Optimal Power Loading for Orthogonal Multiuser Relaying

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Outline

System Model

Multiuser Stream Orthogonalization
  Joint Signal Processing at Sources and Destinations
  Separate Signal Processing at Sources and Destinations

Simulation Results

Conclusions
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Conclusions
\( N_{SD} \) source/destination pairs

\( N_{R} \) antennas of a linear distributed antenna system (LDAS) acting as perfectly phase synchronous *amplify-and-forward* relays → joint signal processing of all received data

\( S_i \) shall transmit to \( D_i \) such that it causes no interference at the other destinations

Complete channel state information (CSI) at all relays

Local compound channel state information at the destinations
System Model

$N_{SD}$ sources $\rightarrow$ $N_R$ relay nodes $\rightarrow$ $N_{SD}$ destinations

$\mathbf{s}; P_S \rightarrow \mathbf{H}_{SR}$ $\rightarrow$ $+\rightarrow \mathbf{G}$$\rightarrow \mathbf{H}_{RD}$$\rightarrow \mathbf{d}$

- No channel knowledge at the sources: entries of $\mathbf{s}$ are i.i.d. complex normal
- $\mathbf{n}_R \sim \mathcal{C}\mathcal{N}(\mathbf{0}, \sigma_{n_R}^2 \mathbf{I}_{N_R})$ and $\mathbf{n}_D \sim \mathcal{C}\mathcal{N}(\mathbf{0}, \sigma_{n_D}^2 \mathbf{I}_{N_{SD}})$ comprise AWGN contributions at relays and destinations, respectively
- **Sum Power Constraint**: No power loading at sources, sum power constraint at relays:

\[
P_R = \mathbb{E}_{\{\mathbf{s}, \mathbf{n}_R\}} [\mathbf{r}^H \mathbf{r}] = \mathbb{E}_{\{\mathbf{s}\}} [\mathbf{s}^H \mathbf{s}] = P_S
\]
Scenario description:

Linear Distributed Antenna System (LDAS) acting as multi-antenna relaying terminal:

- Relays are connected via backbone
- Global signal knowledge: *Joint processing* of the signal at all relays
- Arbitrary gain matrix $G$
- Global channel knowledge
- Global phase reference

Distinguish two grades of cooperation among sources and destination:

1. *Joint signal processing* at sources and destinations (multi-antenna terminals)
2. *Individual signal processing* at sources and destinations (autonomous single-antenna terminals)
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Joint Signal Processing at Sources and Destinations

System model

- Equivalent channel matrix

\[ \mathbf{H}_{SD} = \mathbf{H}_{RD} \mathbf{G} \mathbf{H}_{SR} \]

with singular value decomposition

\[ \mathbf{H}_{SR} = \mathbf{U}_{SR} \mathbf{\Sigma}_{SR} \mathbf{V}_{SR}^H \quad \text{and} \quad \mathbf{H}_{RD} = \mathbf{U}_{RD} \mathbf{\Sigma}_{RD} \mathbf{V}_{RD}^H \]

- Relays orthogonalize the \( N_{SD} \) subchannels when choosing

\[ \mathbf{P}_S = \mathbf{V}_{SR}, \quad \text{and} \quad \mathbf{G} = \mathbf{V}_{RD} \mathbf{D} \mathbf{U}_{SR}^H, \quad \mathbf{P}_D = \mathbf{U}_{RD}^H. \]
Multiuser Stream Orthogonalization

Joint Signal Processing at Sources and Destinations

Equivalent diagonal system model

Source Terminals

\[ \begin{align*}
\mathbf{s} & \quad \mathbf{U}^H_{\text{SR}} \\
\mathbf{\Sigma}_{\text{SR}} & \quad \mathbf{D} \\
\mathbf{\Sigma}_{\text{RD}} & \quad \mathbf{d}
\end{align*} \]

Uplink
Relays (LDAS)
Downlink
Destination Terminals

▷ Equivalent channel matrix

\[ \mathbf{H}_{SD} = \mathbf{H}_{RD} \mathbf{G} \mathbf{H}_{SR} \]

with singular value decomposition

\[ \begin{align*}
\mathbf{H}_{SR} & = \mathbf{U}_{SR} \mathbf{\Sigma}_{SR} \mathbf{V}^H_{SR} \\
\mathbf{H}_{RD} & = \mathbf{U}_{RD} \mathbf{\Sigma}_{RD} \mathbf{V}^H_{RD}
\end{align*} \]

▷ Relays orthogonalize the \( N_{SD} \) subchannels when choosing

\[ \begin{align*}
\mathbf{P}_S & = \mathbf{V}_{SR}, \\
\mathbf{G} & = \mathbf{V}_{RD} \mathbf{D} \mathbf{U}^H_{SR}, \\
\mathbf{P}_D & = \mathbf{U}^H_{RD}.
\end{align*} \]
- **MIMO capacity:**

\[ I_{\text{LDAS, joint}} = \frac{1}{2} \log_2 \det \left( I_{N_R} + \frac{P_S}{N_{SD}} R_n^{-1} H_{SD} H_{SD}^H \right) \]

- **Sum power constraint:**

\[ \mathbb{E}_{\{s,n_R\}} [r_r^H r_r] = \text{tr} \left( \sigma_s^2 G H_{SR} H_{SR}^H G^H + \sigma_{n_R}^2 G G^H \right) \overset{!}{=} P_S \]

- **Convex Optimization Problem [05: Muñoz et al.]**

Minimize

\[ - \frac{1}{2} \log_2 \det \left( I_{N_R} + \frac{P_S}{N_{SD}} R_n^{-1} D D^H \right) \]

subject to

\[ - |d_k|^2 \leq 0, \quad k \in \{1, \ldots, N_{SD}\} \]

and

\[ \sum_{k=1}^{N_{SD}} \left( \frac{P_S}{N_{SD}} \lambda_{SR}^{(k)} + \sigma_{n_R}^2 \right) |d_k|^2 - P_S = 0. \]

\[ \rightarrow \text{Waterfilling solution} \]
Separate Signal Processing at Sources and Destinations

System model

- Equivalent channel matrix
  \[ H_{SD} = H_{RD} G H_{SR} \]

- Relays orthogonalize the \( N_{SD} \) subchannels when choosing
  \[ G = H_{RD}^\dagger D H_{SR}^\dagger, \]

where

\[ H_{SR}^\dagger = \left( H_{SR}^H H_{SR} \right)^{-1} H_{SR}^H \quad \text{and} \quad H_{RD}^\dagger = H_{RD}^H \left( H_{RD} H_{RD}^H \right)^{-1}. \]
Separate Signal Processing at Sources and Destinations

Equivalent diagonal system model

- **Equivalent channel matrix**

\[ H_{SD} = H_{RD} G H_{SR} \]

- Relays orthogonalize the \( N_{SD} \) subchannels when choosing

\[ G = H_{RD}^{\dagger} D H_{SR}^{\dagger}, \]

where

\[ H_{SR}^{\dagger} = (H_{SR}^{H} H_{SR})^{-1} H_{SR}^{H} \quad \text{and} \quad H_{RD}^{\dagger} = H_{RD}^{H} (H_{RD}^{H} H_{RD})^{-1}. \]
MIMO capacity:

\[ I_{\text{LDAS},\text{sep}} = \frac{1}{2} \log_2 \det \left( I_{\text{NR}} + \frac{P_S}{N_{\text{SD}}} (R_n \odot I_{N_{\text{SD}}})^{-1} H_{\text{SD}}H_{\text{SD}}^H \right) \]

Sum power constraint:

\[ E_{\{s,n_R\}} [r^H r] = \text{tr} \left( \sigma_s^2 G H_{\text{SR}} H_{\text{SR}}^H G^H + \sigma_{n_R}^2 G G^H \right) \overset{!}{=} P_S \]

No Convex Optimization Problem

\[
\begin{align*}
\text{Minimize} & \quad - \frac{1}{2} \log_2 \det \left( I_{\text{NR}} + \frac{P_S}{N_{\text{SD}}} (R_n \odot I_{N_{\text{SD}}})^{-1} H_{\text{SD}}H_{\text{SD}}^H \right) \\
\text{subject to} & \quad - |d_k|^2 \leq 0, \quad k \in \{1, \ldots, N_{\text{SD}}\} \\
\text{and} & \quad \sigma_s^2 \sum_{k=1}^{N_{\text{SD}}} b_{kk} |d_k|^2 + \sigma_{n_R}^2 \sum_{i=1}^{N_{\text{SD}}} \sum_{j=1}^{N_{\text{SD}}} b_{ji} a_{ij} d_j^* d_i - P_S = 0.
\end{align*}
\]
Multiuser Stream Orthogonalization
Separate Signal Processing at Sources and Destinations

- **MIMO capacity:**

\[
I_{\text{LDAS,sep}} = \frac{1}{2} \log_2 \det \left( I_{N_R} + \frac{P_S}{N_{SD}} \left( R_n \odot I_{N_{SD}} \right)^{-1} H_{SD} H_{SD}^H \right)
\]

- **Sum power constraint:**

\[
E_{\{s, n_R\}} [r^H r] = \text{tr} \left( \sigma_s^2 G H_{SR} H_{SR}^H G^H + \sigma_n^2 G G^H \right) = P_S
\]

- **For high SNR assumption:** *Convex Optimization* Problem

\[
\text{Minimize} \quad - \frac{1}{2} \log_2 \det \left( I_{N_R} + \frac{P_S}{N_{SD}} \left( R_n \odot I_{N_{SD}} \right)^{-1} H_{SD} H_{SD}^H \right)
\]

subject to

\[
- |d_k|^2 \leq 0, \quad k \in \{1, \ldots, N_{SD}\}
\]

and

\[
\sigma_s^2 \sum_{k=1}^{N_{SD}} b_{kk} |d_k|^2 + \sigma_n^2 \sum_{k=1}^{N_{SD}} b_{kk} a_{kk} |d_k|^2 - P_S = 0.
\]

→ *Waterfilling solution*
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Channels: Frequency flat block fading model

1. *Symmetric* channel conditions: i.i.d. Rayleigh fading with unit variance $\sigma^2_{h} = 1$
2. *Asymmetric* channel conditions: i.i.d. Rayleigh fading with
   - Link 1: variance $\sigma^2_{h} = 1$
   - Other links: variance $\sigma^2_{h} = 0.1$

Noise: AWGN at the relays $n_R \sim CN(0, \sigma^2_n I_{NR})$ and at the destinations $n_D \sim CN(0, \sigma^2_n I_{NR})$

Transmit power $\sigma^2_s$ for each source $S_k$ is calculated such that it would result in a specific SNR for a $1 \times 1 \times 1$ reference scenario
Average sum rate

Joint ('joint') / separate ('sep') signal processing at sources/destinations
Simulation Results

cdf of instantaneous rate for link 1

Joint ('joint') / separate ('sep') signal processing at sources/destinations

![Graph showing cdf of instantaneous rate for link 1.](image)
Average sum rate

Identical ('id') / optimal ('opt') subchannel weighting

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Simulation Results

Average sum rate in bits/s/Hz for different subchannel weights and SNR values.

- $N_{SD} = 4$, optimal ('opt')
- $N_{SD} = 4$, identical ('id')
- $N_{SD} = 2$, optimal ('opt')
- $N_{SD} = 2$, identical ('id')
- $N_{SD} = 1$, optimal ('opt')
- $N_{SD} = 1$, identical ('id')

SNR [dB] vs. Average Sum Rate [bits/s/Hz]
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- **Distributed relay network** with *linear distributed antenna system* acting as relaying architecture
- Global channel knowledge at the relays
- Distinguish between *joint* and *separate* signal processing at sources/destinations
- **Waterfilling** at the relays:
  - Diagonal system model
  - Derivation of the optimal subchannel weighting factors
- **Average sum rate** (uncoded) as figure of merit
  - Schemes exhibit virtually the same average sum rate (uncoded)
  - Equal (uncoded) diversity gain
  - 'joint': Average rates for individual links differ
  - 'sep': Average rates for individual links are equal
- Substantial gain in average sum rate over identical subchannel weighting for asymmetric channel conditions