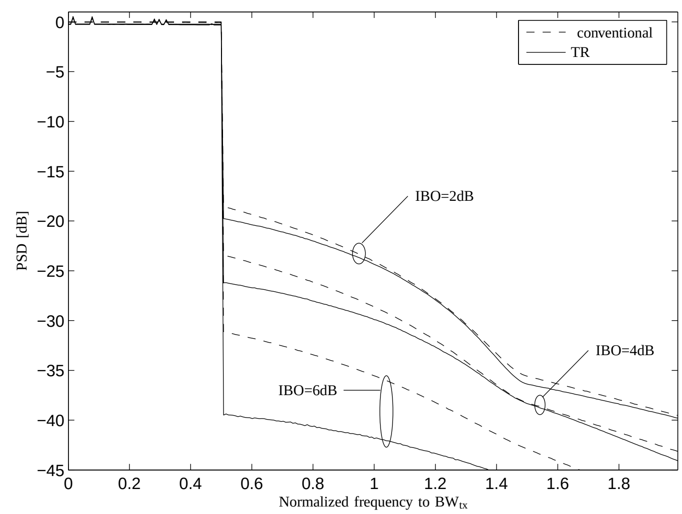

Fig. 5. PSD of a conventional and a TR-based PAPR-reduced OFDM system obtained when a SL is used.

mapping, therefore so it is the PSD in Figure 5. As in ACE, PAPR-reduction in TR is achieved at expenses of increasing the transmitted power which has a direct influence on the BER performance. As it can be appreciated in Figure 6, when QPSK is used the BER performance is only improved for high SNR and 2dB of IBO. When 16-QAM is used the BER performance is only improved at IBOs of 2dB and 4dB. In the other cases a conventional OFDM system with no PAPR-reduction achieves better performance.

C. Partial Transit Sequences

Figures 7 and 8 show the PSD and the BER of a conventional and a PTS-based PAPR-reduced OFDM system obtained when a SL is used. Prior to evaluate its performance improvement capabilities we remark that a perfect knowledge of the side information at the receiver site is assumed. Moreover, we also did not take into account the extra power that we should put in for the side information and, hence, no loss of power efficiency occurs.

Let us consider that QPSK is used and that the amplifier is set to operate at IBO $=$ 6dB. From Figure 8 it can be appreciated that the BER performance when PAPR-reduction is done is scarcely improved compared to a conventional OFDM system. Therefore, if the loss of power efficiency due to the transmission of the side information were taken into account then the BER curves in Figure 8 would be shifted to the right causing a final degradation of the BER performance as occurred to ACE and TR. Moreover, since PTS requires the transmission of side information, an error on this side information would result in a large increase of the BER.

V. Conclusions

In this paper we presented a quantitative study of both the PAPR and the performance of an OFDM system when a clip-

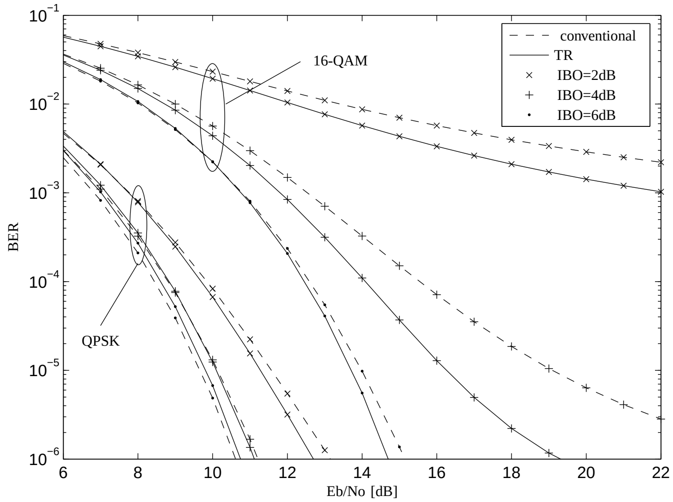

Fig. 6. BER performance of a conventional and a TR-based PAPR-reduced OFDM system obtained when a SL is used.

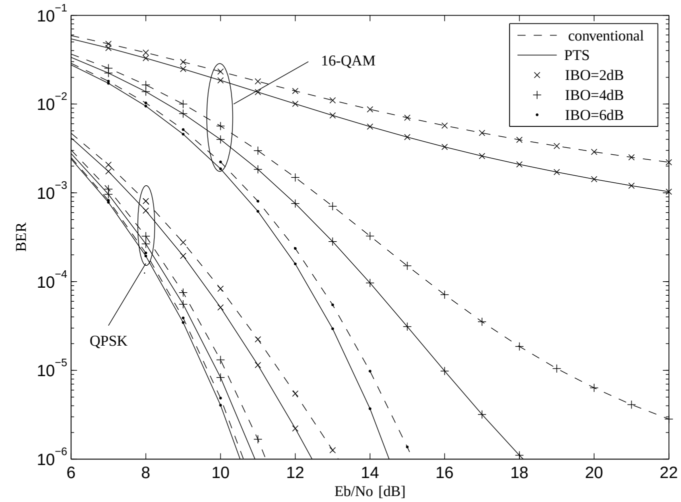

Fig. 8. BER performance of a conventional and an PTS-based PAPR-reduced OFDM system obtained when a SL is used.

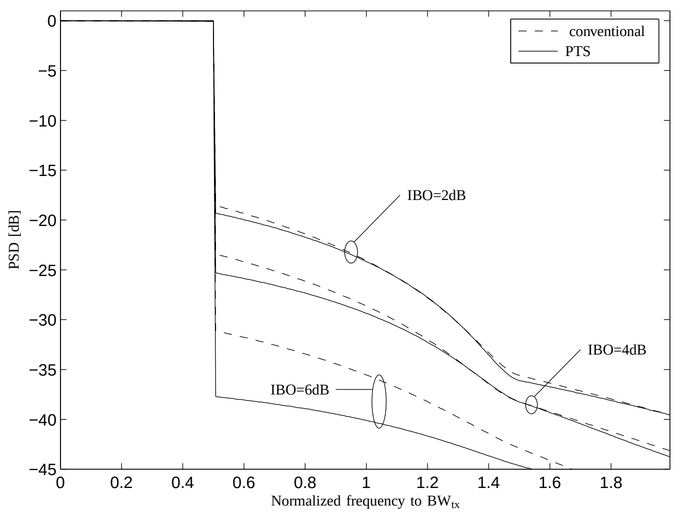

Fig. 7. PSD of a conventional and an PTS-based PAPR-reduced OFDM system obtained when a SL is used.

ping device is present. PAPR-reduction is meant to decrease the distortion introduced by a nonlinearity and, therefore, reduce both the out-of-band radiation and the BER degradation. However, we showed that the effect of a nonlinearity on an OFDM signal is not clearly related to its PAPR. In some recent contributions other measures for the sensitivity of MC systems to nonlinearity have been proposed [8]–[10], which, except for large back-offs, are shown to be more related to the the amount of distortion introduced by a nonlinearity than PAPR [10].

In this paper we also compared the BER performance and the PSD of a conventional OFDM with that of a PAPR-reduced OFDM system. From the presented work, we conclude that spectral spreading is reduced when applying PAPR-reduction but that a BER performance improvement only occurs when the effect of reducing the in-band distortion is more important than the loss of power efficiency. In one hand we know that the smaller the constellation size is the lower its sensitivity to the nonlinear distortion will be, on the other hand, we know that the larger the IBO is the lower the distortion term will be. Hence, in general, for small constellation sizes and high IBO it is more important to maintain a high power efficiency, while for large constellation sizes and low IBO PAPR-reduction is more important. This should be considered when implementing PAPR-reduction to assure that the BER performance of the system is not degraded.

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References

[1] S. H. Han and J. H. Lee, “An overview of peak-to-average power ratio reduction techniques for multicarrier transmission,” *IEEE Wireless Communications Magazine*, vol. 12, pp. 56–65, Apr. 2005.

[2] A. Katz, “Linearization: reducing distortion in power amplifiers,” *IEEE Microwave Magazine*, vol. 2, no. 4, pp. 37 – 49, Dec. 2001.

[3] D. Dardari, V. Tralli, and A. Vaccari, “A theoretical characterization of nonlinear distortion effects in OFDM systems,” *IEEE Transactions on Communications*, vol. 48, pp. 1755–1764, Oct. 2000.

[4] B. S. Krongold and D. L. Jones, “PAR reduction in OFDM via active constellation extension,” *IEEE Transactions on Broadcasting*, vol. 49, no. 3, pp. 258 – 268, Sept. 2003.

[5] J. Tellado and J. Cioffi, “Efficient algorithms for reducing PAR in multicarrier systems,” in *Proc. IEEE International Symposium on Information Theory*, Aug. 1998, p. 191.

[6] L. J. Cimini Jr. and N. R. Sollenberger, “Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences,” *IEEE Communication Letters*, vol. 4, pp. 86–88, Mar. 2000.

[7] M. Deumal, A. Behravan, T. Eriksson, and J. L. Pijoan, “Constrained clipping for Peak Power reduction of multicarrier systems by Tone Reservation,” *Proc. IEEE Vehicular Technology Conference Spring*, accepted for publication, Apr. 2007.

[8] “Comparison of PAR and Cubic Metric for power de-rating,” Motorola, Tech. Rep. R4-040367, 2005.

[9] A. Bahai, M. Singh, A. J. Goldsmith, and B. R. Saltzberg, “A new approach for evaluating clipping distortion in multicarrier systems,” *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1037–1046, June 2002.

[10] A. Behravan and T. Eriksson, “Some Statistical Properties of Multicarrier Signals and Related Measures,” in *Proc. IEEE Vehicular Technology Conference*, Melbourne, Australia, May 2006.