Optimal Power Loading for Orthogonal Multiuser Relaying

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Outline

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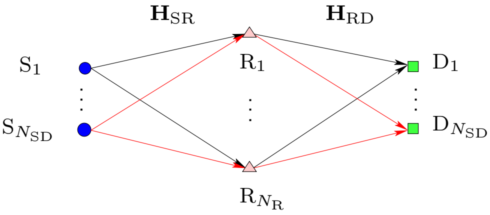
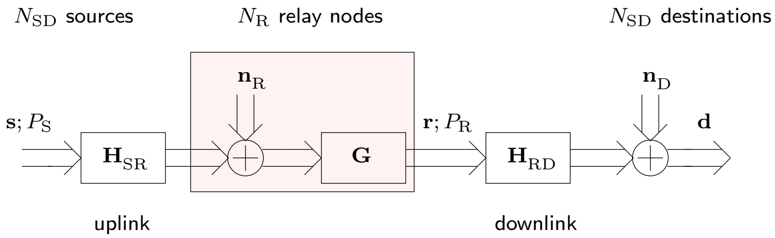


- N_{SD} source/destination pairs
- N_{R} antennas of a linear distributed antenna system (LDAS) acting as perfectly phase synchronous *amplify-and-forward* relays
  $\rightarrow$ joint signal processing of all received data
- S_i shall transmit to D_i such that it causes no interference at the other destinations
- Complete channel state information (CSI) at all relays
- Local compound channel state information at the destinations



- No channel knowledge at the sources: entries of $\mathbf{s}$ are i.i.d. complex normal
- $\mathbf{n}_{\mathrm{R}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2_{n_{\mathrm{R}}} \mathbf{I}_{N_{\mathrm{R}}}\right)$ and $\mathbf{n}_{\mathrm{D}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2_{n_{\mathrm{D}}} \mathbf{I}_{N_{\mathrm{SD}}}\right)$ comprise AWGN contributions at relays and destinations, respectively
- *Sum Power Constraint*: No power loading at sources, sum power constraint at relays:

$$P_{\mathrm{R}} = \mathrm{E}_{\{\mathbf{s},\mathbf{n}_{\mathrm{R}}\}}\left[\mathbf{r}^{\mathrm{H}}\mathbf{r}\right] \overset{!}{=} \mathrm{E}_{\{\mathbf{s}\}}\left[\mathbf{s}^{\mathrm{H}}\mathbf{s}\right] = P_{\mathrm{S}}$$

Scenario description:

Linear Distributed Antenna System (LDAS) acting as multi-antenna relaying terminal:

- Relays are connected via backbone
- Global signal knowledge: *Joint processing* of the signal at all relays
- Arbitrary gain matrix $\mathbf{G}$
- Global channel knowledge
- Global phase reference

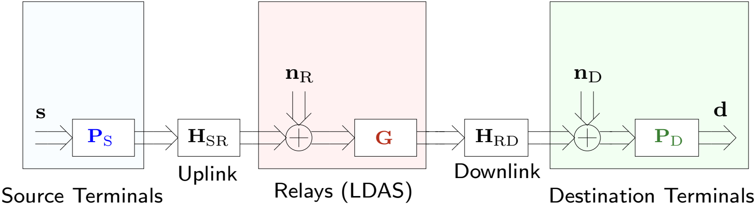
Distinguish two grades of cooperation among sources and destination:

1. *Joint signal processing* at sources and destinations (multi-antenna terminals)
2. *Individual signal processing* at sources and destinations (autonomous single-antenna terminals)

Outline

Joint Signal Processing at Sources and Destinations

System model



- Equivalent channel matrix

$$\mathbf{H}_{\mathrm{SD}} = \mathbf{H}_{\mathrm{RD}}\mathbf{G}\mathbf{H}_{\mathrm{SR}}$$

with singular value decomposition

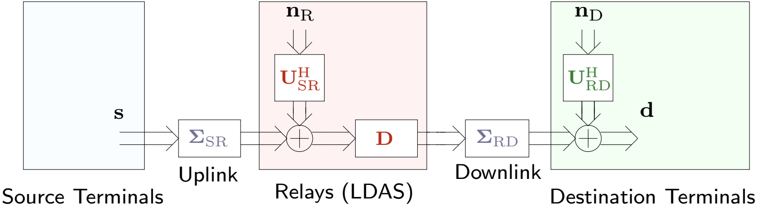
$$\mathbf{H}_{\mathrm{SR}} = \mathbf{U}_{\mathrm{SR}}\mathbf{\Sigma}_{\mathrm{SR}}\mathbf{V}_{\mathrm{SR}}^{\mathrm{H}} \quad \text{and} \quad \mathbf{H}_{\mathrm{RD}} = \mathbf{U}_{\mathrm{RD}}\mathbf{\Sigma}_{\mathrm{RD}}\mathbf{V}_{\mathrm{RD}}^{\mathrm{H}}$$

- Relays orthogonalize the N_{SD} subchannels when choosing

$$\mathbf{P}_{\mathrm{S}} = \mathbf{V}_{\mathrm{SR}}, \quad \text{and} \quad \mathbf{G} = \mathbf{V}_{\mathrm{RD}}\mathbf{D}\mathbf{U}_{\mathrm{SR}}^{\mathrm{H}}, \quad \mathbf{P}_{\mathrm{D}} = \mathbf{U}_{\mathrm{RD}}^{\mathrm{H}}.$$

Joint Signal Processing at Sources and Destinations

Equivalent diagonal system model



- Equivalent channel matrix

$$\mathbf{H}_{\mathrm{SD}} = \mathbf{H}_{\mathrm{RD}}\mathbf{G}\mathbf{H}_{\mathrm{SR}}$$

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- Relays orthogonalize the N_{SD} subchannels when choosing

$$\mathbf{P}_{\mathrm{S}} = \mathbf{V}_{\mathrm{SR}}, \quad \text{and} \quad \mathbf{G} = \mathbf{V}_{\mathrm{RD}}\mathbf{D}\mathbf{U}_{\mathrm{SR}}^{\mathrm{H}}, \quad \mathbf{P}_{\mathrm{D}} = \mathbf{U}_{\mathrm{RD}}^{\mathrm{H}}.$$

- MIMO capacity:

$$I_{\mathrm{LDAS,joint}} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_{N_\mathrm{R}} + \frac{P_\mathrm{S}}{N_\mathrm{SD}}\mathbf{R}_\mathrm{n}^{-1}\mathbf{H}_\mathrm{SD}\mathbf{H}_\mathrm{SD}^\mathrm{H}\right)$$

- Sum power constraint:

$$\mathrm{E}_{\{\mathbf{s},\mathbf{n}_\mathrm{R}\}}\left[\mathbf{r}^\mathrm{H}\mathbf{r}\right] = \mathrm{tr}\left(\sigma_\mathrm{s}^2\mathbf{G}\mathbf{H}_\mathrm{SR}\mathbf{H}_\mathrm{SR}^\mathrm{H}\mathbf{G}^\mathrm{H} + \sigma_{n_\mathrm{R}}^2\mathbf{G}\mathbf{G}^\mathrm{H}\right) \overset{!}{=} P_\mathrm{S}$$

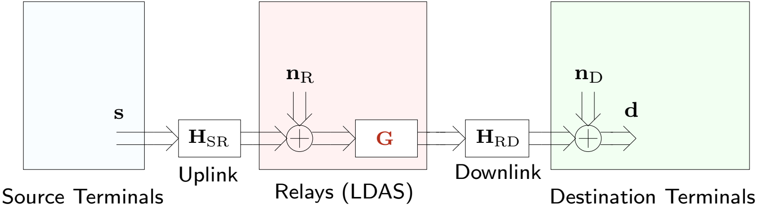
- *Convex Optimization* Problem [05: Muñoz et al.]

$$\begin{aligned}
\text{Minimize} \quad & -\frac{1}{2}\log_2\det\left(\mathbf{I}_{N_\mathrm{R}} + \frac{P_\mathrm{S}}{N_\mathrm{SD}}\mathbf{R}_\mathrm{n}^{-1}\mathbf{D}\mathbf{D}^\mathrm{H}\right)\\
\text{subject to} \quad & -\left|d_k\right|^2 \le 0, \quad k \in \{1,\ldots,N_{\mathcal{SD}}\}\\
\text{and} \quad & \sum_{k=1}^{N_\mathrm{SD}}\left(\frac{P_\mathrm{S}}{N_\mathrm{SD}}\lambda_\mathrm{SR}^{(k)} + \sigma_{n_\mathrm{R}}^2\right)\left|d_k\right|^2 - P_\mathrm{S} = 0.
\end{aligned}$$

$\rightarrow$ Waterfilling solution

Separate Signal Processing at Sources and Destinations

System model



- Equivalent channel matrix

$$\mathbf{H}_{\mathrm{SD}} = \mathbf{H}_{\mathrm{RD}}\mathbf{G}\mathbf{H}_{\mathrm{SR}}$$

- Relays orthogonalize the N_{SD} subchannels when choosing

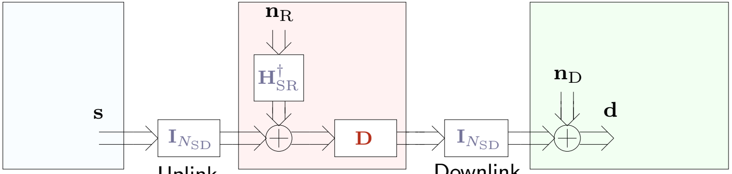
$$\mathbf{G} = \mathbf{H}_{\mathrm{RD}}^{\dagger}\mathbf{D}\mathbf{H}_{\mathrm{SR}}^{\dagger},$$

where

$$\mathbf{H}_{\mathrm{SR}}^{\dagger} = \left(\mathbf{H}_{\mathrm{SR}}^{\mathrm{H}}\mathbf{H}_{\mathrm{SR}}\right)^{-1}\mathbf{H}_{\mathrm{SR}}^{\mathrm{H}} \quad \text{and} \quad \mathbf{H}_{\mathrm{RD}}^{\dagger} = \mathbf{H}_{\mathrm{RD}}^{\mathrm{H}}\left(\mathbf{H}_{\mathrm{RD}}\mathbf{H}_{\mathrm{RD}}^{\mathrm{H}}\right)^{-1}.$$

Separate Signal Processing at Sources and Destinations

Equivalent diagonal system model



- Equivalent channel matrix

$$\mathbf{H}_\mathrm{SD} = \mathbf{H}_\mathrm{RD}\mathbf{G}\mathbf{H}_\mathrm{SR}$$

- Relays orthogonalize the N_SD subchannels when choosing

$$\mathbf{G} = \mathbf{H}_\mathrm{RD}^\dagger \mathbf{D} \mathbf{H}_\mathrm{SR}^\dagger,$$

where

$$\mathbf{H}_\mathrm{SR}^\dagger = \left(\mathbf{H}_\mathrm{SR}^\mathrm{H}\mathbf{H}_\mathrm{SR}\right)^{-1}\mathbf{H}_\mathrm{SR}^\mathrm{H} \quad \text{and} \quad \mathbf{H}_\mathrm{RD}^\dagger = \mathbf{H}_\mathrm{RD}^\mathrm{H}\left(\mathbf{H}_\mathrm{RD}\mathbf{H}_\mathrm{RD}^\mathrm{H}\right)^{-1}.$$

- MIMO capacity:

$$I_{\mathrm{LDAS,sep}} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_{N_\mathrm{R}} + \frac{P_\mathrm{S}}{N_\mathrm{SD}}\left(\mathbf{R}_\mathrm{n} \odot \mathbf{I}_{N_\mathrm{SD}}\right)^{-1}\mathbf{H}_\mathrm{SD}\mathbf{H}_\mathrm{SD}^\mathrm{H}\right)$$

- Sum power constraint:

$$\mathrm{E}_{\{\mathbf{s},\mathbf{n}_\mathrm{R}\}}\left[\mathbf{r}^\mathrm{H}\mathbf{r}\right] = \mathrm{tr}\left(\sigma_\mathrm{s}^2\mathbf{G}\mathbf{H}_\mathrm{SR}\mathbf{H}_\mathrm{SR}^\mathrm{H}\mathbf{G}^\mathrm{H} + \sigma_{n_\mathrm{R}}^2\mathbf{G}\mathbf{G}^\mathrm{H}\right) \stackrel{!}{=} P_\mathrm{S}$$

- No *Convex Optimization* Problem

$$\begin{aligned}
\text{Minimize} \quad & -\frac{1}{2}\log_2 \det\left(\mathbf{I}_{N_\mathrm{R}} + \frac{P_\mathrm{S}}{N_\mathrm{SD}}\left(\mathbf{R}_\mathrm{n} \odot \mathbf{I}_{N_\mathrm{SD}}\right)^{-1}\mathbf{H}_\mathrm{SD}\mathbf{H}_\mathrm{SD}^\mathrm{H}\right) \\
\text{subject to} \quad & -\left|d_k\right|^2 \leq 0, \quad k \in \{1, \ldots, N_{\mathcal{SD}}\} \\
\text{and} \quad & \sigma_\mathrm{s}^2 \sum_{k=1}^{N_{\mathcal{SD}}} b_{kk}\left|d_k\right|^2 + \sigma_\mathrm{n}^2 \sum_{i=1}^{N_{\mathcal{SD}}}\sum_{j=1}^{N_{\mathcal{SD}}} b_{ji}a_{ij}d_j^* d_i - P_{\mathcal{S}} = 0.
\end{aligned}$$

- MIMO capacity:

$$I_{\mathrm{LDAS,sep}} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_{N_{\mathrm{R}}} + \frac{P_{\mathrm{S}}}{N_{\mathrm{SD}}}\left(\mathbf{R}_{\mathrm{n}} \odot \mathbf{I}_{N_{\mathrm{SD}}}\right)^{-1}\mathbf{H}_{\mathrm{SD}}\mathbf{H}_{\mathrm{SD}}^{\mathrm{H}}\right)$$

- Sum power constraint:

$$\mathrm{E}_{\{\mathbf{s},\mathbf{n}_{\mathrm{R}}\}}\left[\mathbf{r}^{\mathrm{H}}\mathbf{r}\right] = \mathrm{tr}\left(\sigma_{\mathrm{s}}^2\mathbf{G}\mathbf{H}_{\mathrm{SR}}\mathbf{H}_{\mathrm{SR}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}} + \sigma_{n_{\mathrm{R}}}^2\mathbf{G}\mathbf{G}^{\mathrm{H}}\right) \stackrel{!}{=} P_{\mathrm{S}}$$

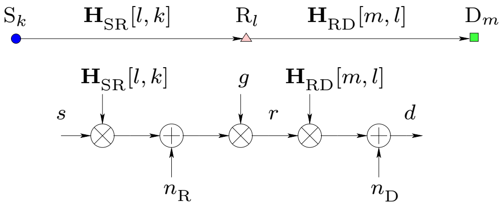
- For high SNR assumption: *Convex Optimization* Problem

$$\begin{aligned}
\text{Minimize} \quad & -\frac{1}{2}\log_2 \det\left(\mathbf{I}_{N_{\mathrm{R}}} + \frac{P_{\mathrm{S}}}{N_{\mathrm{SD}}}\left(\mathbf{R}_{\mathrm{n}} \odot \mathbf{I}_{N_{\mathrm{SD}}}\right)^{-1}\mathbf{H}_{\mathrm{SD}}\mathbf{H}_{\mathrm{SD}}^{\mathrm{H}}\right) \\
\text{subject to} \quad & -\left|d_k\right|^2 \leq 0, \quad k \in \{1,\ldots,N_{\mathcal{SD}}\} \\
\text{and} \quad & \sigma_{\mathrm{s}}^2\sum_{k=1}^{N_{\mathcal{SD}}} b_{kk}\left|d_k\right|^2 + \sigma_{\mathrm{n}}^2\sum_{k=1}^{N_{\mathcal{SD}}} b_{kk}a_{kk}\left|d_k\right|^2 - P_{\mathcal{S}} = 0.
\end{aligned}$$

$\rightarrow$ Waterfilling solution

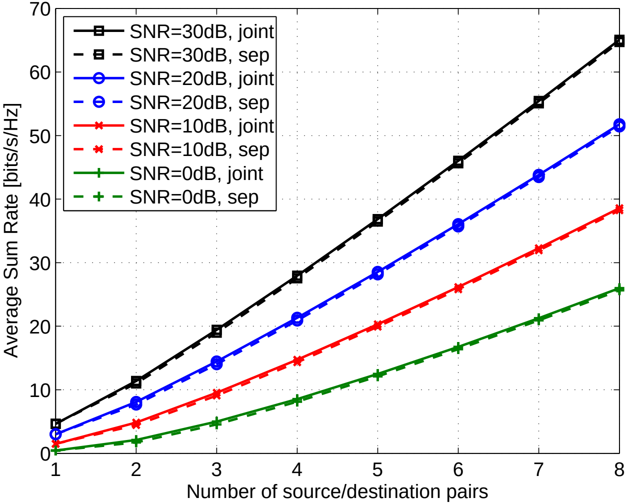
Outline

- **Channels**: Frequency flat block fading model
  1. *Symmetric* channel conditions: i.i.d. Rayleigh fading with unit variance $\sigma_\mathrm{h}^2 = 1$
  2. *Asymmetric* channel conditions: i.i.d. Rayleigh fading with
     - Link 1: variance $\sigma_\mathrm{h}^2 = 1$
     - Other links: variance $\sigma_\mathrm{h}^2 = 0.1$
- **Noise**: AWGN at the relays $\mathbf{n}_\mathrm{R} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_\mathrm{R}})$ and at the destinations $n_\mathrm{D} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_\mathrm{R}})$
- **Transmit power** σ_s^2 for each source S_k is calculated such that it would result in a specific SNR for a $1 \times 1 \times 1$ reference scenario



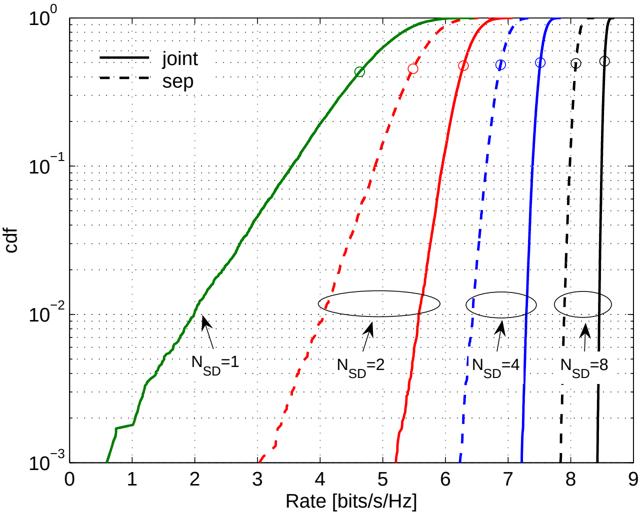
Average sum rate

Joint ('joint') / separate ('sep') signal processing at sources/destinations



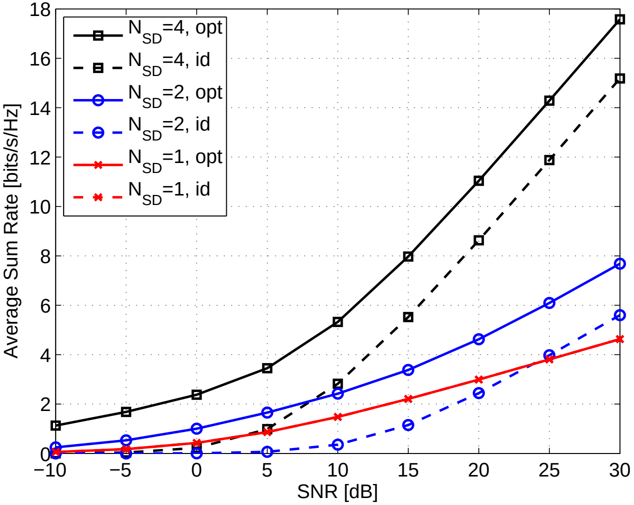
cdf of instantaneous rate for link 1

Joint ('joint') / separate ('sep') signal processing at sources/destinations



Average sum rate

Identical ('id') / optimal ('opt') subchannel weighting



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Conclusions

- Distributed relay network with *linear distributed antenna system* acting as relaying architecture
- Global channel knowledge at the relays
- Distinguish between joint and separate signal processing at sources/destinations
- Waterfilling at the relays:
  - Diagonal system model
  - Derivation of the optimal subchannel weighting factors
- Average sum rate (uncoded) as figure of merit
  - Schemes exhibit virtually the same average sum rate (uncoded)
  - Equal (uncoded) diversity gain
  - 'joint': Average rates for individual links differ
  - 'sep': Average rates for individual links are equal
- Substantial gain in average sum rate over identical subchannel weighting for asymmetric channel conditions