Adaptive Code Allocation for Interference Exploitation on the Downlink of MC-CDMA Systems

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***Abstract* – This paper presents a new technique based on adaptive code-to-user allocation for interference management on the downlink of BPSK based MC-CDMA systems. The principle of the proposed technique is to exploit the dependency of multiple access interference on the instantaneous symbol values of the active users. The objective is to adaptively allocate the available spreading sequences to users on a symbol-by-symbol basis to optimize the decision variables at the downlink receivers. The resulting SINR improvement happens by making use of the energy that is already in the system so the performance benefit is attained with no additional power-per-user investment. The presented simulations show a significant BER performance improvement with the proposed technique while the adaptation overhead is kept less than 10% of the available bandwidth.**

I. INTRODUCTION

The use of orthogonal Walsh-Hadamard spreading sequences provides excellent performance for the downlink of multiple carrier code division multiple access (MC-CDMA) systems [1] in additive white Gaussian noise (AWGN) channels. However, the hostile nature of the wireless channel can severely degrade the orthogonality of such sequences and unless compensated for at the receiver it will result in significant multiple access interference (MAI). Optimizing the signature waveforms for transmission in MC-CDMA can greatly benefit a wireless communication system. Many researchers have proposed optimization of the spreading codes towards orthogonalizing the users in multipath scenarios which involved waveform design of the codes used taking into account the characteristics of the channel encountered (see e.g. [3], [4] & [5]). This paper proposes a different approach to code optimization in which, instead of performing code waveform design, the spreading sequences available in the system are used unmodified but are adaptively allocated to the users on a symbol-by-symbol basis. Secondly, the optimization is not done according to the channel encountered, but rather to the data to be transmitted. In addition, instead of total interference rejection, as adopted in conventional techniques, the primary objective of the adaptive code-to-user allocation is to influence and exploit the constructive interference inherent in the system to deliver an enhanced signal to interference-plus-noise ratio (SINR) at the receiver. More specifically, the MAI experienced by the different users depends on the cross-correlations of the users' codes as well as the instantaneous values of the users' data symbols to be transmitted. Hence, by appropriately redistributing the codes and consequently the cross-correlation values amongst the users taking into account the values of the data symbols to be transmitted at each symbol period, MAI can be manipulated. The reallocation is done in such way that the destructive component of MAI is minimized while the constructive component is enhanced to provide optimized decision variables at the mobile unites' (MUs') receivers towards making detection more reliable. This is the objective here and constitutes the adaptation criteria of the proposed method. It should be noted that with this method the improvement in the received SINR is attained without the need for additional per-user-power investment at the transmitter, as energy inherent in the system is exploited.

It may be clear by now that the proposed technique entails some overhead in the form of transmitting side information (SI) in order to inform the MUs' receivers of their code allocation at each symbol period to achieve correct de-spreading. It will be demonstrated in the results section that the bandwidth efficiency reduction due to the SI transmission can be maintained at less than 10% of the bandwidth. This is worthwhile as the achieved bit error rate (BER) improvements are significant compared to the non-adaptive case.

II. SYSTEM DESCRIPTION

Consider the downlink transmission in a discrete-time synchronous frequency selective MC-CDMA system of K equal power users, where all codes and channels are assumed normalized to unit energy and the spreading gain is equal to L. For simplicity we assume that the number of OFDM subcarriers $M = L$. The use of cyclic prefix is presupposed so that the ISI is completely suppressed. Assuming pre-equalization at the transmitter the received signal at the *u-th* MU can be expressed in matrix form as:

$$\mathbf{r_{iu}} = [\mathbf{x_i} \cdot \mathbf{A} \cdot (\mathbf{C} \circ \mathbf{E})] \circ \mathbf{H_u} + \mathbf{N_u} \qquad (1)$$

where $\mathbf{x_i}=[\ x_{1i}\ x_{2i}\ .\ .\ .\ x_{Ki}\ ]$ is the $1\times K$ matrix containing all users' data for the i-th symbol period, $\mathbf{A}=\mathrm{diag}([\ a_1\ a_2\ .\ .\ .\ a_K\ ])$ is the $K\times K$ diagonal matrix of amplitudes, $\mathbf{C}=[\mathbf{C_1}\ \mathbf{C_2}\ .\ .\ .\ \mathbf{C_K}]^T$ and $\mathbf{E}=[\mathbf{E_1}\ \mathbf{E_2}\ .\ .\ .\ \mathbf{E_K}]^T$ are the $K\times L$ matrices containing the users' codes and equalization coefficients for each subcarrier while $\mathbf{H_u}$ and $\mathbf{N_u}$ are the u-th MU's channel and noise matrix of size $1\times L$. In (1) the notation ° is used to denote element-by-element matrix multiplication. To attain initial reference results, three simple single user pre-equalization schemes are considered in this paper, namely, Maximum Ratio Combining (MRC), Equal Gain Combining (EGC) and Minimum Mean

Square Error (MMSE) pre-equalization [2] given by $\mathbf{E_u}=\mathbf{H_u}^*$, $\mathbf{E_u}=\mathbf{H_u}^*/|\mathbf{H_u}|$, $\mathbf{E_u}=\mathbf{H_u}^*/((K\text{-}1)\cdot|\mathbf{H_u}|^2/L+\sigma_n^2)$ respectively. The above are not optimal for the downlink transmission, as, due to the knowledge of all users' data at the base station (BS), more sophisticated precoding techniques can be applied. It can be proved however that the proposed method can offer performance gains with more complex MC-CDMA pre-equalization schemes. Following (1), at the receiver the data is extracted as

$$d_{iu} = \mathbf{r_{iu}} \cdot \mathbf{C_u^H} = \rho_{uu} x_{iu} + \sum_{k=1,k\neq u}^{K} \rho_{ku} x_{ik} + n_{iu} \quad (2)$$

where

$$\rho_{pq}=(\mathbf{C_p}\circ\mathbf{E_p}\circ\mathbf{H_p}).\mathbf{C_q}^{\mathrm{H}} \quad (3)$$

In (2) $\rho_{uu}x_{iu}$ is the desired user's signal, $\sum_{k=1,k\neq u}^{K} \rho_{ku}x_{ik} = MAI_{iu}$ is the Multiple Access Interference caused by the other K-1 users and n_{iu} is the noise component. As regards the users' crosscorrelations, it is evident from (3) that even if orthogonal codes are used where $\mathbf{C}\cdot\mathbf{C}^{\mathrm{H}}=\mathbf{I}$, the resulting crosscorreltation of the codes viewed at the receiver is non-zero due to the channel distortion.

It can be seen in (2) that given the channel state information (CSI) and data knowledge readily available at the BS the decision variables at the receiver can be pre-estimated. By selecting the appropriate code allocation for transmission at each symbol period the factors ρ_{ku} can be influenced and hence the distribution of the d_{iu} values in (2) for all users can be improved to offer enhanced reliability in the detection.

To clarify this we present a simple example of K=5 users as shown in Table 1. Here the distributions of the decision variables for eight different code allocation patterns are depicted, for two different transmitted symbols combinations (*a,b*). It can be seen that by changing the code-to-user allocation and hence the users' cross-correlations, the decision variables are dramatically affected. It can also be viewed that some code allocations, e.g. *s*=3, *s*=6 for (*a*), deliver better decision variables' distributions than others, e.g. *s*=2, *s*=8. As mentioned above this is a derivative of the difference in the resulting interference for different code allocations. Consequently the detection can be made more reliable by optimizing the code-to-user allocation to be employed at each symbol period. Additionally, it is apparent that for (*b*), with a different symbol combination, different code allocations (e.g. *s*=4, *s*=7) provide improved decision variables' distributions. This is why in the proposed method the code allocation to be used is dynamically adjusted to the symbols x_{ik} to be transmitted at the period i of interest.

This is a way of "fine tuning" the users' symbols and codes so that the energy in the channel be used constructively instead of being wasted because of data misalignment as in conventional methods. As a result the effective received SINR can be increased and improved decision variables can be delivered at the MUs' receivers without the need to increase the transmitted per-user-power.

III. Code to User Allocation Method (CUA)

In order to limit the amount of SI needed, the adaptive code-user allocation is performed by selecting the code-to-user allocation for every symbol period from a limited number, p_c, of allocation patterns, which are known at both the transmitter and the receiver. By doing so, only the index of the allocation pattern needs to be conveyed to the receivers. The selected allocation pattern is chosen at the transmitter according to a certain optimization criterion which is explained in the subsection below.

A) Method Analysis

For reasons of simplicity, the analysis presented assumes Matched Fliter (MF) detection but the process for the case of Multiuser Detection (MUD) is easily deduced by analogy. Firstly, a number p_c of allocation patterns is formed for an initial set of $N_c \geq K$ codes by randomly shuffling them amongst the users. In order to choose the appropriate code allocation pattern prior to transmission the expected decision variables at the MUs for all the available code-to-user allocation combinations need to be determined at the transmitter using the instantaneous symbol values for the active users. Hence, in the proposed method, the estimated effective crosscorrelation matrix $\hat{\mathbf{R}}_s$ of dimensions $K\times K$ is formed for each allocation pattern at the BS from the estimated $\hat{\rho}_{ku}$ given by (3) in which the estimated channel coefficients $\hat{H}_p$ are used:

$$\hat{\mathbf{R}}_{\mathbf{s}} = \begin{pmatrix} \hat{\rho}_{11} & \hat{\rho}_{12} \cdots\cdots & \hat{\rho}_{1K} \\ \hat{\rho}_{21} & \hat{\rho}_{22} \cdots\cdots & \hat{\rho}_{2K} \\ \vdots & \ddots\ddots & \vdots \\ \hat{\rho}_{K1} & \hat{\rho}_{K2} \cdots & \hat{\rho}_{KK} \end{pmatrix} \quad (4)$$

The decision variables at the MUs outputs for the *i-th* symbol period for the *s-th* code-to-user allocation pattern can then be pre-estimated as:

$$\hat{\mathbf{d}}_{\mathbf{i,s}} = \left[\hat{d}_{i1,s} \quad \hat{d}_{i2,s} \quad \ldots \quad \hat{d}_{iK,s}\right] = \mathbf{x_i}\mathbf{A}\hat{\mathbf{R}}_{\mathbf{s}} = \mathbf{d_{i,s}} + \mathbf{e_{i,s}} \quad (5)$$

where e_i is the decision variable pre-estimation error due to inaccurate channel estimation. The proposed algorithm works as shown in the diagram in Fig. 1. Assuming $N_c=K$, at each symbol period and using the instantaneous values of x_i, the decision variables' distribution for each of the p_c allocation patterns is evaluated using (5) and the optimal pattern is chosen according to a selection criteria that will be presented below. Since the receiver has knowledge of all the p_c available patterns it only needs to be informed about the index (s) of which specific pattern is used at each symbol-period by a control signal of $\lceil \log_2(p_c) \rceil$ bits transmitted at a different frequency/time slot as SI. By recognizing the correct pattern each MU can find the new code assigned to it for the current symbol detection as well as the remaining codes of the rest of the users to utilize for mutliuser detection (MUD).

	x_1	x_2	x_3	x_4	x_5
	1	-1	1	1	-1
allocation pattern No.	d_1	d_2	d_3	d_4	d_5
s=1	0.75	-1	0.5	0.75	-1
s=2	0.5	-1.75	-0.5	1.25	-2
s=3	2.5	-2.75	2.25	3	-0.5
s=4	0.5	-1	0.75	0.75	-1
s=5	0.75	-0.5	-0.25	0	-1
s=6	2.25	-0.5	3.25	2.5	-2.5
s=7	-0.25	-0.5	0.75	0	-1
s=8	1.25	-2	0.5	-0.5	-1.75

a

	x_1	x_2	x_3	x_4	x_5
	-1	-1	1	-1	1
allocation pattern No.	d_1	d_2	d_3	d_4	d_5
s=1	-0.25	0	0.5	-0.25	-1
s=2	-1	-1.25	1	-0.25	-0.5
s=3	-0.5	0.25	0.25	0	-0.5
s=4	-3	-1.75	3.25	-2.75	-2.25
s=5	0.25	0	0.25	-0.5	-0.5
s=6	-0.25	-1	0.5	0	-0.25
s=7	-2.75	-3	3.25	-2.5	-1.5
s=8	0.75	-1.5	1	-1	-1.25

b

Table 1: Noiseless decision variables' distributions for different allocation patterns of a system of K=5 users with random codes of L=16 for p_c=8

An alternative route towards performance improvement, also considered here for the sake of comparison, is using a larger number of codes (N_c>K) to attain a greater variety of interference distribution, but this would lead to the requirement for increased system resources, i.e. more available signature waveforms. As will be shown the performance enhancement attained by this is insignificant.

It is noteworthy to highlight that the SI bits do not need to be spread as the information they convey is common to all users and furthermore this would be more bandwidth efficient. If the number of SI bits is not a power of two or if the SI is to be forward error correction encoded, then a frame based approach can be adopted as depicted in Fig. 2. Here the allocation procedure is run for all the symbols in the frame prior to transmission and the control bits for all symbols are transmitted in the beginning of the frame. Each symbol in the frame is a CDMA-multiplexed symbol for K users.

B) Allocation criteria

It is intuitive from equation (5) and Table 1 that a number of criteria can be extracted for the selection of the optimum available code allocation pattern based on the instantaneous interference amongst users and the distribution of the resulting values of $\hat{d}_i$. Since the performance of the worst user

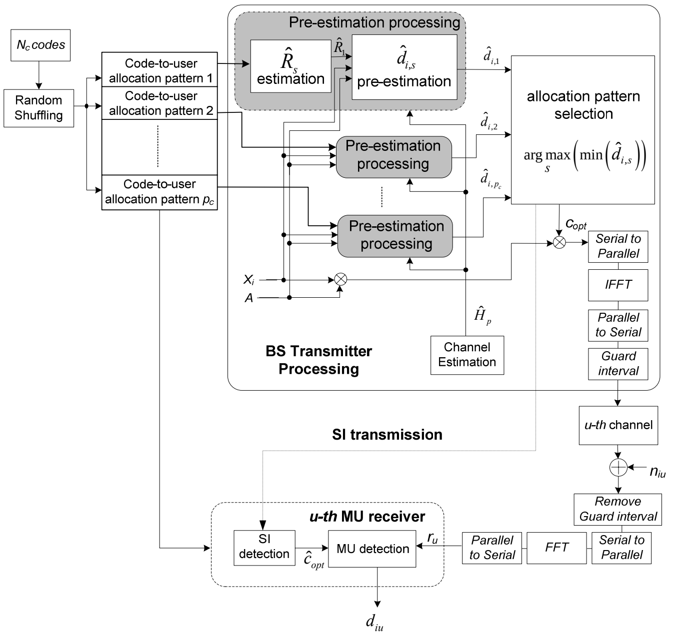

Fig. 1. Flow diagram of code allocation technique

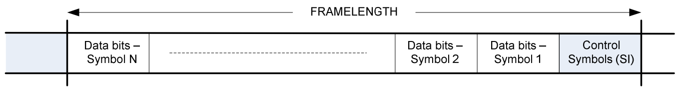

Fig. 2. Frame-based transmission structure for the code allocation technique

catalytically affects the overall system BER, the following code pattern selection criteria is proposed and examined here:

$$\arg\max_{s}\left(\min\left(\hat{\mathbf{d}}_{\mathbf{i,s}}\right)\right) \qquad (6)$$

In more detail, $\min\left(\hat{\mathbf{d}}_{\mathbf{i,s}}\right)$ determines the MU output that is the most prone to decision errors for each code allocation. From the p_c available distributions of $\hat{d}_i$ according to the p_c available code allocation patterns, the optimum is chosen as the one that maximizes the minimum of $\hat{d}_i$ which denotes the decision variable of the worst user at each symbol period for each distribution of $\hat{d}_i$. Hence, the code allocation selected (c_{opt}) is the one that delivers the highest decision variable for the worst user. In the case where two different allocation patterns offer the same minimum, the second minimum is considered and so on. For the example depicted in Table 1 this criteria would indicate allocation pattern s=3 for (*a*) and pattern s=4 for (*b*). Evidently, this is one bit error rate (BER) optimization approach that favors the users that are more susceptible to errors. By inspection of this criteria and Table 1, the comparison to other available code allocation patterns shows that by this optimization a favorable constructive to destructive MAI ratio is chosen which evidently delivers a higher SINR and boosts performance. Constructive interference is enhanced while destructive interference is minimized.

IV. NUMERICAL AND SIMULATION RESULTS

Monte Carlo simulations for various combinations of the proposed technique with conventional methods have been performed for frequency selective fading channels with AWGN. Walsh-Hadamard codes of variable lengths have been used. The multipath channel considered here is a chip-spaced 4-path Rayleigh frequency selective fading with unity gain and equal average power per channel's path (uniform channel power-profile). The channel is implemented in the form of a 4-tap delay line with an independent complex-number Gaussian distributed coefficient per tap to represent each path's phase and amplitude. In order to provide a good average performance a new set of the tap coefficients are generated at every symbol period. The channels' characteristics are assumed to be perfectly known. As in the mathematical analysis presented above, the use of cyclic prefix is presupposed so that the ISI is completely suppressed. In all simulations it is assumed that the number of OFDM subcarriers $M = L$. For reasons of efficiency the SI bits are QPSK encoded. Performance of MRC, EGC and MMSE pre-equalization schemes for MC-CDMA with and without adaptive code-user allocation (CUA) are compared. Results for systems utilizing successive interference cancellation (SIC) detectors [6] are also presented.

In Fig. 3 the performance of the proposed method with p_c=16 allocation patterns for N_c=K is depicted for all three pre-equalization schemes and compared to the system without CUA. The number of users is K=20 and the spreading gain L=32. The channel is Rayleigh fading with P=4 paths. It can be seen in the figure that for low SNR values the proposed method performs worse due to unreliable SI detection. However, for higher SNRs all three types of pre-equalization benefit from a significant performance improvement when combined with the proposed technique. This is due to the enhanced effective SINR that is attained with the adaptive allocation selection. For EGC it can be seen that the performance enhancement reaches an order of magnitude. As regards the SI overhead, the SI bits in this case are $\lceil \log_2(p_c) \rceil = 4$ equivalent to 2 QPSK symbols. This yields a symbol transmission rate efficiency of 20/22=91%. Although a 9% efficiency reduction is not insignificant it is obvious from this figure that this method produces a significant BER improvement. Using a larger number of users or higher order modulation would further reduce the efficiency loss (e.g. in the case of 40 users or using 16PSK modulation the efficiency loss would be ~5%).

Fig. 4 shows the comparison of both MF and SIC receiver schemes with and without CUA for the same system as in Fig. 3 using EGC pre-equalization. The case of N_c>K and specifically N_c=32 is also included to investigate any potential performance improvement. Simulation results with perfect SI detection for the proposed technique are also presented to show how the reliability of SI detection affects the overall performance. Perfect SI detection refers to a genie-type detection of the SI bits. It can be seen that the results with imperfect SI detection converge to the case of perfect SI for high SNR as the SI detection becomes much more reliable. In both receiver cases significant performance improvement is attained. As for the case of N_c=32 it can be seen that a small performance benefit is gained for a specific SNR area.

Fig. 5 depicts the comparison for the case of the same channel for a fully loaded system of K=16, L=16 and p_c=16 patterns. Performance for EGC pre-equalization for MF and SIC detection is depicted to show the proposed method's superiority. In all cases the proposed technique offers a significant performance enhancement.

In Fig. 6 the BER for different values of p_c at SNR=7dB is depicted for MRC and EGC pre-equalization for K=16, L=16 in the same multipath to show the BER performance improvement as a function different values for p_c. For both systems increasing p_c further than a specific value yields limited benefit. For the cases depicted on the figure, values of p_c>16 would provide no significant performance improvement while an unnecessary SI overhead increase would be required.

V. CONCLUSION

In conventional schemes the users' codes' misalignment leads to waist of critical useful energy inherent in the transmission medium. By "fine tuning" the users' codes so that constructive interference be exploited, we have shown that the adaptive code allocation technique can improve performance for both MF and MUD in MC-CDMA by up to an order of magnitude. This comes with no need for additional per-user-power investment as existent energy is exploited. The trade-off is the need for transmission of side information which imposes an overhead on the detection processing.

Further work can be focused on improving the distribution of codes between users instead of randomly shuffling. Moreover the selection criteria could be investigated towards further optimization. Though the constructive MAI analysis applies only to PSK modulation, further work can be done towards applying code shuffling to PAM and QAM modulation with the use of adaptive decision thresholds.

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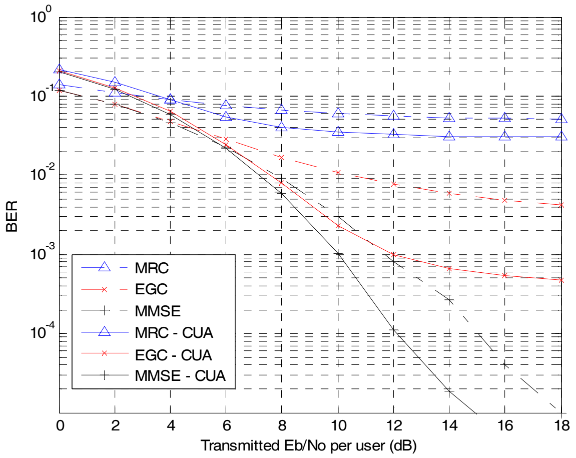

Fig. 3. BER for conventional and code allocation using MRC, EGC, MMSE pre-equalization with N_c=K, p_c=16, K=20, L=32 in an Rayleigh channel of P=4

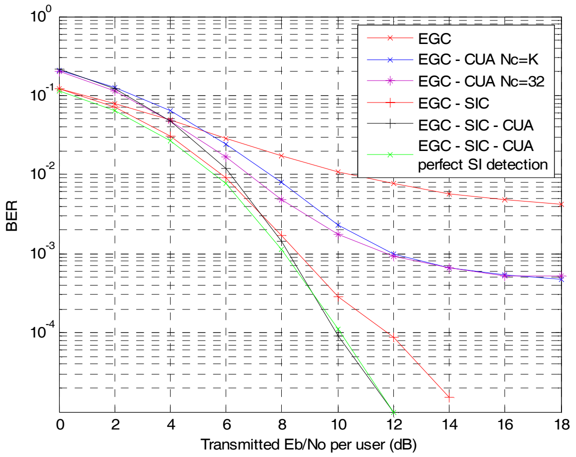

Fig. 4. BER performance of conventional EGC pre-equalizing and EGC-SIC methods and the methods using the code allocation technique with N_c=K *and* N_c=*32* for p_c=16 in an Rayleigh multipath channel of P=4 with K=20, L=32

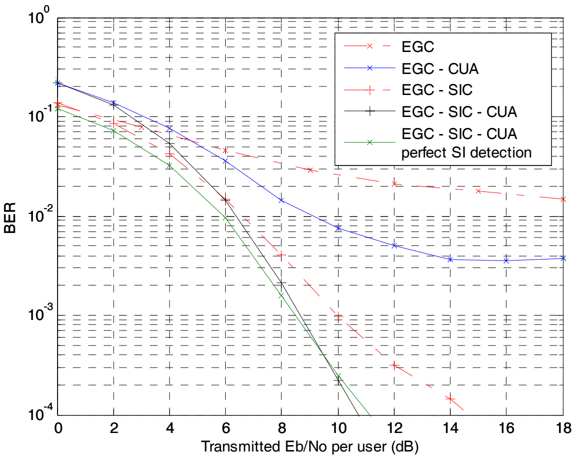

Fig. 5. BER performance of conventional EGC pre-equalizing and EGC-SIC methods and the methods using the code allocation technique with N_c=K, p_c=16 in an Rayleigh multipath channel of P=4 with K=16, L=16

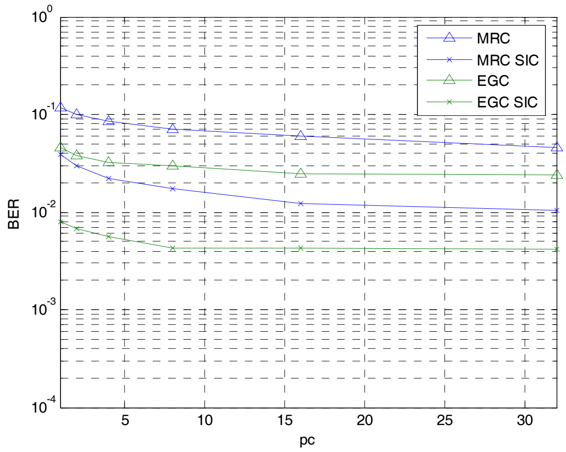

Fig. 6. BER vs. p_c performance for the cases of MRC and EGC pre-equalization with MF and SIC detection for K=16, L=16 in Rayleigh P=3 channel employing code allocation for SNR=7dB