Performances of Convolutionally-Coded QAM Mapping Non-Coded Bits

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Abstract
The paper analyzes the BER and spectral efficiency performances of convolutionally-coded square QAM constellations that map non-coded bits, besides the coded ones, in an OFDM environment that imposes a user bin with only 81 QAM payload-symbols. It shows that the QAM constellations that map only coded bits using coding rates higher than 2/3 are outperformed , i.e. higher coding gains and/or soectral efficiencies, by configurations that map non-coded bits besides the coded bits using coding rates not greater than 2/3.

Introduction

One way to increase the spectral efficiency ensured by coded QAM constellations is to map some non-coded bits, besides the coded ones on each QAM symbol. This approach would bring an increase coding rate for the coded configuration, i.e. the assembly of the code and QAM constellation, at the expense of a smaller coding gain.
But with an appropriate bit-mapping, the error-rate of the non-coded bits might be close to the one of the coded ones, leading to a global error-rate comparable to the one of the coded bits and to an increased spectral efficiency at the same SNR value.
The performances of such coded configurations are analyzed for short messages employed in mobile communications.

Initial Assumptions

The messagge length (user-bin) is chosen to have 81 QAM payload symbols, for reasons to be explained later in this paper.
The number of bits/symbol of configuration i, n_i, is split into n_{ci} coded bits (taken of the output of the convolutional encoder) and n_{ni} non-coded information bits, i.e:

$$n_i = n_{ci} + n_{ni} \tag{1}$$

In order to allow the mapping of at least two non-coded bits and still to ensure at least two coded bits/QAM symbol, only constellations with at least n_i = 4 bits/symbol would be considered. The paper analyzes the performances of the configurations presented in table 1, built on the 16 and 64-QAM constellations. The configurations with all bits coded are included for reference; their performances are presented in [1]

n_i	4	4	6	6	6
$_{ci}$	4	2	6	2	4
n_{ni}	0	2	0	4	2

Table 1.
Structures of the coded configurations employed

The n_c coded bits are mapped according the Mapping by Set Partitioning (MSP) method [2], which maximizes the free Euclidean distance d_{Efree} and the n_n non-coded bits are mapped using a Gray mapping .
The convolutional codes employed have the following paramters:

- are based on the "parent" R_p=1/2, K = 9 (256-state trellis), G = $[561,753]_8$
- the codes with R = 2/3, ¾, 4/5, 5/6 are obtained by puncturing the parent code with the puncturing patterns of [3].

To ensure the non-correlation between successive bins, which might be transmitted with different modulations and codes under different channel conditions, each bin is finished with a number of termination information bits which would bring the trellis to the "all-zero" state. The number of termination bits n_t for the R_p = ½ and codes with R = m/(m+1) is:

$$n_t = \begin{cases} [(K-1)/R]+1 & \text{for} \quad (K-1)/R \notin N \\ (K-1)/R & \text{for} \quad (K-1)/R \in N \end{cases} \tag{2}$$

The convolutional codes are decoded with the Viterbi algorithm with the following properties:

- the metric employed is the *a posteriori* probability of each bit, p(1/r,) p(0/r), [4]

- the "cumulative distance" is computed by the product of the a posteriori probabilities of the bits corresponding to each path.
- the "missing" bits of the punctured codes are replaced, according to the puncturing pattern, by *a posteriori* probabilities equaling 1

The values of the *a posteriori probabilities* are obtained by a soft demapping procedure defined in [5].

Soft-decision of the non-coded bits

To ensure a smaller BER of the non-coded bits, they are decided employing a soft-decision procedure, similar to the one defined in [2]. It performs basically the following steps:

- due to the MSP mapping, the coded bits define the 2^{nc} subsets and the non-coded bits define the vectors within the subsets;
- once the coded bits are known from the Viterbi decoding, the subset is known; then the vector at the smallest d_E from the received vector is chosen and its non-coded bits are the decided bits.

Coded Modulations Studied

The coded modulations studied are the 16-QAM, with n_{ci} = 4 or 2 and 64-QAM with n_{ci} = 6,4 (table 1) coded with the convolutional codes mentioned above having the coding rates R=1/2, 2/3, ¾, 4/5 and 5/6.

Rate of the coded configurations

Considering the 81 QAM payload-symbols bin that carry the $n = n_c + n_n$ bits/symbol and the n_t trellis-termination bits, the coded configuration rate is expressed by:

$$R_{cfgi} = \frac{[81 \cdot n_{ci} \cdot R] - n_{ti} + 81 \cdot n_{ni}}{81 \cdot n_i} \qquad (3)$$

Table 2 shows the rates of the studied coded configurations.

n_i, n_{ci}, n_{ni} ↓ R→	½	2/3	¾	4/5	5/6	6/7*
4 4 0	0.450	0.630	0.716	0.769	0.802	0.826
4 2 2	0.701	0.797	0.841	0.869	0.885	0.897
6 6 0	0.467	0.642	0.727	0.779	0.813	0.857
6 4 2	0.633	0.753	0.811	0.846	0.868	0.884
6 2 4	0.800	0.864	0.894	0.910	0.924	0.931

Table 2 Coding rates of the studied configurations

The coding rates of table 2 show that the mapping of non-code bits increase the coding rate of the configurations. They also suggest that a combination of coded bits, coded with a smaller coding rate, more powerful, and non-coded bits, offers about the same coding rate as a configuration that maps only coded bits coded with a higher coding rate, less powerful, see the arrows on the table 1; e.g. for the 16-QAM, the configuration 422 R = ½ offers about the same coding rate as the 440 R = ¾.

Table 2 also shows that the codes with rates higher than 2/3 may be replaced by codes with R = ½ or 2/3 as follows:

- for the 16-QAM:

- the 440 R=1/2 or 2/3 could provide coding rates up to 0.63
- the 422 R = ½ or 2/3 could provide coding rates between 0.7 up to 0.8 replacing the 440 coded with R=3/4, 4/5 and 5/6

  - for the 64-QAM:

- the 660 R=1/2 could provide R_{cfg}=0.467
- the 642 R = ½ , 2/3 could provide R_{cgf} = 0.633 or 0.75
- the 624 R = ½ and 2/3 could provide R_{cf} = 0.800 or 0.864 (replacing even the 660 R=1/2)

* the configurations coded with the R=6/7 punctured code would be no longer studied since their coding gain is very small.

Erorr Probabilities of the Studied Configurations – Evaluation Methods

Due to the fact that the two types of bits mapped on the QAM symbol are decoded in different manners, their BER vs. SNR performances should be analyzed separately.

The global BER, p_g, is expressed, in terms of the coded bits BER p_{ec} and of the non-coded bits p_{en}, by:

$$p_g = \frac{p_{eni} \cdot n_{ni} + p_{eci} \cdot n_{ci}}{n_i} \qquad (4)$$

The BER vs. SNR performances of the convolutionally-coded QAM constellations that map non-coded bits are evaluated both theoretically and by computer simulation.

I. Theoretical evaluation of the BER

The error probability of these coded constellations could be computed using the approach proposed by [2]. This involves the analysis of the trellis diagram to establish the d_{Efree}, which is a complex operation for K =9.

The paper proposes an approximate method to evaluate the BER of these constellations:

I.a Evaluation of p_{ec}

The BER of the coded bits requires a previous evaluation of the coding gain provided by the coded modulation, C_G. Then, the BER is computed using (5) for the equivalent non-coded transmission which operates at a SNR_{eq} that equals SNR + C_G, [6]:

$$p_{ec} = \frac{4}{n_{ci}} Q\left(\sqrt{\frac{3}{N-1}\rho}\right) = \frac{2}{n_{ci}} \mathrm{erfc}\left(\sqrt{\frac{3}{N-1}\rho}\right); \quad N = 2^{n_i}; \quad \rho = 10^{(SNR+C_G)/10} \tag{5}$$

I.b, Evaluation of p_{en}

The non-coded bits may be in error in two situations:

b.1 the coded bits are correctly decoded (1-p_{ec}), i.e. the correct subset is chosen and the soft-decision chooses the wrong vector within the subset. This is equivalent to the error-probability p' of a QAM constellation having a minimum distance $d_p = d_0 \cdot 2^{n_{ci}/2}$, where d_0 denotes the minimum distance in the QAM constellation.

$$p' = \frac{4}{n_{ni}} Q\left(\frac{d_0 \cdot 2^{n_{ci}/2}}{\sigma}\right) = \frac{2}{n_{ni}} \mathrm{erfc}\left(\frac{d_0 \cdot 2^{n_{ci}/2}}{\sqrt{2} \cdot \sigma}\right) \tag{6}$$

b.2 the coded bits are in error, p_{ce}, and the non-coded bits are decided from the wrong subset; then the probability of error is p'':

$$p'' = p_{ec} \cdot \frac{2^{n_{ni}+1}}{n_{ni}(2^{n_{ni}} - 1)} \tag{7}$$

The error-probability of the non-coded bits is obtained from (6) and (7):

$$p_{en} = (1 - p_{ec})p' + p_{ec} \frac{2^{n_{ni}+1}}{n_{ni}(2^{n_{ni}} - 1)} \tag{8}$$

The global BER vs. SNR curve of the 16-QAM with n_c = 2, n_n = 2 coded with R=1/2 is shown in figure 1 for the theoretical evaluation (using 4-8) and in figure 2 for the computer simulation evaluation (1 million bits for every SNR value).

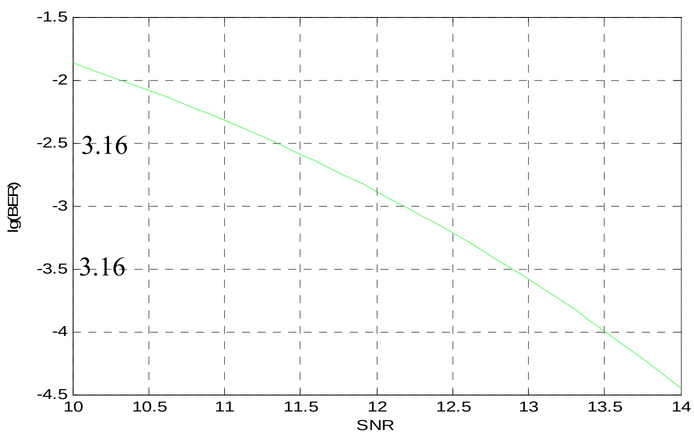

Figure 1 BER vs. SNR of the 422 R=1/2 (theoretical evaluation)

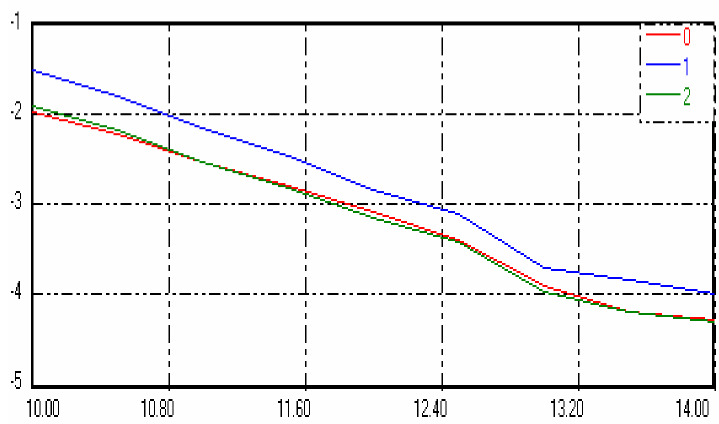

Figure 2 BER vs. SNR of the 422 R=1/2 (computer simulation) red – p_{en}; blue – p_{ec}; green - p_{eg}

The global BER vs. SNR curve of the 64-QAM with n_c = 4, n_n = 2 coded with R=1/2 is shown in figure 3 for the theoretical evaluation (using 4-8) and in figure 4 for the computer simulation evaluation.

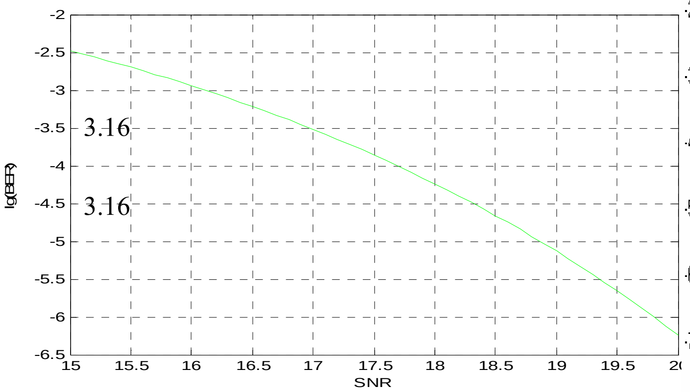

Figure 3 BER vs. SNR of the 642 R=1/2 (theoretical evaluation)

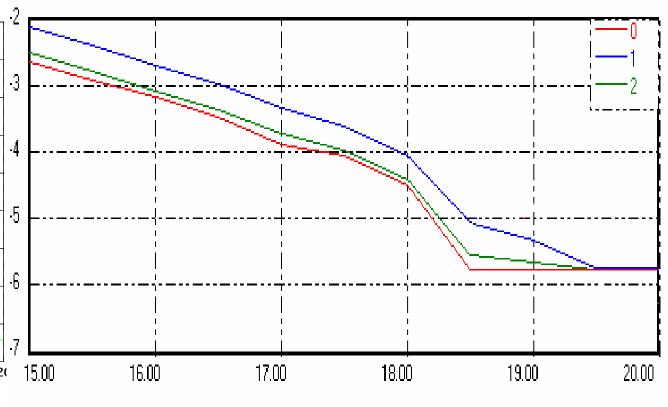

Figure 4 BER vs. SNR of the 642 R=1/2 (computer simulation) red – p_{en}; blue – p_{ec}; green - p_{eg}

The results of figures 1-4 and additional comparisons performed by the author show that the BER values provided by the approximate computation method (4-8) are reasonably close to the ones delivered by the computer simulations.

The approximate method for the BER computation of the coded configurations is accurate enough to provide a correct value of the spectral efficiency ensured by these constellations (to be explained later) and more simple than the rigorous method.

The simulations and separate theoretical computations of (4-8) show that the error rate of the non-coded bits is smaller than the one of the coded bits. Still this property should be more thoroughly analyzed.

BER Performances the Studied Configurations

The BER of the studied configurations were evaluated by computer simulations using the platform described in [7]. The coding gains were evaluated at the SNR values required to ensure BER = $1\cdot10^{-5}$. The SNR required by the non-coded 16 and 64-QAM to ensure the same BER are 19.5 dB and 25.5 dB, so the coding gains can be computed by referring to these figures.

The evaluation was made for BER higher or equal to 10^{-5} because the coded configurations are to be employed in transmissions with packets of 81 QAM-symbols, i.e. 324 or 486 bits, with a packet error-rate smaller than $1\cdot10^{-2}$ - $5\cdot10^{-3}$.

Table 3 presents the SNR by the studied configurations required to ensure BER = $1\cdot10^{-5}$.

n_i, n_{ci}, n_{ni} ↓ R→	½	2/3	¾	4/5	5/6	1-nc
4 4 0	12	15	16	17.5	18	19.5
4 2 2	15	16.5	17	-	-	-
6 6 0	16.3	20.5	22	23.5	24	25.5
6 4 2	19	21.6	22.2	23.5	24.6	-
6 2 4	21	22.5	23.5	-	-	-

Table 3. SNR values required to ensure BER = 10^{-5} by the studied coded configurations

As expected, the coding gains decrease with the increase of the coding rate. The same equivalencies that were shown in table 2 occur for the BER performances, i.e. codes with rates higher than 2/3 may be replaced by codes with R = ½ or 2/3 using non-coded bits as follows, see the arrows in table 3:

- for rates R = ½ and 2/3

- the 440 R = ½ ensures a coding gain of about 7.5 dB (curve 0 in figure 5)
- the 440 R=2/3 offers about the same performances as 422 R=1/2, (curves 1,2 in figure 5)
- the 660 R = 2/3 offers better performances as the 642 R = ½ (curves 3, 4 in figure 5)
- the 440 R=3/4 configuration may be replaced by the 422 R=2/3, for BER ≥ 10^{-5}, while the 440 R =4/5 ensures a small coding gain (see figure 6).
- the 660 R=1/2 offers about the same performances as the constellations above (see figure 6)
- Configurations 660 R=3/4, 642 R=2/3 and 624 R =1/2 have similar performances (figure 7)
- Configurations 660 R=4/5, 642 R=3/4 and 624 R =2/3 have similar performances (figure 7)
- The 660 R=5/6 may be replaced by 642 R=4/5 or even better by 624 R=3/4 (see figure 8)

The results of table 3 and figures 5-8 show that the coded configurations that employ only coded bits, n_{ci} = 4 or 6 and n_{ni} =0, and high coding rates R = 5/6, 4/5 and even ¾ may be replaced, with about the same coding gains, by configurations that employ n_{ni} = 2 or even 4 non-coded bits.

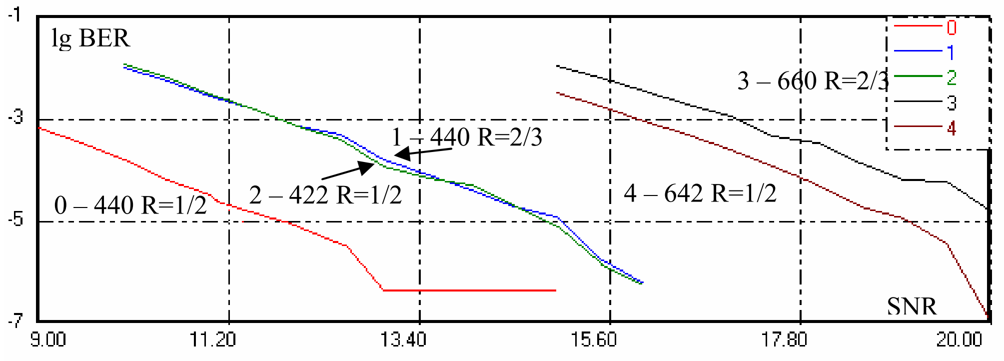

Figure 5. BER vs.SNR – Replacement of the R = 2/3, n_{ci}=0 by the R = 1/2, n_{ci} = 2 for 16 and 64-QAM

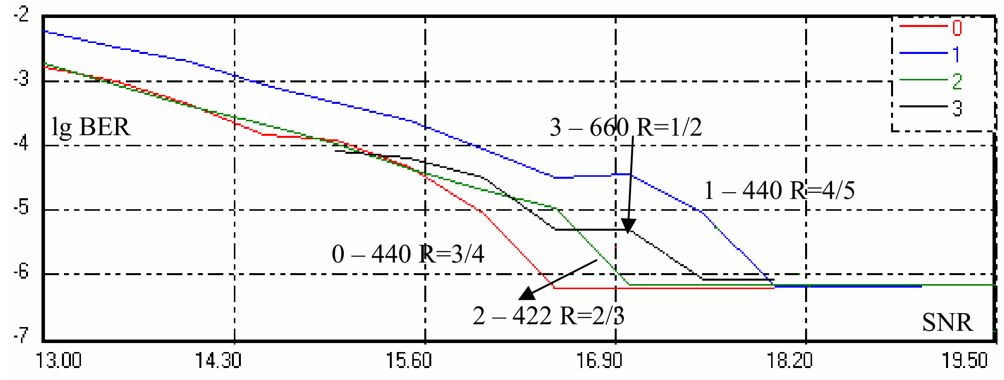

Figure 6. BER vs. SNR – Replacement of the R = 3/4, 4/5, n_{ci}=0 by the R = 2/3 , n_{ci} = 2 for 16-QAM and by R=1/2 n_c =0, 64-QAM

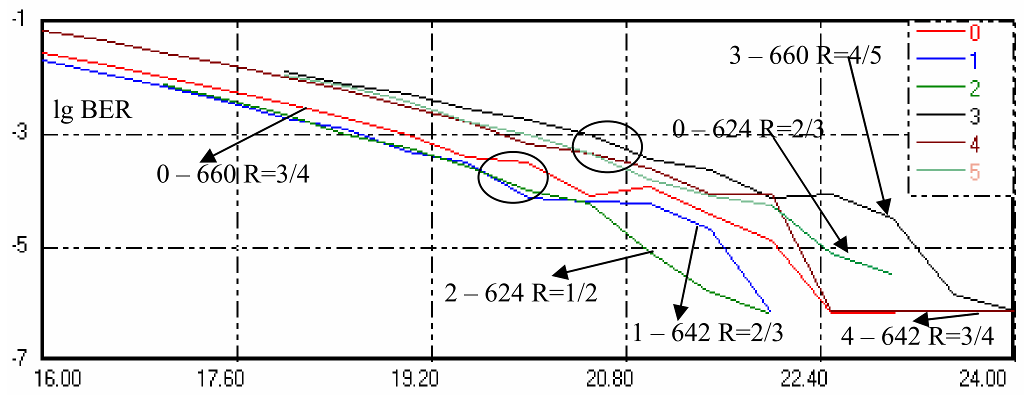

Figure 7. BER vs. SNR – replacement of the R = 4/5 and ¾ for 64-QAM

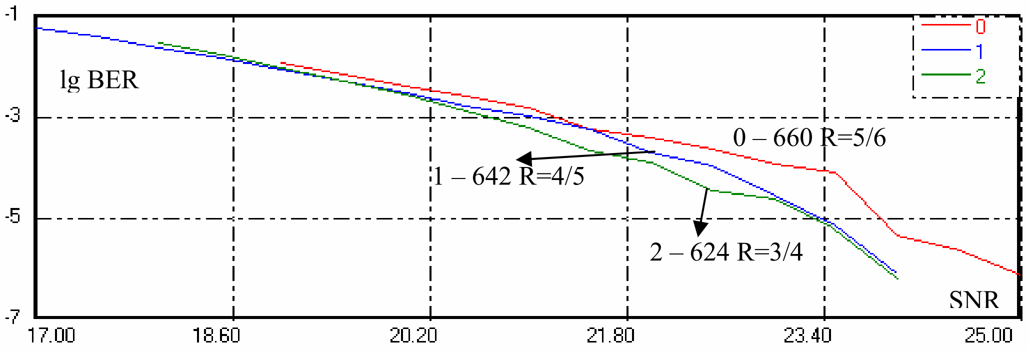

Figure 8. BER vs. SNR – replacement of the R = 5/6 and 4/5 for 64-QAM

This approach would require fewer puncturing patterns and a simpler implementation of the decoding algorithm, at the expense of splitting the information bits into two flows and an MSP-type mapping.

Spectral efficiencies of the studied configurations

The spectral efficiencies are studied in an OFDM scheme with the following parameters of the user-bin [8]:

- Sub carrier spacing: f_s = 39062.5 Hz
- OFDM symbol length (excluding guard time): T_s = 25.60 microsec.
- Guard time/cyclic prefix: G_{ET} = 3.20 microseconds (G = 0.125 - 1/8 OFDM symbol)
- Physical bin size: 312.5 KHz x 345.6 microseconds (BW_b = 312.5 KHz; T_b = 345.6µs)
- Bin size in QAM-symbols: 8 x 12 = 96 = V_s; payload symbols P_s = 81; "service" symbols/bin S_s = 15.

The bin rate B_R, including the guard interval and the nominal bit rate of a transmission using a configuration with n_i bits/symbol are expressed by (9) and (10), respectively:

$$B_R = \frac{f_s}{(1+G)\cdot 12} \quad (9) \qquad D_{ci} = B_R \cdot 81 \cdot n_i \cdot R_{cfgi}\,(\text{bit/s}) \quad (10)$$

and the throughput is obtained by:

$$\Theta_{ci}(SNR) = B_R \cdot 81 \cdot n_i \cdot R_{cfgi}(1-BinER_i(SINR)) \quad (11)$$

In (11) $BinER_i$ denotes the chunk error rate, when configuration i is employed, i.e.:

$$BinER_i(SINR) = 1-(1-p_{gi}(SNR))^{ni} \quad (12)$$

P_{gi} being the bit error rate ensured by the configuration i (QAM const, convolutional code, n_{ci} and n_{ni}) for that value of SNR.
The spectral efficiency is obtained dividing the throughput by the bin bandwidth:

$$\eta_{ci}(SNR) = \frac{\Theta_{ci}(SNR)}{BW_b} = \frac{f_s \cdot 81 \cdot n_i \cdot R_{cfgi}}{12(1+1/8)} \cdot \frac{1}{8 \cdot f_s} \cdot (1 - BinER_{ci}(SNR))\ (\text{bps/Hz}) \quad (13)$$

Replacing now the numerical values in (9), (10), (11) and (12) we get for the spectral efficiency:

$$\eta_{ci}(SNR) = 0.75 \cdot n_i \cdot R_{cfgi} \cdot (1 - BinER_{ci}(SNR)) = \eta_{cinom} \cdot (1 - BinER_{ci}(SNR)) \quad (14)$$

For an 81 QAM symbol bin, the $BinE_i < 0.5 \cdot 10^{-3}$ - $1 \cdot 10^{-2}$ would not affect significantly the spectral efficiency, both for transmissions governed or not by an H-ARQ protocol. This would require a bit-error rate smaller than about $5 \cdot 10^{-4}$ - $1 \cdot 10^{-5}$. The nominal spectral efficiencies of the studied configurations are presented in table 4 together with their configuration rates and SNR value required to ensure a $p_{gi} = 1 \cdot 10^{-5}$. Note that the throughput can be computed by multiplying the η_{inom} by the bin-bandwidth, i.e. 312.5 kHz (13).

n_i, n_{ci}, n_{ni} ↓ R→		½	2/3	¾	4/5	5/6	1-nc
4 4 0	**SNR**	12	15	16	17.5	18	19.5
	$Rcfg_i$	0.450	0.630	0.716	0.769	0.802	1
	η_{inom} bps/Hz	1.35	1.89	2.15	2.31	2.41	3
4 2 2	**SNR**	15	16.5	17	-	-	-
	$Rcfg_i$	0.701	0.797	0.841	0.869	0.885	-
	η_{inom} bps/Hz	2.1	2.39	2.52	2.61	2.65	-
6 6 0	**SNR**	16.3	20.5	22	23.5	24	25.5
	$Rcfg_i$	0.467	0.642	0.727	0.779	0.813	1
	η_{inom} bps/Hz	2.1	2.89	3.27	3.50	3.66	4.5
6 4 2	**SNR**	19	21.6	22.2	23.5	24.6	-
	$Rcfg_i$	0.633	0.753	0.811	0.846	0.868	-
	η_{inom} bps/Hz	2.85	3.39	3.65	3.81	3.91	-
6 2 4	**SNR**	21	22.5	23.5	-	-	-
	$Rcfg_i$	0.800	0.864	0.894	0.910	0.924	-
	η_{inom} bps/Hz	3.6	3.89	4.02	4.11	4.16	-

Table 4 Performances of the studied configurations – SNR for BER=10^{-5}, configuration rate and nominal spectral efficiency

Practical conclusions (see the arrows in table 4)

- the 440 R=2/3 and ¾ may be replaced by 422 R=1/2
- the 440 R=4/5 and 5/6 may be replaced by 422 R=2/3

Figure 9 shows the η_i vs. SNR of the 16-QAM coded configurations. The 422 configurations have higher spectral efficiencies than the 440 with R > ½, in about the same SNR domains, groups 2 and 3.

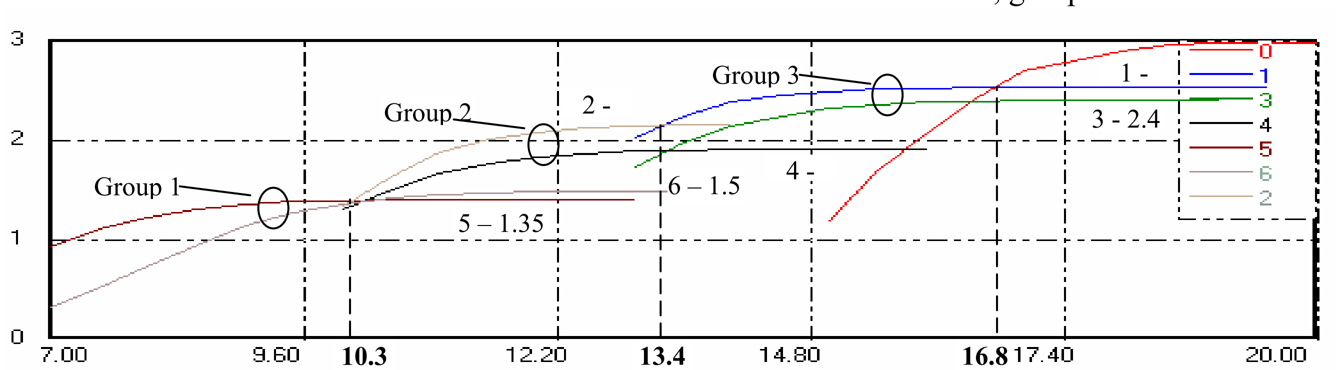

Figure 9 Spectral Efficiencies of the 16-QAM configurations
0- 404; 1 – 422 ¾ 2 – 422 ½ 3- 440 5/6; 4- 440 2/3; 5 – 440 ½; 6 -202

- the 660 R = 2/3 may be replaced by 642 R=1/2
- the 660 R =3/4, 4/5, 5/6 and 642 R=1/2 and ¾ may be replaced by 624 R=1/2
- the 642 R=4/5 and 5/6 may be replaced 624 R=2/3

Figure 10 shows the spectral efficiencies of the 64-QAM configurations. In group5 and 6, according the SNR domain, there is a 624 configuration that outperforms a 642 and a 660 configurations. In group 4, a 642 configuration outperforms a 660 and the 404 configurations.

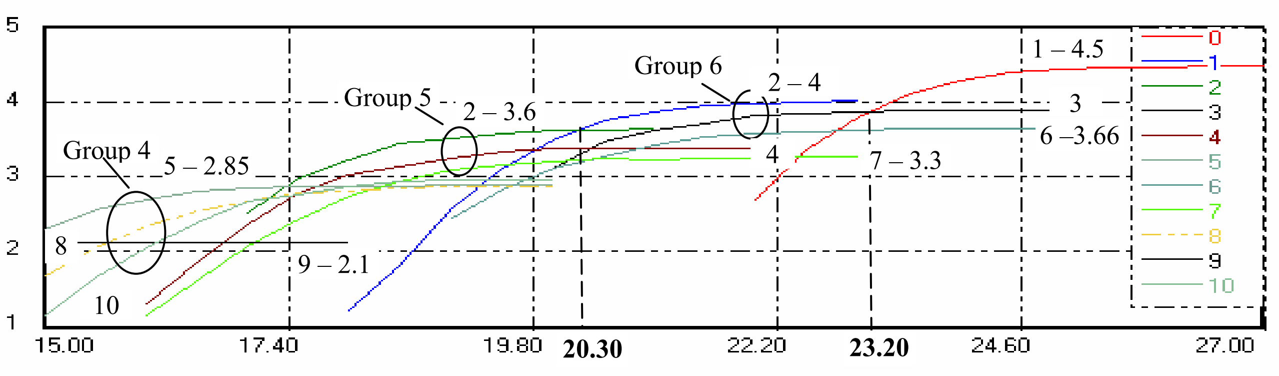

Figure 10 Spectral Efficiencies of the 64-QAM configs

0 – 606; 1- 624 3/4 ; 2 – 624 ½; 3 – 642 5/6; 4 – 642 2/3; 5 – 642 ½; 6 – 660 5/6; 7 – 660 ¾; 8 660 2/3; 9 – 660 ½; 10- 404

Comparing figures 9 and 10, note that the 660 R=1/2 may be replaced by 422 R=2/3

As a thumb rule we may conclude that coded configurations with only R=1/2 and 2/3 together with an appropriate combination between the numbers of coded and non-coded bits may ensure better or equivalent spectral efficiencies in at lower SNR domains, with about 1 dB.

The subsequent spectral efficiency increases are of about 0.3-0.35 bps/Hz, which, considering the BW_b = 312.5 kHz, lead to about some 100 kbps throughput increases, i.e. about 10% of the nominal throughput ensured by the coded configurations which map only coded bits.

Global set of configurations employing the 16 and 64-QAM that also map non-coded bits

A set of configurations, selected from the configurations studied above, that ensure the highest spectral efficiencies is shown in figure 11. The figure also contains the thresholds of the SNR domains where each configuration ensures a maximum spectral efficiency and should be adaptively employed. The 660 R=1/2 is shown for comparison and below 7 dB a coded 4-QAM should be employed.

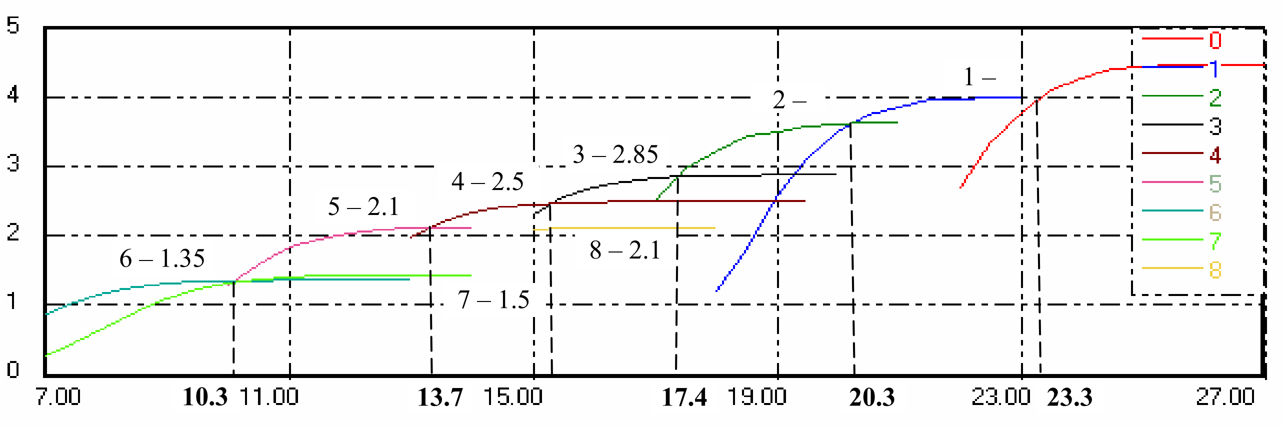

Figure 11 Spectral Efficiencies of a selected set of 16 and 64-QAM coded configurations with non-coded bits

0 – 606; 1- 624 ¾; 2- 624 ½; 3- 642 – 1/2 ; 4 – 422 3/4 ; 5 422 1/2 ; 6 – 440 ½; 7 – 202; 8 - 660 ½

Conclusions

The coded configurations with non-coded bits offer about the same, or sometimes higher coding gains, than the coded configurations mapping only coded bits, at about the same configuration rate, because they allow the employment of a smaller coding rate and ensure an error-rate of the non-coded bits which is close to the one of the coded bits.

They also ensure higher spectral efficiencies than the ones employing only coded bits.

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