On OFDM systems with low sensitivity to nonlinear amplification

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Outline

- Orthogonal Frequency Division Multiplexing (OFDM)
- Nonlinear models for high power amplifiers
- Multicarrier systems with nonlinear amplifiers
- Overview of nonlinear distortion compensation techniques in OFDM
- Conclusions

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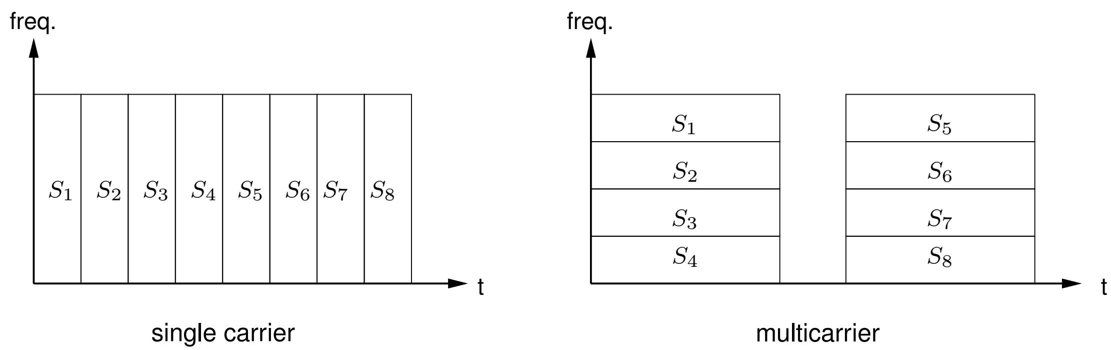
I. OFDM

Multicarrier modulations

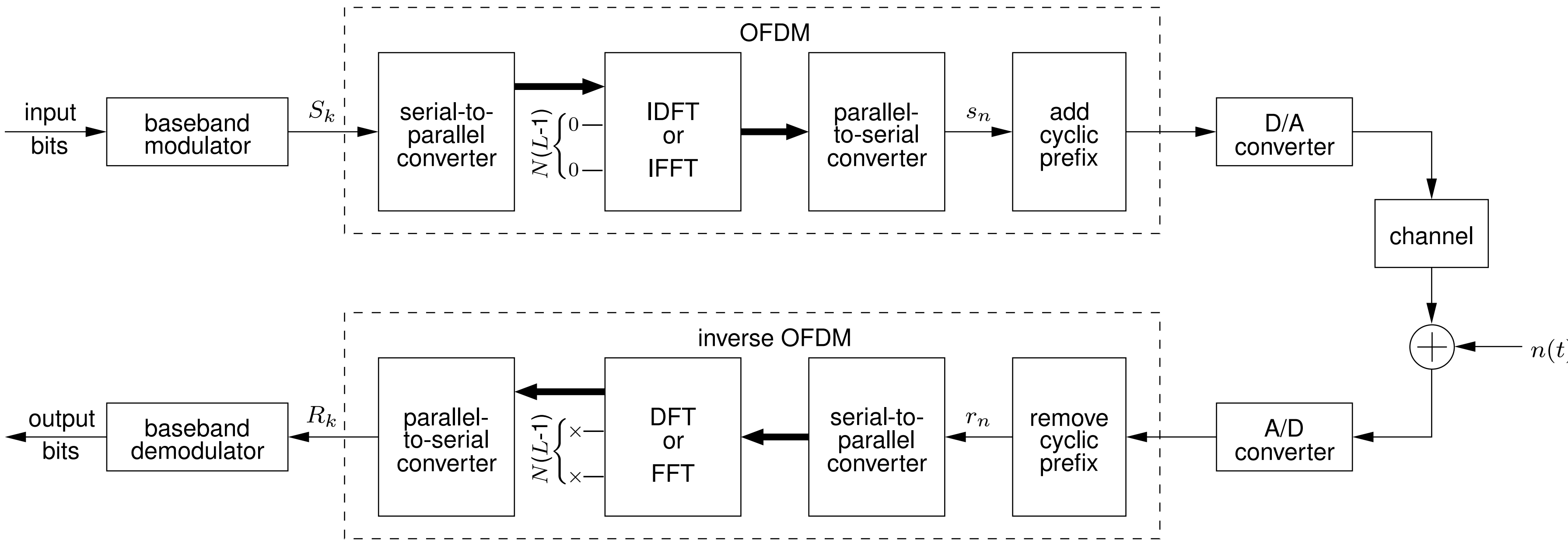
- Multicarrier transmission consists on splitting a high-rate data stream into many parallel low-rate substreams that are transmitted in different frequency channels, i.e. in different subcarriers.



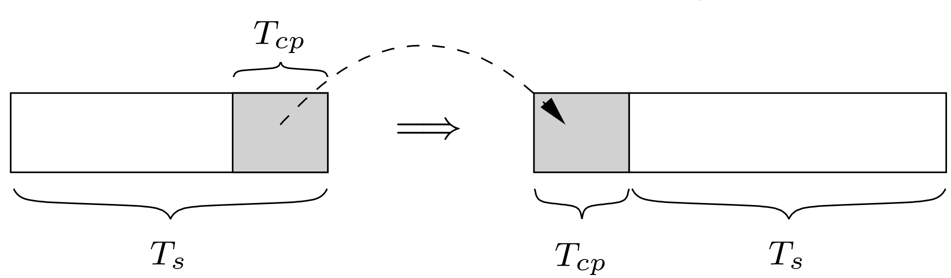
- Motivations for using such a technique:
  - Complex equalization can be avoided by using many subcarriers with a sufficiently low symbol rate such that the effect of the delay spread decreases significantly.
  - Transmission rates close to capacity can be achieved by using a sufficiently large number of subcarriers with a relatively narrow bandwidth each and by allocating the transmitted power, constellation size and coding rate to each subcarrier efficiently.
- OFDM is a low complexity technique to efficiently modulate multiple subcarriers by using digital signal processing [Bingham,ComMag,1990].

I. OFDM

Block diagram of an OFDM system



⇒ The cyclic prefix is used to avoid ISI without introducing ICI.



I. OFDM

Advantages and drawbacks

- Advantages
  - Low computational complexity implementation by means of the FFT operation.
  - Robustness against frequency selective fading and time dispersion.
  - Transmission rates close to capacity can be achieved.
  - Each substream may be independently coded and modulated according to the transmission conditions on each subcarrier.
  - Flexibility to allocate different users by assigning to them different groups of subcarriers (OFDMA).
- Drawbacks
  - Sensitivity to frequency offset, which introduces ICI and therefore increases the BER.
  - Sensitivity to nonlinear amplification.
  - Sensitivity to the resolution and dynamic range of the D/A and A/D converters.
  - Loss in power and spectral efficiency due to the guard interval.
  - The phase noise introduced by the transmitter and receiver oscillators influences the system performance.

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II. Power amplifiers

- Two major type of power amplifiers are used in communication systems:
  - *Traveling wave tube amplifiers* (TWTA)
  - *Solid state power amplifiers* (SSPA)
- A common characteristic of both devices is that the signal at their output is a nonlinear function of the input signal at both the present and previous instants.
- Volterra series allow us to model both the HPA nonlinear and the memory effects:

$$
\begin{aligned}
y(t) &= \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \cdots \int_{-\infty}^{\infty} d\tau_n h_n(\tau_1, \tau_2, \ldots, \tau_n) \prod_{i=1}^{n} x(t-\tau_i) \\
&= h_0 + \int_{-\infty}^{\infty} d\tau_1 \; h_1(\tau_1) x(t-\tau_1) \\
&+ \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \; h_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) \\
&+ \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \int_{-\infty}^{\infty} d\tau_3 \; h_3(\tau_1, \tau_2, \tau_3) x(t-\tau_1) x(t-\tau_2) x(t-\tau_3) + \ldots
\end{aligned}
$$

where $\begin{cases} h_n(\tau_1, \cdots, \tau_n) \text{ is the } n\text{-th order impulse response of the system} \\ \{h_0, h_1(\tau_1), h_2(\tau_1, \tau_2), \ldots\} \text{ are called the Volterra kernels of the system} \end{cases}$

$\Rightarrow$ **Problem**: The computation necessary to produce an output sample from the n-th kernel is the number of operations required for $n = 1$, raised to the n-th power

II. Power amplifiers

Memoryless models (derivation of higher level expressions)

- The behavior of memoryless models can be obtained by particularizing the Volterra series for memoryless systems. Steps:
  - Use the baseband equivalent: $b_x(t) = u_x(t)e^{j\alpha_x(t)} \Longrightarrow x(t) = \text{Re}\{b_x(t)e^{jw_c t}\}$
  - Memoryless assumption: The duration of the kernels is much shorter than the time span given by the inverse of the bandwidth of the input baseband signal:

$$|\tau_i| \ll \frac{1}{B_x} \; i = 1, 2, \ldots, n \Longrightarrow b_x(t - \tau_i) \approx b_x(t)$$

  - Express the product of the n complex conjugated terms as a double sum. This will allow us to discard the high order terms that correspond to the harmonics.
  - Just consider the signal that is around the carrier frequency (first-zone of the amplifier).
  - Finally we obtain:

$$y_1(t) = \text{Re}\left\{ G\Big(u_x(t)\Big) e^{j\left[\alpha_x(t) + \Phi\left(u_x(t)\right)\right]} e^{jw_c t} \right\} \quad \text{where} \begin{cases} G(\cdot) \text{ is the AM/AM characteristic} \\ \Phi(\cdot) \text{ is the AM/PM characteristic} \end{cases}$$

- Baseband modeling of a nonlinearity (memoryless HPA):

$$b_x = u_x e^{j\alpha_x} \longrightarrow \boxed{\text{HPA}} \longrightarrow b_{y_1} = G(u_x)e^{j[\alpha_x + \Phi(u_x)]}$$

II. Power amplifiers

Memoryless models

- **Saleh model** [Saleh,TransCom,1981]:

  AM/AM: $G(u_x) = \frac{\kappa_G \cdot u_x}{1+\chi_G \cdot u_x^2}$

  AM/PM: $\Phi(u_x) = \frac{\kappa_\Phi \cdot u_x^2}{1+\chi_\Phi u_x^2}$

  - Commonly used for TWTA modeling.

- **Rapp model** [Rapp,SatConf,1991]:

  AM/AM: $G(u_x) = \frac{\kappa_G \cdot u_x}{(1+(\frac{u_x}{O_{sat}})^{2s})^{\frac{1}{2s}}}$

  AM/PM: $\Phi(u_x) = 0$

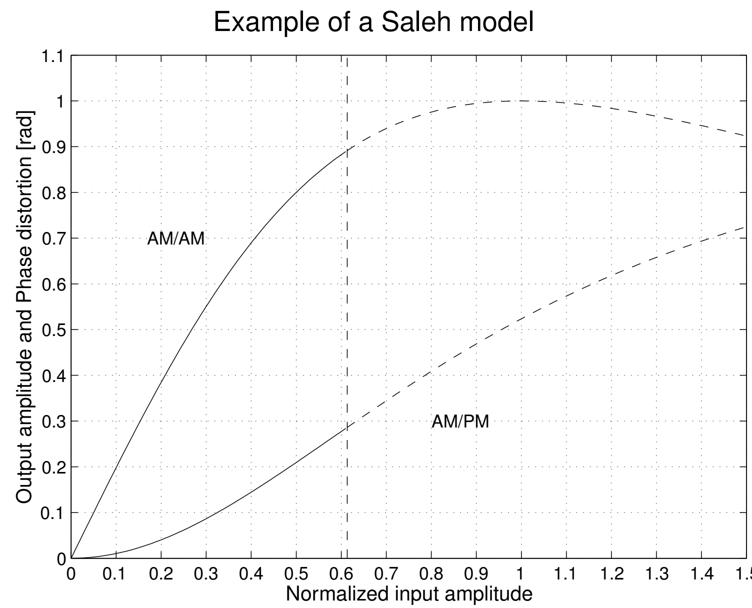
  - Commonly used for SSPA modeling.

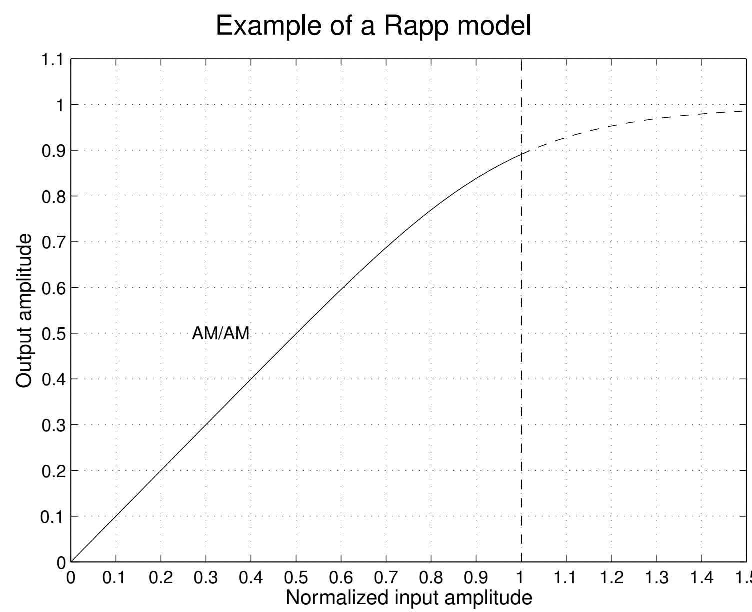
- **Ghorbani model** [Ghorbani,DPSC,1991]:

  AM/AM: $G(u_x) = \kappa_{G_1} \cdot u_x + \frac{\kappa_{G_2} \cdot {u_x}^{\xi_G}}{1+\chi_G \cdot {u_x}^{\xi_G}}$

  AM/PM: $\Phi(u_x) = \kappa_{\Phi_1} \cdot u_x + \frac{\kappa_{\Phi_2} \cdot {u_x}^{\xi_\Phi}}{1+\chi_\Phi \cdot {u_x}^{\xi_\Phi}}$

  - Commonly used for SSPA modeling.

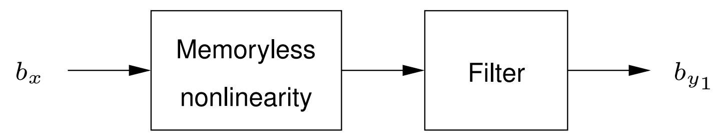




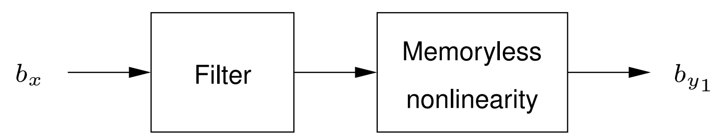
II. Power amplifiers

Models with memory

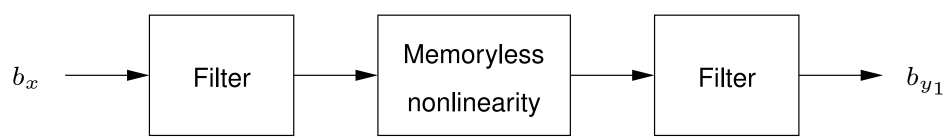
- Memoryless models are only suitable for narrow-band signals since over any relatively small portion of the band, the transfer characteristic looks nearly frequency independent.
- For wideband signals the transfer function is frequency dependent. One can add the memory effect by introducing a filter to the the previous memoryless models.
  - **Hammerstein model** [Abuelmaatti,TransCom,1984]:



  - **Wiener model** [Schetzen,ProcIEEE,1981]:



  - **Filter-NL-Filter model** [Jeruchim,Kluwer,2000]:



II. Power amplifiers

Operating point

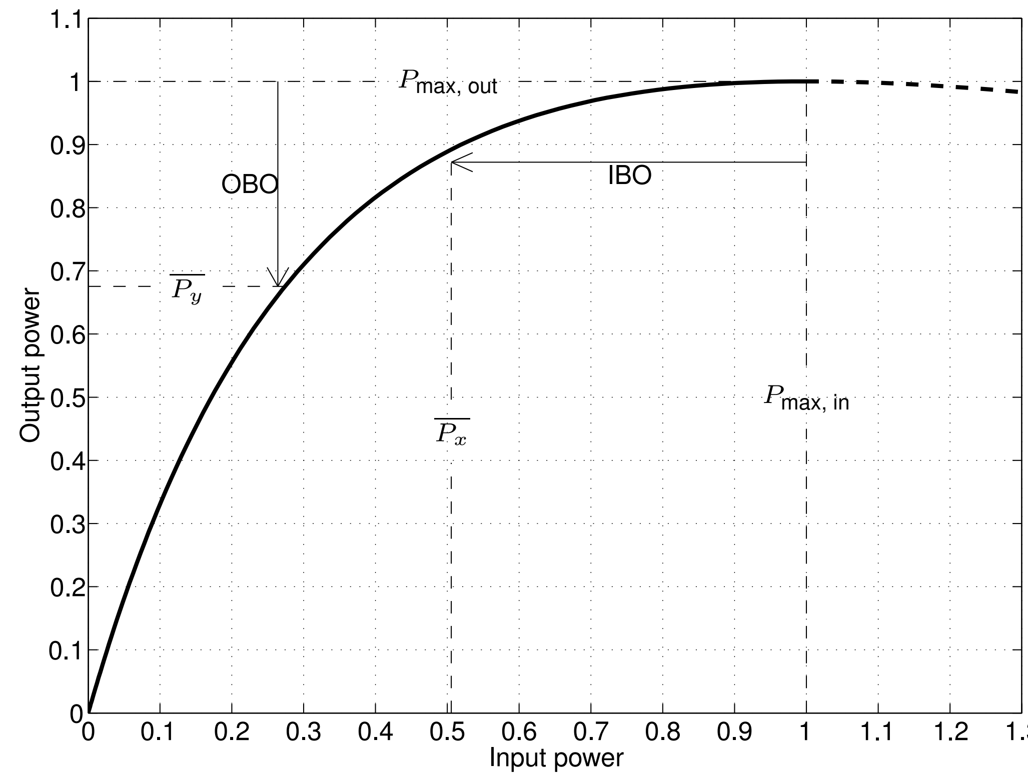
The operating point of the amplifier is determined by the ratio between the saturation power of the amplifier and the average power of the signal:

- *Input back-off* (IBO)

$$\text{IBO} = 10\log_{10}\left(\frac{P_{\text{max,in}}}{\overline{P_x}}\right) \quad [\text{dB}]$$

- *Output back-off* (OBO)

$$\text{OBO} = 10\log_{10}\left(\frac{P_{\text{max,out}}}{\overline{P_y}}\right) \quad [\text{dB}]$$



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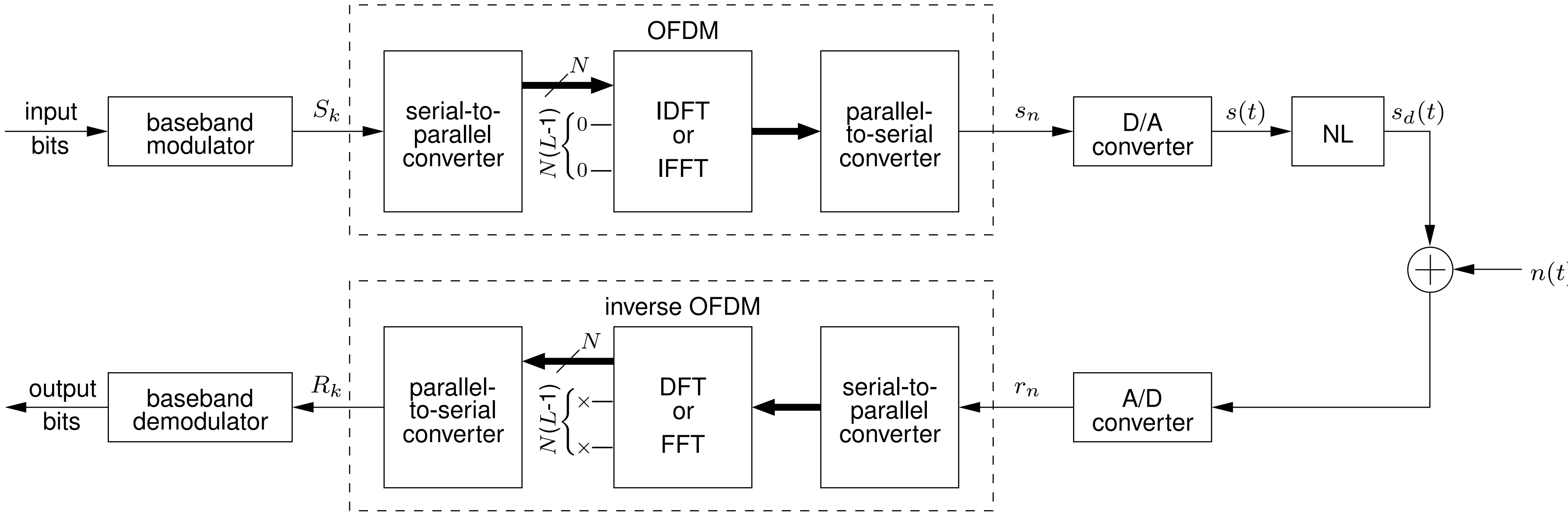
III. OFDM with nonlinearities

Block diagram



- We will only consider the nonlinear effect of the amplifier.
- We will only use AWGN channels $\Rightarrow$ the cyclic prefix is unnecessary.
- To analyze the effect of nonlinearities we will study the OFDM signal in three places:
  - At the input of the nonlinearity
  - At the output of the nonlinearity
  - At the input of the baseband demodulator

III. OFDM with nonlinearities

Signal at the input of the nonlinearity

We remind that the baseband OFDM signal can be expressed as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi f_k t}, \; 0 \leq t < T_s,$$

independent and identically distributed random variables

According to the central limit theorem, if the number of subcarriers is large enough, the signal can be approximated as a complex Gaussian distributed random variable.
The envelope of an OFDM signal follows a Rayleigh distribution as:

$$f_X(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}},$$

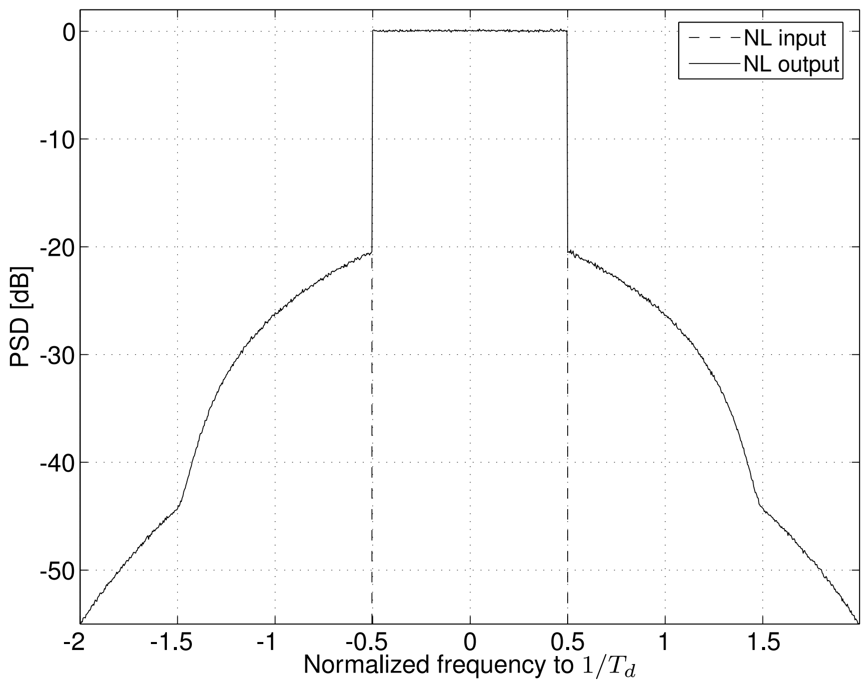
with

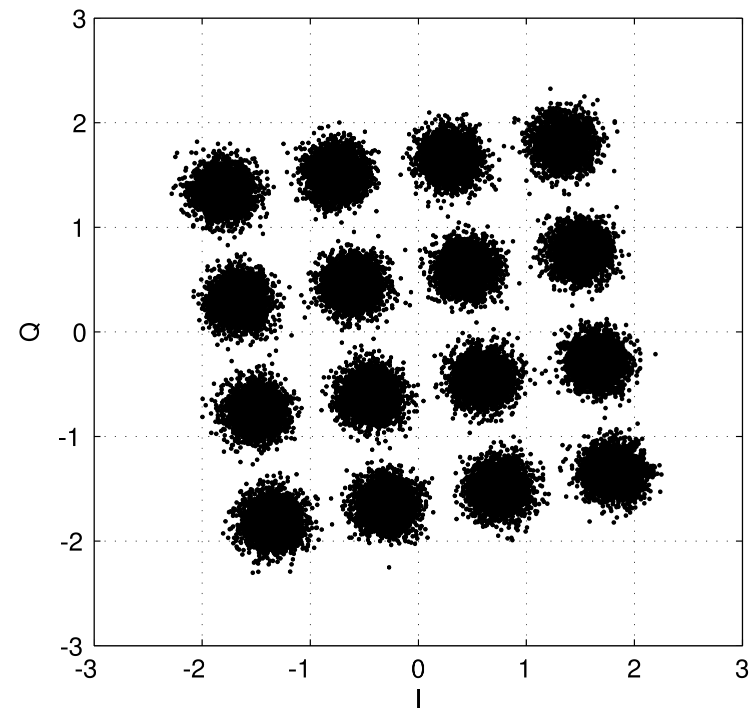
$$\begin{aligned} E[X] &= \sigma \frac{\sqrt{\pi}}{2} \\ \text{var}[X] &= \sigma^2 (1 - \frac{\pi}{4}), \end{aligned}$$

where the variance of the real and imaginary parts of the signal is assumed to be $\sigma^2/2$.

III. OFDM with nonlinearities

Signal at the output of the nonlinearity





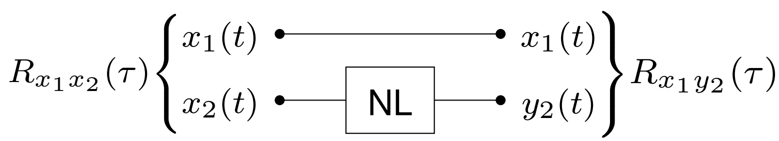
- Spectral outgrowth.
  - Generated by the intermodulation products at frequencies $|mf_1 \pm nf_2|$ that lay outside the transmission bandwidth.
  - It interferes neighboring communication systems.
- The signal constellation is largely distorted, we appreciate 2 phenomena: rotation and clouding.
  - Generated by the in-band intermodulation products.
  - It increases the error rate.

III. OFDM with nonlinearities

Signal at the output of the nonlinearity (cont.)

- We use the Bussgang theorem to analyze the signal at the output of the nonlinearity
  - **Bussgang theorem** [Bussgang,TechReport,1952]:

    For two Gaussian signals $x_1(t)$ and $x_2(t)$, the cross-correlation function taken after one of them (e.g. $x_2(t)$) has undergone nonlinear amplitude distortion ($R_{x_1 y_2}$) is identical, except for a factor of proportionality α, to the cross-correlation function taken before the distortion ($R_{x_1 x_2}$):



where $R_{x_1 y_2}(\tau) = \alpha R_{x_1 x_2}(\tau)$

  - In particular, if $x_1(t) = x_2(t)$, then $R_{xy}(\tau) = \alpha R_{xx}(\tau)$
  - An interesting result is that the output $y(t)$ of a NL with Gaussian input $x(t)$ can be written as [Dardari,TransCom,2000]:

$$y(t) = \alpha x(t) + d(t), \qquad \text{on } \alpha = \frac{R_{xy}(\tau_1)}{R_{xx}(\tau_1)}.$$

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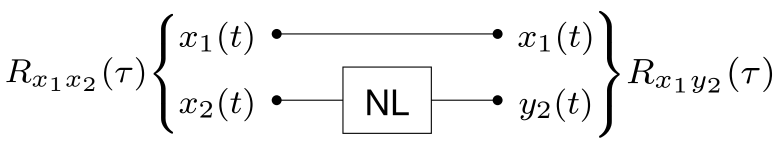

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$$\boxed{\text{OFDM is Gaussian distributed} \implies \text{we can use the Bussgang theorem}}$$

III. OFDM with nonlinearities

Signal at the output of the nonlinearity (cont.)

$$y(t) = \alpha x(t) + d(t), \qquad \text{where } \alpha = \frac{R_{xy}(\tau_1)}{R_{xx}(\tau_1)}.$$

attenuation and rotation: compensated by the synchr.

distortion: in the frequency domain:

$$\mathbf{D} = \mathbf{D}^{(in)} + \mathbf{D}^{(out)},$$

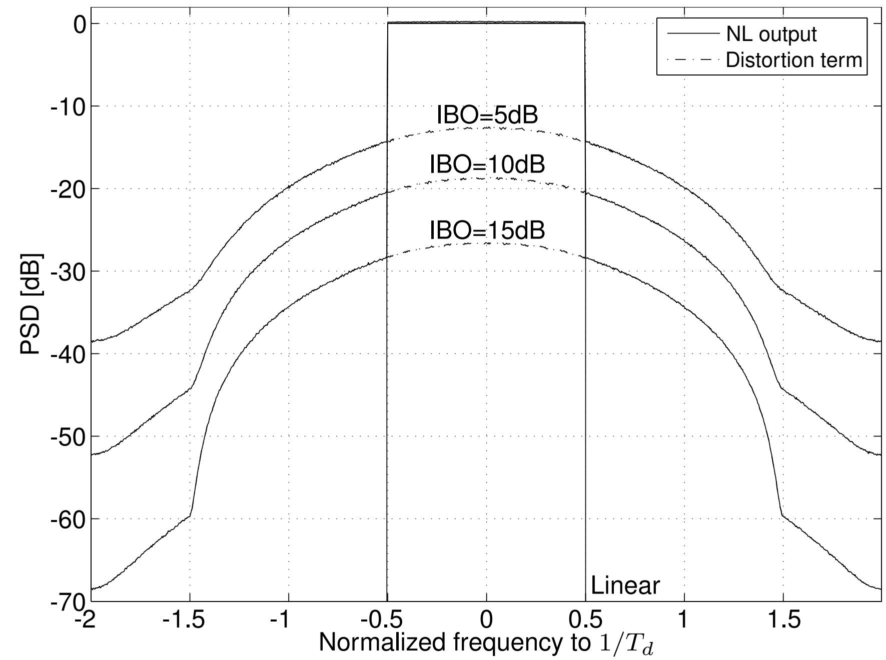
with

$$D_k^{(in)} = \begin{cases} D_k & \text{if } k = 0, \ldots, N-1 \\ 0 & \text{otherwise} \end{cases}$$

introduces an in-band noise that increases the error probability.

$$D_k^{(out)} = \begin{cases} D_k & \text{if } k = N, \ldots, LN-1, \\ 0 & \text{otherwise} \end{cases}$$

is the out-of-band radiation.



- Independent of both the baseband modulation and the number of subcarriers.
- It is interesting to notice that in order to avoid aliasing the out-of-band distortion into the data bearing tones, sufficient oversampling (normally $L \geq 4$) is required.

III. OFDM with nonlinearities

Signal at the input of the baseband demodulator

The decision variables at the input of the demodulator are:

$$R_k = \alpha S_k + D_k + W_k \qquad k = 0, \ldots, N-1,$$

- αS_k: uniformly attenuated and rotated constellation.
- W_k: Gaussian noise with variance $\sigma_W^2 = N_0 N / T_s$.
  $\Rightarrow$ This is the AWGN $n(t)$ with $\sigma_n^2 = N_0$ added at the receiver, after the FFT block:

$$W_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{LN-1} n_n e^{-j2\pi kn/LN},$$

- D_k: in-band nonlinear distortion noise component, after the FFT block:

$$D_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{LN-1} d_n e^{-j2\pi kn/LN}, \qquad k = 0, \ldots, N-1.$$

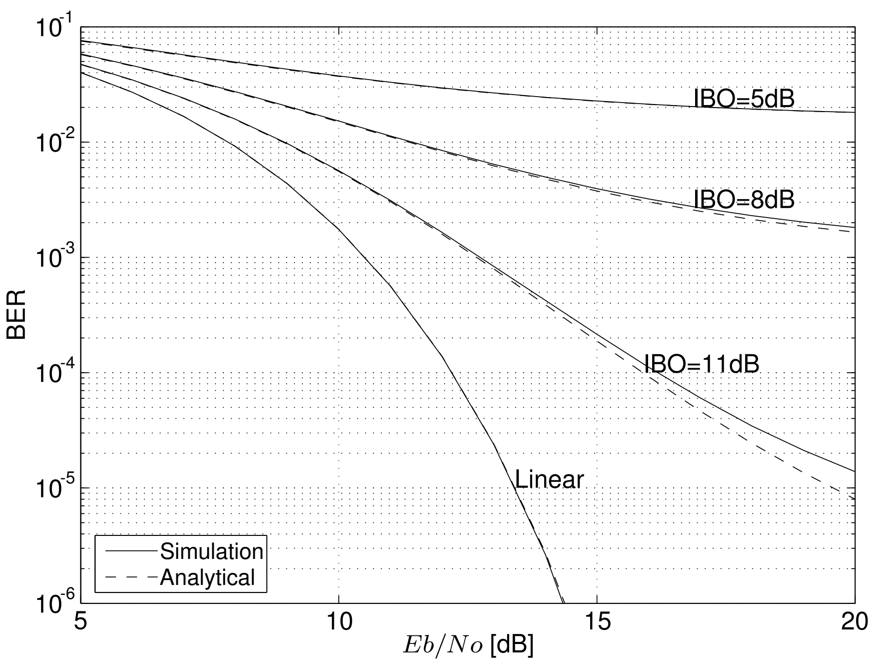
  $\Rightarrow$ Since it is the sum of N identically distributed random variables, we can assume it to be complex Gaussian distributed even though the terms of the sum are not uncorrelated [Dardari,TransCom,2000]. ($\mu_D = 0$ i $\sigma_D^2 = E[|S_d|^2] - |\alpha|^2 E[|S|^2]$)

III. OFDM with nonlinearities

Signal at the input of the baseband demodulator (cont.)

Notice that due to the Gaussianity of both W_k and D_k we can evaluate analytically the BER as a function of the modulation and the SNDR, which is defined as:

$$\text{SNDR} = \frac{|\alpha|^2 E[|S|^2]}{\sigma_W^2 + \sigma_D^2}.$$



OFDM with 16-QAM baseband modulation.

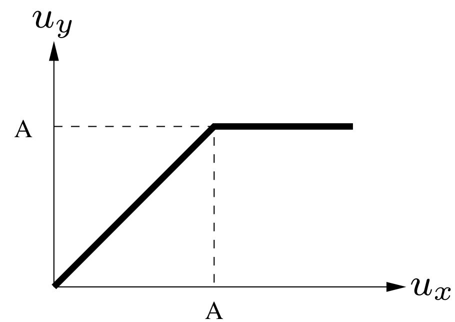
⇒ The amplifier is more linear for large IBOs, as a result less distortion is introduced. The fact that less distortion components exist implies that the distortion is less Gaussian distributed and, thus, that the analytical result becomes less exact.

III. OFDM with nonlinearities

Signal at the input of the baseband demodulator (cont.)

- A soft limiter (SL) is defined as:

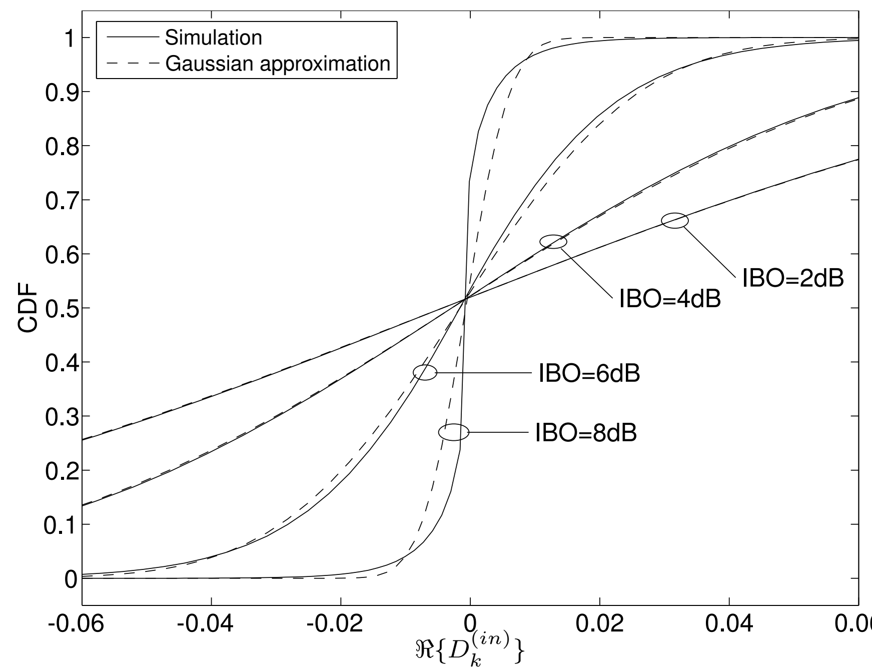
$$b_{y_1} = \begin{cases} b_x & \text{if } u_x \leq A \\ Ae^{j\alpha_x} & \text{otherwise} \end{cases}$$



When a SL operating at high IBO's is used, the distortion term is not Gaussian distributed anymore [Deumal,TransWCom,2006].

- $\alpha \simeq 1$ and the distortion term, in the time domain, is only formed by few delta functions (typically one or two).
- The Gaussianity of a random variable can be determined by its kurtosis kurt$(x) = E[x^4] - 3(E[x^2])^2$:

IBO=2dB	IBO=4dB	IBO=6dB	IBO=8dB	IBO=10dB
0.037	0.125	0.976	14.496	386.084



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IV. Compensation techniques

Digital techniques for nonlinear distortion compensation in OFDM

- **Linearization**
  - Linearize the amplifier response
  - Performed at the transmitter side: both the spectral outgrowth and the BER performance degradation may be reduced.
- **Post-processing**
  - Processes the received signal with the aim to eliminate the distortion term introduced by the transmitter nonlinearity.
  - Performed at the receiver: only the BER performance degradation is reduced.
- **Reduce the envelope fluctuations of the transmitted signal**
  - OFDM-based multicarrier signals suffer from large envelope fluctuations. This requires backing-off the power amplifier significantly in order to avoid the signal to be largely non-linearly amplified.
  - By reducing the envelope fluctuations we reduce the performance degradation.
  - Performed at the transmitter side: both the spectral outgrowth and the BER performance degradation may be reduced.

IV. Compensation techniques

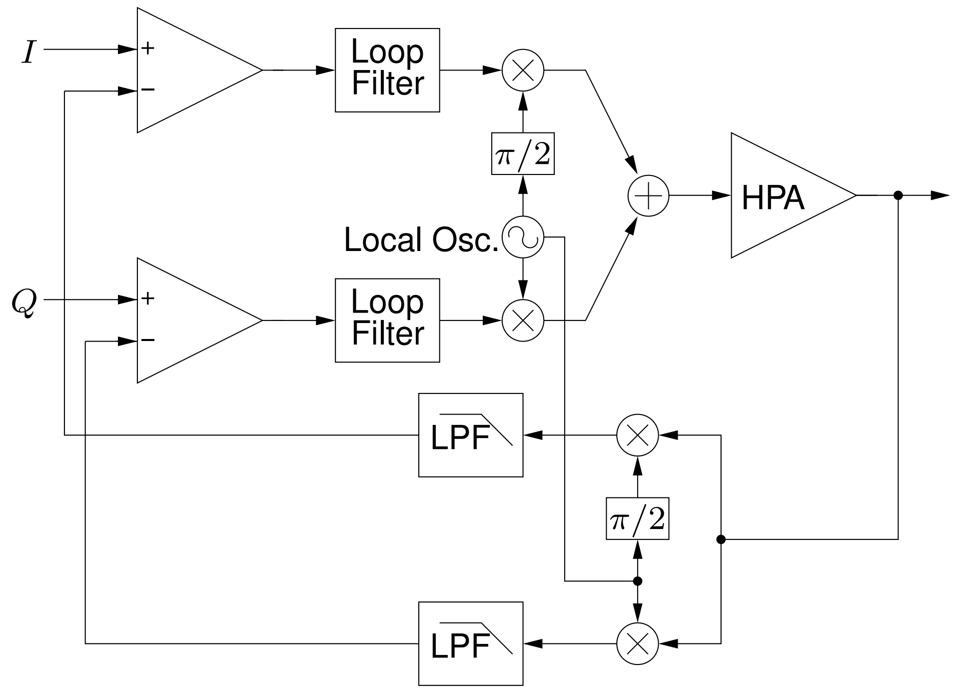
Linearization

- **Feedback**
- **Feedforward**
- **Predistorion**

IV. Compensation techniques

Linearization

- **Feedback**
  - It requires analog circuit design.



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<u>Linearization</u>

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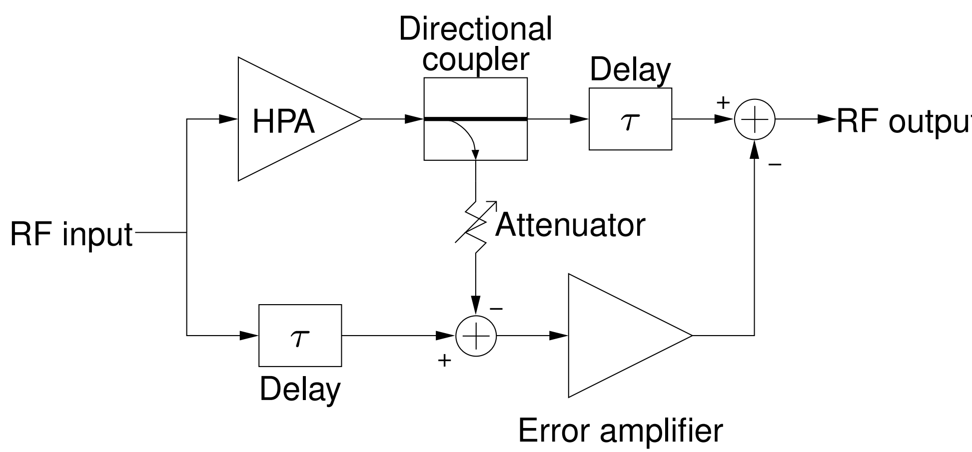
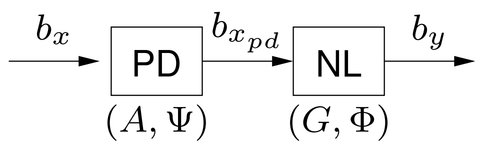


IV. Compensation techniques

Linearization

- **Feedback**
- **Feedforward**
- **Predistorion**
  - It can be implemented fully digitally: *Digital predistortion*.

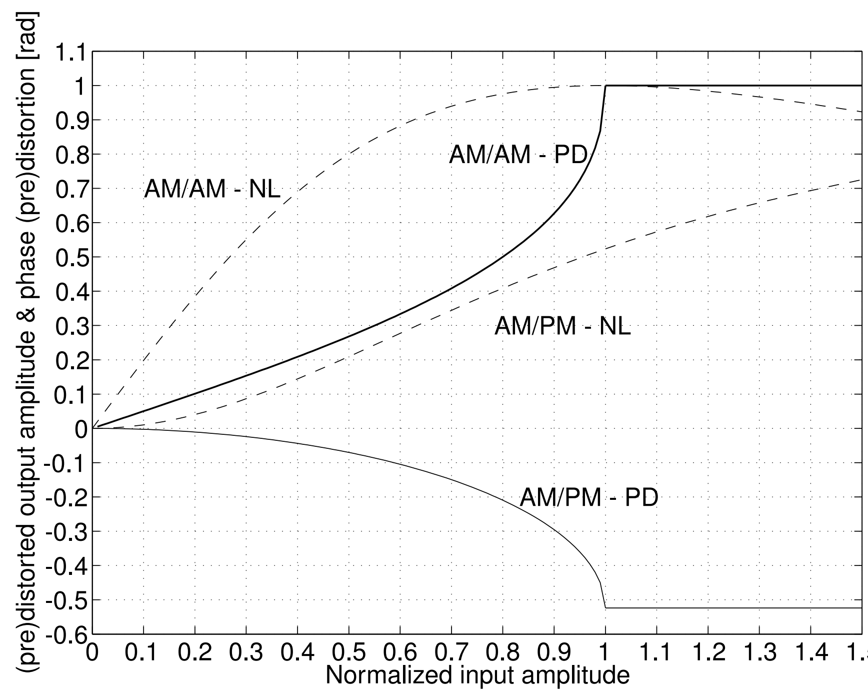


$\Rightarrow$ The AM/AM and AM/PM characteristics of the predistorter must satisfy:

$$A(u) = G^{-1}(u),$$
$$\Psi(u) = -\Phi(G^{-1}(u))$$

$\Rightarrow$ After combining the PD and the NL we obtain a SL:

$$\begin{aligned} G(A(u)) &= \begin{cases} u & \text{if } u \leq 1 \\ 1 & \text{otherwise} \end{cases} \\ \Psi(u) + \Phi(A(u)) &= 0 \end{aligned}$$



IV. Compensation techniques

Post-processing

- The aim is to eliminate the distortion term introduced by the transmitter NL.
  - We saw that the in-band NL distortion can be explained from the in-band intermodulation products of the different subcarriers.
  - Therefore, to reduce the in-band distortion at the receiver side we need to make the decision based on all data symbols that are transmitted parallelly in each OFDM symbol (sequence detection).
- Consider a multipath fading channel, then, form the Bussgang theorem the decision variables at the input of the demodulation stage can be expressed as

$$R_k = H_k(\alpha S_k + D_k^{(\mathbf{S})}) + W_k \qquad k = 0, \ldots, N-1,$$

The maximum likelihood (ML) sequence detector must solve

$$\hat{\mathbf{S}} = \arg\min_{\forall \overline{\mathbf{S}}} \sum_{k=0}^{N-1} \left| R_k - H_k(\alpha \overline{S}_k + D_k^{(\overline{\mathbf{S}})}) \right|^2 .$$

$\boxed{\text{Too complex, we need to find a suboptimal solution with reduced complexity.}}$

IV. Compensation techniques

Post-processing (cont.)

- Let us assume that the receiver can compute $\mathbf{D}^{\mathbf{S}}$ and that $\mathbf{H}$ is known. Then, since the distortion term can be eliminated from the decision variables [Tellado,TransCom,2003]

$$\tilde{R}_k = R_k - H_k D_k^{(\mathbf{S})} = \alpha H_k S_k + W_k \qquad k = 0, \ldots, N-1,$$

the deterministic term $D_k^{(\overline{\mathbf{S}})}$ in the ML detector becomes unnecessary,

$$\hat{\mathbf{S}} = \arg \min_{\forall \overline{\mathbf{S}}} \sum_{k=0}^{N-1} \left| \tilde{R}_k - \alpha H_k \overline{S}_k \right|^2 = \arg \min_{\forall \overline{\mathbf{S}}} \sum_{k=0}^{N-1} \left| \alpha H_k \left(\left[\frac{R_k}{\alpha H_k} - \frac{D_k^{(\mathbf{S})}}{\alpha} \right] - \overline{S}_k \right) \right|^2 .$$

- How can the receiver compute $\mathbf{D}^{\mathbf{S}}$?
  - One possible solution to obtain $\mathbf{D}^{\mathbf{S}}$ is by transmitting additional information to the receiver.

    $\Rightarrow$ An accurate reception of $\mathbf{D}^{\mathbf{S}}$ requires transmitting a large amount of additional information.

    $\Rightarrow$ This solution reduces the throughput of the communication system considerably.
  - The receiver can compute an estimate of $\mathbf{D}^{\mathbf{S}}$, denoted as $\hat{\mathbf{D}}^{\mathbf{S}}$, from the received symbol vector $\mathbf{R}$, if it knows the transmit nonlinear function $f(\cdot)$ and has an estimate of the transmit symbol vector [Tellado,TransCom,2003].

IV. Compensation techniques

Post-processing (cont.)

- Simple iterative quasi-ML algorithm based on hard decoding [Tellado,TransCom,2003].
  1. Compute a hard-decision of the received vector as
  $$\mathbf{S}^{(q)} = \left[\left\langle \frac{R_0}{\alpha H_0} - \frac{D_0^{(\mathbf{S}^{(q-1)})}}{\alpha} \right\rangle \cdots \left\langle \frac{R_{N-1}}{\alpha H_{N-1}} - \frac{D_{N-1}^{(\mathbf{S}^{(q-1)})}}{\alpha} \right\rangle \right]^T = \left\langle \frac{\mathbf{R}}{\alpha \mathbf{H}} - \frac{\mathbf{D}^{(\mathbf{S}^{(q-1)})}}{\alpha} \right\rangle$$
  where $\mathbf{D}^{(\mathbf{S}^{(0)})} = [\overbrace{00 \cdots 0}^{N}]$ and starting with $q = 1$.
  2. Assuming that $\mathbf{S}^{(q)}$ was transmitted compute the corresponding distortion term as
  $$\mathbf{D}^{\mathbf{S}^{(q)}} = \text{DFT}\left(\mathbf{d}^{\mathbf{S}^{(q)}}\right) = \text{DFT}\left(f(\mathbf{s}^{(q)}) - \alpha \cdot \mathbf{s}^{(q)}\right),$$
  where $\mathbf{s}^{(q)} = \text{IDFT}\left(\mathbf{S}^{(q)}\right)$.
  3. Go to step 1 and compute $\mathbf{S}^{(q+1)}$ by considering both the received symbol vector and the estimated distortion term.
- Similar versions have been proposed in [Chen,ComLet,2003] and [Behravan,PhD,2006]. Moreover, in [Behravan,PhD,2006] soft decision decoding is also considered and shown to offer better performance.
- A different decision-directed iterative nonlinear decoder was proposed in [Kim,ComLet,1999], however this is not ML.

IV. Compensation techniques

Reducing the envelope fluctuations of the transmitted signal

- The most common measure is the so-called peak-to-average power ratio (PAPR).
  - Let $\mathbf{s}^{(m)}$ be the m-th OFDM symbol of length LN, the PAPR of $\mathbf{s}^{(m)}$ is defined as the ratio between its peak power and the average power of all OFDM symbols:

$$\text{PAPR}_m = \text{PAPR}(\mathbf{s}^{(m)}) = \frac{||\mathbf{s}^{(m)}||_\infty^2}{E_n[||\mathbf{s}^{(n)}||^2]/LN}$$

  - PAPR-reduction is evaluated in terms of its CCDF:

$$\text{CCDF(PAPR)} = Pr(\text{PAPR}_m > \gamma_0)$$

  - The problem of PAPR-reduction consists on modifying the envelope of the signal such that the CCDF(PAPR) is reduced.
- Several PAPR-reduction techniques that have been proposed in recent years are:
  - Clipping
  - Active Constellation Extension
  - Tone Injection
  - Tone Reservation
  - Selected Mapping
  - Partial Transmit Sequences
  - Interleaving
  - Controlled Spectral Outgrowth
  - Coding

IV. Compensation techniques

Clipping

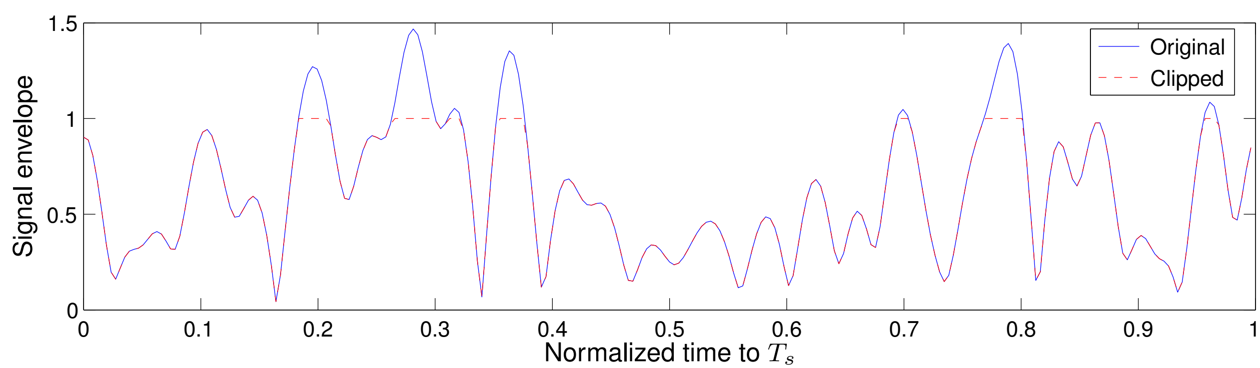
- In clipping technique all samples exceeding a given threshold are clipped to this maximum value [Li,ComLet,1998][Ochiai,TransCom,2002].

√ Advantage: Assures that PAPR is always reduced to a desired level.

× Drawback: Increases both the out-of-band radiation and the BER.

  - Clipping a signal can be seen as passing it through a soft-limiter nonlinearity.

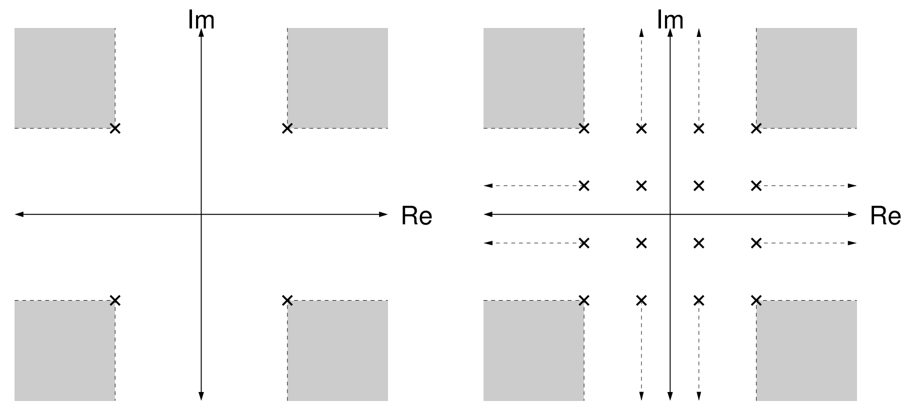
- Filtering and windowing can be introduced to control the performance degradation [Li,ComLet,1998][Armstrong,ElectroLet,2002].
  - These methods try to balance the compromise between reducing the PAPR of a signal and scarcely increasing their performance degradation.



IV. Compensation techniques

Active Constellation Extension (ACE)

- In ACE, at each OFDM block, some of the outer signal constellation points are extended towards outside of the constellation such that the PAPR of the resulting block is reduced [Krongold,TransBroad,2003].



√ Advantages:

- It is transparent to the receiver.
- There is no loss of data rate.
- No side information is required.

× Drawbacks:

- The increase in the average energy per bit might be higher than the BER improvement.
- The larger the constellation size is the lower the number of extensible constellation points will be.

- Optimal PAPR-reduction can be obtained by solving a convex optimization problem, particularly a quadratically-constrained quadratic program (QCQP).
  - In [Krongold,TransBroad,2003] a low complexity method is proposed, $\mathcal{O}(N \log N)$.

IV. Compensation techniques

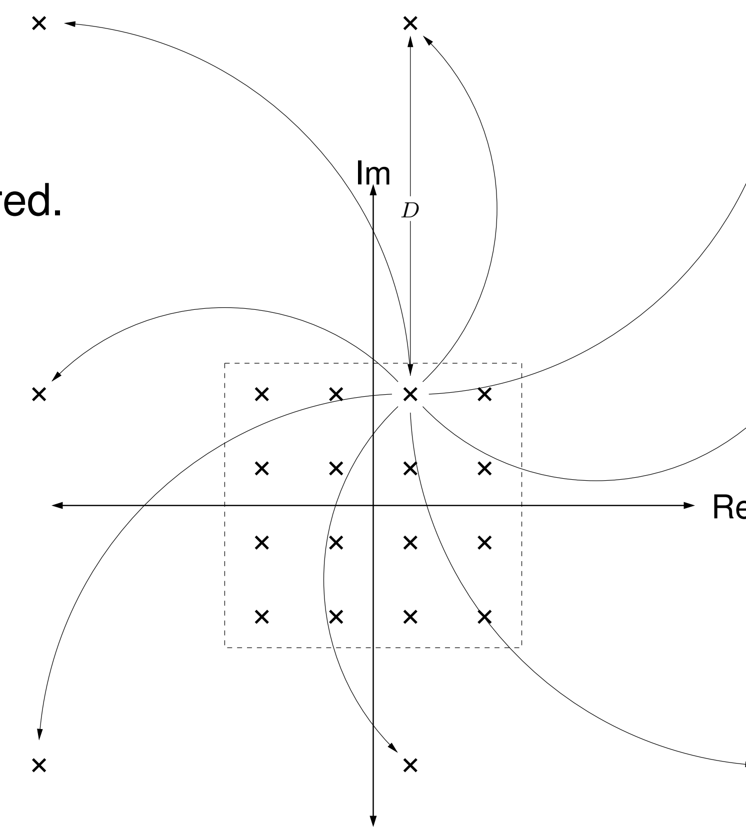
Tone Injection (TI)

- The idea is to increase the constellation size so that each of the points in the original constellation can be mapped into several equivalent points in the expanded constellation. These extra degrees of freedom can be exploited for PAPR-reduction [Tellado,PhD,1999].
  - By choosing $D_k \geq d_k\sqrt{M_k}$, the error probability is not increased.
  - To recover the original symbol points a simple *modulo*-D_k operation is required.

√ Advantages:
  - There is no loss of data rate.
  - No transmission of side information is required.

× Drawbacks:
  - The increase in the average energy per bit might be higher than the BER improvement.

- Optimal solution requires solving an integer programing problem (exponent. complexity).
  - [Tellado,PhD,1999] proposes a less computationally demanding algorithm.

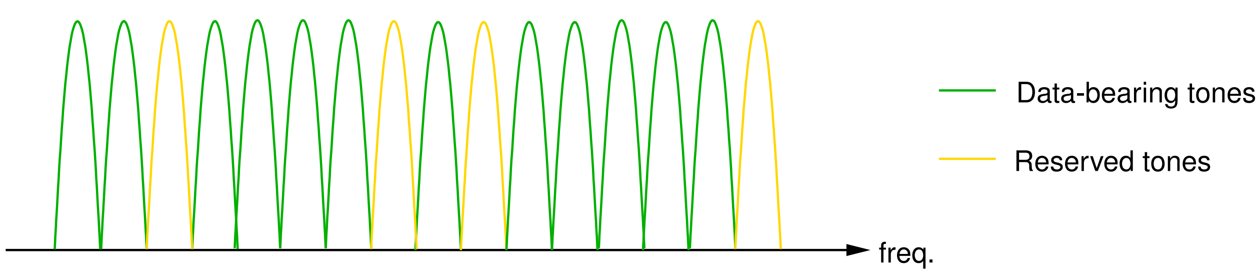


IV. Compensation techniques

Tone Reservation (TR)

- TR consists on reducing the PAPR by reserving a few tones (PRT) within the transmitted bandwidth and assign them the appropriate values [Tellado,PhD,1999].
  - Consider $\mathbf{S}$ as the length-N OFDM symbol vector in the frequency domain with nonzero tones at positions $\mathcal{I}_M = \{i_1, i_2, \ldots, i_M\}$.
  - Consider $\mathbf{C}$ as the length-N peak-reducing signal in the frequency domain with nonzero tones (PRT) at positions $\mathcal{J}_R = \{j_1, j_2, \ldots, j_R\}$
  - $\mathcal{I}_M$ and $\mathcal{J}_R$ are disjoint sets and $R + M = N$.
  - The peak-reduced OFDM symbol in the time domain is computed as:

$$\bar{s}_{n/L} = s_{n/L} + c_{n/L} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (S_k + C_k) e^{j \frac{2\pi k n}{NL}}$$



IV. Compensation techniques

Tone Reservation (TR) (cont.)

√ Advantages:

- Since $\mathcal{I}_M$ and $\mathcal{J}_R$ are disjoint sets, $\bar{s}_{n/L}$ is created without introducing any distortion to the data bearing tones
- No transmission of side information is required.

× Drawbacks:

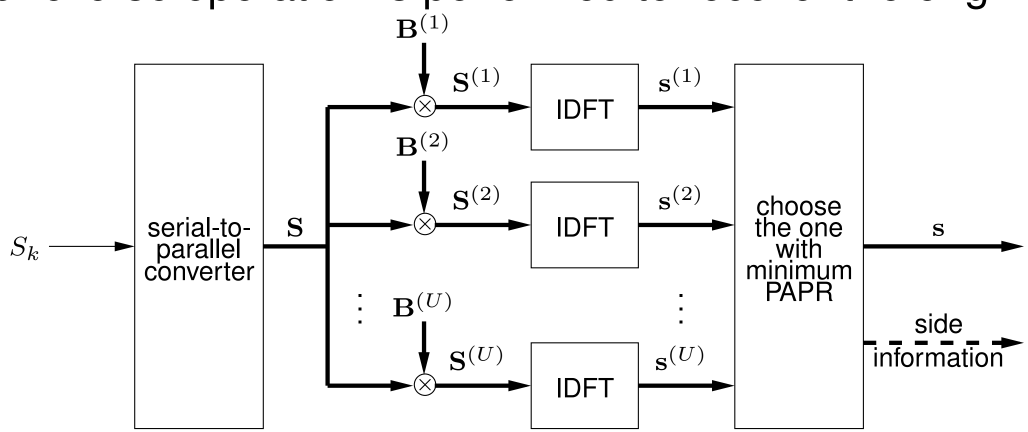
- Increase in the average energy per bit which may reduce the BER performance.
- Loss of spectral efficiency due to the tone reservation.

- The optimal PRT solution requires solving a QCQP.
  - In [Tellado,PhD,1999], [Krongold,TransSigProc,2004] and [Deumal,VTC,2007] several low complexity methods have been proposed.
    - These methods have similar complexity requirements, $\mathcal{O}(N)$, but different convergence speed and PAPR-reduction capabilities.

IV. Compensation techniques

Selected Mapping (SLM)

- In SLM, from the original data block several candidate data blocks are generated and the one with the lowest PAPR is transmitted [Bauml,ElectroLet,1996] i [Breiling,ComLet,2001].
- At the receiver the reverse operation is performed to recover the original data block.



√ Advantage: No distortion is introduced.

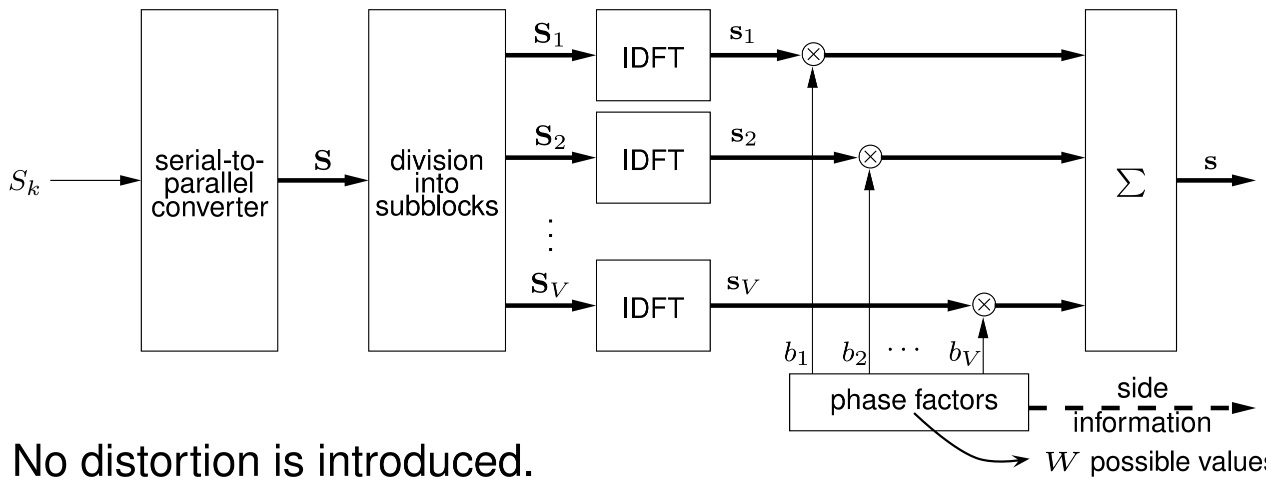
× Drawback: It requires transmitting $\lceil \log_2 U \rceil$ bits of side information per OFDM symbol.
$\Rightarrow$ It is crucial that side information is received without errors.
$\Rightarrow$ side information has to be heavily protected.

- SLM has a complexity of U IFFT operations and U complex vector multiplications.
- The amount of PAPR-reduction depends on U and the design of the phase sequences.

IV. Compensation techniques

Partial Transmit Sequences (PTS)

- The original data block is partitioned into V disjoint subblocks. The subcarriers in each subblock are rotated by the same phase factor such that the PAPR of the combination is minimized. [Muller,ElectroLet,1997], [Tellambura,ElectroLet,1998] and [Cimini,ComLet,2000].
- At the receiver the reverse operation is performed to recover the original data block.



√ Advantage: No distortion is introduced.

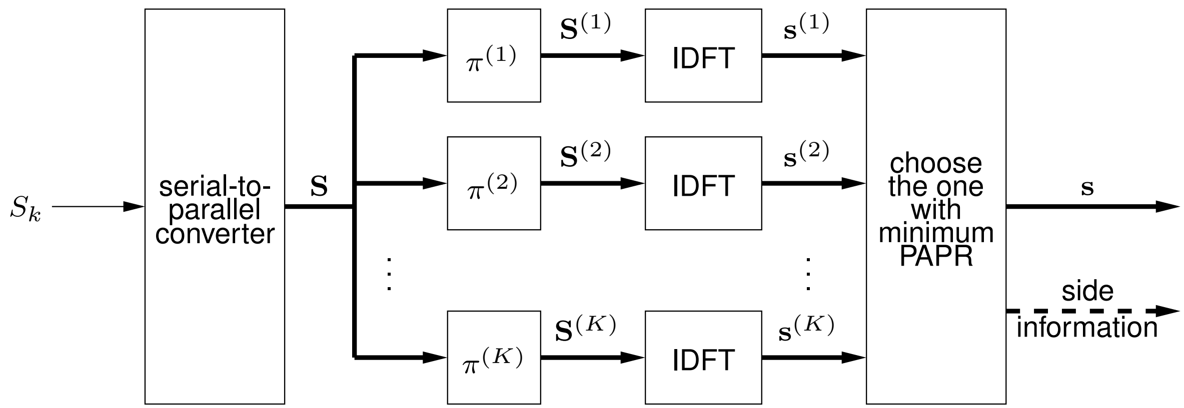
× Drawback: It requires transmitting $\lceil \log_2 W^{(V-1)} \rceil$ bits of side information.

- PTS has a complexity of V IFFT operations, $(V-1)W^{(V-1)}$ complex vector multiplications and $(V-1)W^{(V-1)}$ complex vector sums.
- The amount of PAPR-reduction depends on V, W and the subblock partitioning.

IV. Compensation techniques

Interleaving

- Similar to SLM, but candidate data blocks are generated by interleaving the original data block. The one with lowest PAPR is transmission [Eetvelt,ElectroLet,1996], [Hill,ElectroLet,2000] i [Jayalath,ElectroLet,2000].



✓ Advantage: No distortion is introduced.

× Drawback: It requires transmitting $\lceil \log_2(K) \rceil$ bits of side information.

- It has a complexity of K IFFT operations plus the complexity associated to $k-1$ interleavings.
- The amount of PAPR-reduction depends on K and the design of the interleavers.

IV. Compensation techniques

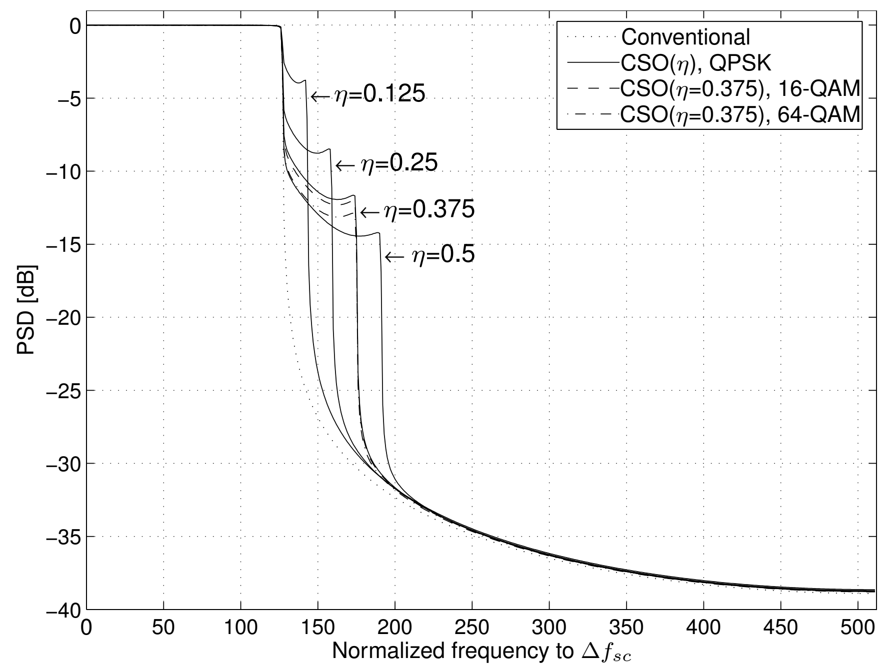
Controlled Spectral Outgrowth (CSO)

- In CSO PAPR is reduced by increasing the out-of-band radiation of the nearby subcarriers in a controlled way [Deumal,ICASSP,2006].

√ Advantages:

- It is transparent to the receiver.
- No side information is required.
- There is no loss of spectral efficiency, as long as an spectral mask is used.
- It scarcely increases the average energy per bit.

Power increase (dB)	QPSK	16-QAM	64-QAM
η = 0.125	0.238	0.227	0.202
η = 0.25	0.172	0.159	0.133
η = 0.375	0.136	0.124	0.101
η = 0.5	0.111	0.101	0.080



× Drawback: It must be specifically designed to match the different standard requirements.

- In [Deumal,ICASSP,2006] a low complexity method is proposed, $\mathcal{O}(N \log N)$.

IV. Compensation techniques

Coding

- This technique consists on using block coding to transmit across the carriers only those sequences with small PAPR [Jones,ElectroLet,1994] and [Wilkinson,VTC,1995].
- [Davis,TransInfoTheory,1999] takes advantage from both the relation between Golay sequences and Reed-Muller codes and their properties:
  - Consider two Golay sequences [Golay,TransInfoTheory,1961]:

$$\left.\begin{aligned} \mathbf{a} &= [a_0, a_1, \ldots, a_{n-1}], \\ \mathbf{b} &= [b_0, b_1, \ldots, b_{n-1}], \end{aligned}\right\} \text{where } a_i, b_i \in \mathbb{Z}_H \text{ (integer ring of size } H\text{)}$$

    If the aperiodic autocorrelation of $\mathbf{a}$ and $\mathbf{b}$ at displacement u

$$C_a(u) = \sum_{i=0}^{n-1} e^{j2\pi(a_i - a_{i+u})/H},$$

    satisfies that

$$C_a(u) + C_b(u) = 0, \text{ for each } u \neq 0,$$

    the sequences $\mathbf{a}$ and $\mathbf{b}$ are called a Golay complementary pair over $\mathbb{Z}_H$ of length n [Davis,TransInfoTheory,1999].
    - The advantage of using Golay sequences in an OFDM system is that they have a maximum PAPR of 3dB [Popovic,TransCom,1991].

IV. Compensation techniques

Coding (cont.)

- Reed-Muller codes are claimed to have good error correction properties and be easy to decode.
- Each of the $m!/2$ cosets of $\mathcal{RM}(1,m)$ in $\mathcal{RM}(2,m)$ having a coset representative of the form $\sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)}$ comprises 2^{m+1} binary Golay sequences of length 2^m, where π is a permutation of the symbols $\{1, 2, \ldots, m\}$ [Davis,TransInfoTheory,1999].

√ Advantages: Good error correction capabilities and maximum PAPR of 3dB.

× Drawbacks: For practical number of subcarriers (larger than 32) the code rate is too low.

m	# of subcarriers	$\mathrm{PAPR}_{\mathrm{MAX}}$	k	code rate	$\delta(\mathcal{C})$	$\rho(\mathcal{C})$
3	8	3dB	5	62.5000%	2	0
4	16	3dB	8	50.0000%	4	1
5	32	3dB	11	34.3750%	8	3
6	64	3dB	15	23.4375%	16	7
7	128	3dB	19	14.8438%	32	15
8	256	3dB	23	8.9844%	64	31
9	512	3dB	27	5.2734%	128	63
10	1024	3dB	31	3.0273%	256	127

IV. Compensation techniques

Some considerations on PAPR reduction

- PAPR is a very well known measure of the envelope fluctuations of multicarrier signals and has become the figure of merit used to define the goodness of a method.
  - The problem of reducing the envelope fluctuations with the aim to increase the system performance has turned to reducing PAPR.
- Is PAPR an appropriate measure of the envelope fluctuations?
  - In order to improve the system performance, PAPR should predict the amount of distortion introduced by the nonlinear amplifier.
  - The effect of a nonlinearity on an OFDM signal is not clearly related to its PAPR [Deumal,TransWCom,2006].
    - PAPR is increased by the number of carriers in the OFDM signal. However, the performance degradation due to the nonlinearity is not.
    - PAPR is dependent on the constellation size while the distortion introduced by the nonlinearity is not.

      $\Rightarrow$ The distortion term and the uniform attenuation and rotation of the constellation only depend on the back-off of the nonlinearity.

IV. Compensation techniques

Some considerations on PAPR reduction (cont.)

- Another important aspect is to determine which are the performance improvement capabilities of PAPR-reducing methods [Deumal,TransWCom,2006].
  - For most of the PAPR-reducing techniques the peak-reduced signal can be formulated as

$$\bar{s}_{n/L} = s_{n/L} + c_{n/L} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (S_k + C_k) e^{j\frac{2\pi kn}{NL}},$$

  - The effective energy per bit of this signal at the input of the nonlinearity is

$$E_b^{(eff)} = \frac{E_o}{K} \frac{\sigma_{\mathbf{s}}^2}{\sigma_{\mathbf{s}+\mathbf{c}}^2} = \frac{E_o}{K} \eta_p.$$

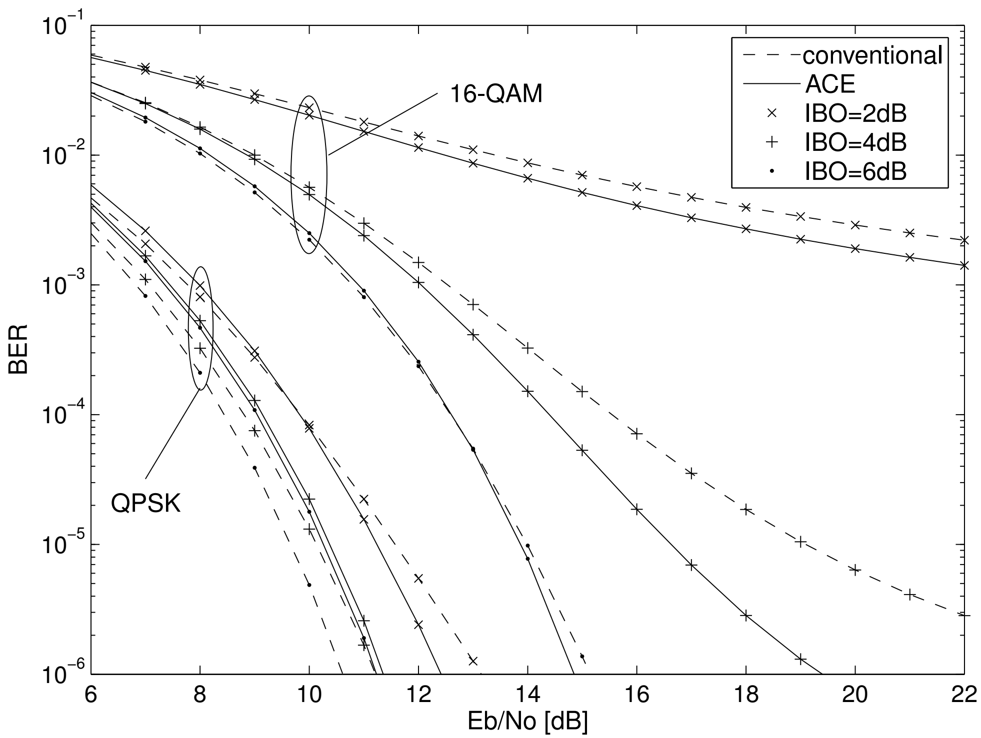
    where E_o is the average energy of the signal at the input of the NL, K is the number of bits per symbol and η_p is the power efficiency.
  - There will only be a BER performance improvement when the effect of reducing the in-band distortion becomes noticeable and more important than the loss of power efficiency due to the addition of the correcting signal.
  - This is not taken into account in the majority of PAPR-reducing methods.

IV. Compensation techniques

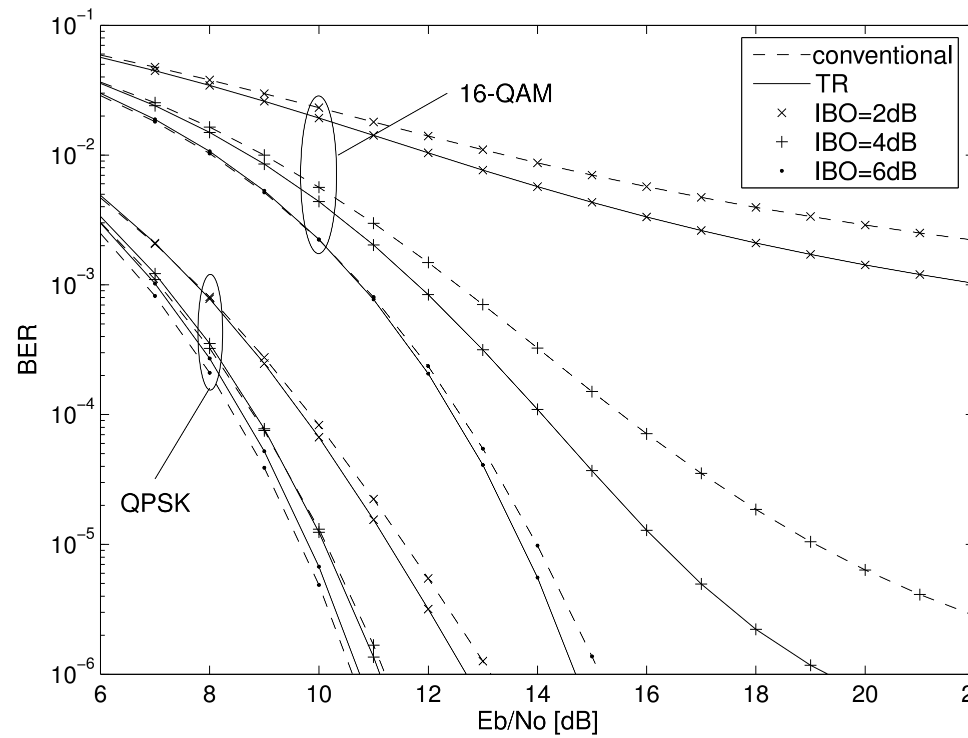
Some considerations on PAPR reduction (cont.)

- BER performance of a conventional and a ACE/TR-based PAPR-reduced OFDM system obtained when a soft limiter is used.

Active constellation extension



Tone reservation



IV. Compensation techniques

Some considerations on PAPR reduction (cont.)

- In some recent contributions, other measures of the envelope fluctuations of multicarrier signals have been proposed:
  - The **cubic metric** [Motorola,TechReport,2005] is obtained by first normalizing the signal to a rms value of 1.0, and then computing the rms value of the cubed signal

$$\mathrm{CM} = \sqrt{E\left[\left(\left|\frac{s_n}{\sqrt{E[|s_n|^2]}}\right|^3\right)^2\right]}.$$

    - Motivation: CM relies on the fact that the major distortion is caused by the third-order intermodulation product.
  - The **variance of the instantaneous power** [Behravan,VTC,2006] is defined as

$$\mathrm{VP} = \mathrm{var}(|s_n|^2).$$

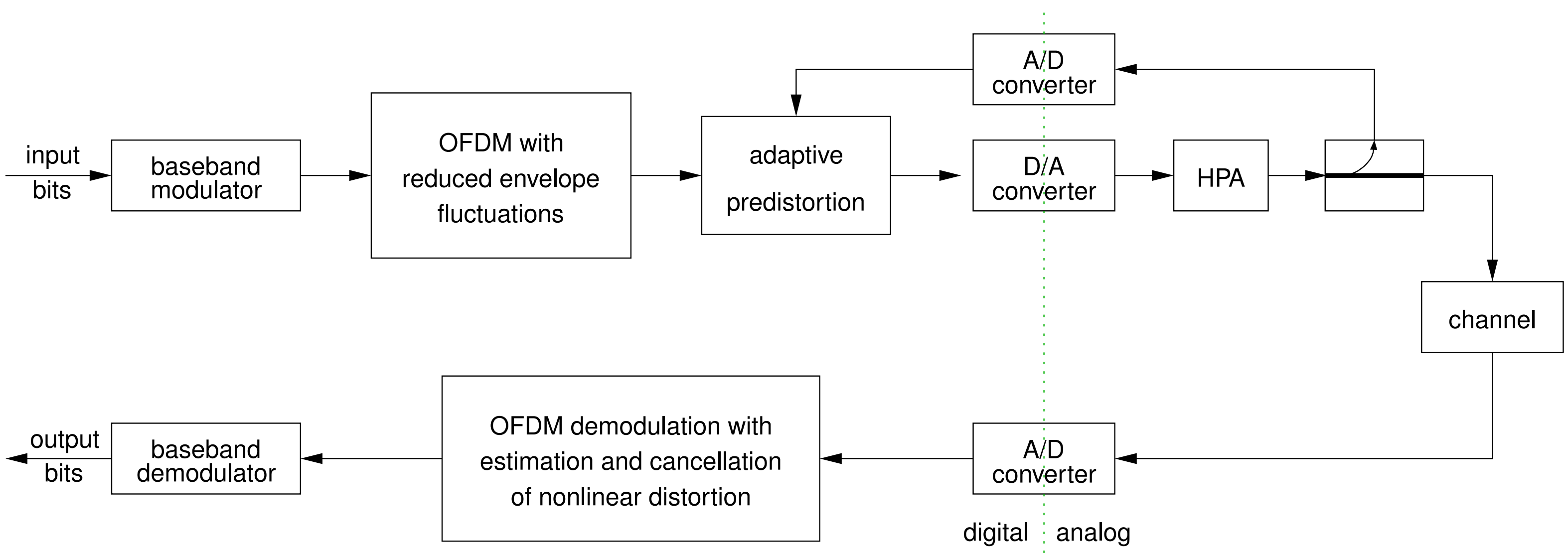
    - Motivation: VP directly reduces the envelope fluctuations.
- In [Behravan,VTC,2006] a study of PAPR, CM and VP shows that except for large back-off values, PAPR is less related to the amount of distortion introduced by a nonlinearity than the other metrics.

Outline

- Orthogonal Frequency Division Multiplexing (OFDM)
- Nonlinear models for high power amplifiers
- Multicarrier systems with nonlinear amplifiers
- Overview of nonlinear distortion compensation techniques in OFDM
- Conclusions

V. Conclusions

- In this tutorial we have basically ...
  - analyzed the sensitivity of OFDM systems to nonlinear amplification.
    - Performance degradation: spectral outgrowth and increase of the error probability.
    - The performance degradation is due to a distortion term introduced by the NL that can generally be modeled as a complex Gaussian random variable.
  - presented an overview of the strategies that are commonly used to reduce the performance degradation of OFDM systems when nonlinear amplifiers are present.



V. Conclusions

- We saw that PAPR is the most common metric of the envelope fluctuations and presented several PAPR-reduction techniques that have been proposed recently.

  The major characteristics of PAPR-reducing techniques are:

  - Whether they are transparent to the receiver.
    - Transparent: Clipping, ACE and CSO.
    - Non-transparent: TI, TR, SLM, PTS, Interleaving and Coding.
  - Whether they introduce distortion to the transmitted signal.
    - Distortionless: ACE, TI, TR, SLM, PTS, Interleaving, CSO and Coding.
    - Non-distortionless: Clipping.
  - Whether they require the transmission of additional information:
    - Requiring: SLM, PTS, Interleaving.
    - Non-requiring: Clipping, ACE, TI, TR, CSO and Coding.
  - Whether they reduce the spectral efficiency:
    - Lossy: TR, SLM, PTS, Interleaving
    - Lossless: Clipping, ACE, TI, Coding and CSO (with spectral mask).

V. Conclusions

- PAPR might not always be the most appropriate measure.
  - The effect of a nonlinearity on an OFDM system is not clearly related to its PAPR.
    - In some cases applying PAPR-reduction can even lead to increasing the performance degradation.
  - In some recent contributions other measures of the envelope fluctuations such as the cubic metric and the variance of the instantaneous power have been proposed.
    - CM relies on the fact that the major distortion is caused by the third-order intermodulation product.
    - VP directly reduces the envelope fluctuations.

V. Conclusions

- Future research topics:
  - VP, CM and other possible metrics should be evaluated to see if they offer better performance than PAPR.
    - Most of the PAPR-reduction techniques should be redefined to consider these new metrics.
    - Their performance should be compared to the performance offered by PAPR-reduction.
  - Joint solutions should be considered. Evaluate OFDM systems exploiting:
    - linearization and post-processing.
    - envelope fluctuations reduction and linearization.
    - envelope fluctuations reduction and post-processing.
    - envelope fluctuations reduction, linearization and post-processing.
  - Some contributions consider PAPR-reduction in OFDM systems with multiple antennas (MIMO).
    - This should be generalized not only to envelope fluctuations reduction but also to linearization and post processing.

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Thanks