# Whitened Matched Filter versus Channel Shortening for ST-BICM over MIMO ISI Channels

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## I. INTRODUCTION

The astonishing capacity gains predicted by the theory [1] [2] have recently generated a quite intensive interest for Multiple-Input Multiple-Output (MIMO) communications and opened new exciting research areas. Prominent among them is the design of good space-time codes. Space-Time Bit-Interleaved Coded Modulation (STBICM), when used in conjunction with efficient Iterative Decoding (ID) are proved to be a simple and robust MIMO signaling scheme for ergodic MIMO flat fading channel, as first conjectured by [3]. Indeed, this scheme has the potential to perform very close to the Shannon theoretical limits for a wide range of MIMO channel models (see, for example, [4] [5] and the reference therein). However, the main drawback of STBICM-ID lies in the complexity involved at the receiver side. As well-known, optimal joint decoding of STBICM is an intractable issue. An efficient way of approaching it is to scatter the problem in two distinct functions, namely MIMO ISI detection and channel decoding, and to apply the turbo-principle [6]. Unfortunately, the complexity of APP MIMO ISI detection is directly related to the number of channel trellis states which grows exponentially with the number of transmit antennas and with the channel memory.

The purpose of this paper is to derive and two different iterative approaches to decode ST-BICM transmitted over MIMO ISI channels and to compare them in terms of trade off between complexity and performance.

#### II. COMMUNICATION MODEL

In this paper, we consider a MIMO *P*-block fading multipath AWGN channel with *T* transmit and *R* receive antennas and memory *M*. The channel is attacked by a STBICM. Symbol-spaced taps  $\mathbf{H}^{p}[0], ..., \mathbf{H}^{p}[M]$  for channel  $\mathbf{H}^{p}$  are  $R \times T$  complex random matrices with zero-mean and mean power satisfying the normalization constraints

$$\mathbb{E}\left[\operatorname{diag}\left\{\sum_{k=0}^{M}\mathbf{H}^{p}\left[k\right]\mathbf{H}^{p}\left[k\right]^{\dagger}\right\}\right] = T\mathbf{I}$$
(1)

in the case of an equal power system.

## III. TURBO-DETECTION WITH A MIMO CHANNEL SHORTENING FRONT-END

The shortening approach proposes to design a receive prefilter which concentrates the energy of the channel in a small number of taps. The equalizer sees a shortened channel and thus needs a smaller number of states. The advantages of this method lies in the fact that it allows MAP post-detection algorithm For each possible delay  $\Delta$ , the shortening prefilter **F** is derived using the projection theorem. As the performance of the BCJR MAP equalizer closely depends on the Signal to Noise Ratio (SNR)  $\gamma_{\Delta}$  at the output of the shortening prefilter, the optimum shortened channel is then computed in order to maximize this output SNR which expression is given by  $\gamma_{\Delta} = \text{tr} \{\mathbf{H}_{s} \mathbf{H}_{s}^{-1}\}$ .

The shortened channel  $\mathbf{H}_s$  can have a maximum of virtual receive antennas  $R_s \leq T.(M_s + 1)$  which can be larger than R. We can present this as a transformation of time diversity to receive antenna diversity [8]. We should observe that increasing  $R_s$  has no impact on the MAP detector state complexity.

Since resulting  $\mathbf{R}_{ee}$  is a diagonal matrix, the noise at the output of the prefilter is spatially white but still temporally colored. However, we know by experience that neglecting temporal whitening has a limited effect on the performances [8].

## IV. TURBO-DETECTION WITH A MIMO WHITENED MATCHED-FILTER FRONT-END

The second receiver is based on the structure of the optimal MIMO receiver presented in [7]. The MIMO ZF-WMF (Zero-Forcing) makes the MIMO channel "minimum-phase". Such a front-end helps to fight back the error propagation induced by Per Survivor Processing.

When  $R \ge T$ , the ZF-WMF exists and is given by  $\mathbf{F}(z) = \mathbf{H}^{\dagger}(z^{-1}) \times \mathbf{W}(z)$  where  $\mathbf{W}(z)$  is a whitening filter for a process with spectrum  $\mathbf{S}(z) = \mathbf{H}^{\dagger}(z^{-1}) \mathbf{H}(z)$ . However, the *Spectral Factorization theorem* states that  $\mathbf{W}(z)$  may not exist if  $\mathbf{S}(z)$  is singular on the unit circle. Therefore, we prefer implementing the Mean-Squared Whitened Matched Filter (MS-WMF) [9]. It always exists independently of the number of transmit and receive antennas.  $\mathbf{W}(z)$  becomes in this case a whitening filter for a process with spectrum  $\mathbf{S}(z) = \mathbf{H}^{\dagger}(z^{-1}) \mathbf{H}(z) + \sigma^{2}\mathbf{I}$ . The channel is, thus, deprived of its minimum phase property. The MS-WMF approaches a matched filter as the SNR goes to zero and the WMF (when it exists) as the SNR goes to infinity.

Computing an approximation of the IIR anticausal filter  $\mathbf{W}(z)$  by a FIR filter is possible using the linear prediction theory [10]. It is easy to demonstrate that spatial whitening is possible if and only if  $R \leq T$ . This is clearly a limitation of the MS-WMF approach compared to the channel shortening one. Note that, the MIMO minimum phase property is not equivalent to having each radio-link " minimum phase". Such property is more suitable to limit the error propagation induced by PSP techniques.

## V. PREFILTER COMPLEXITY

In this section, we estimate the computational complexity of both approaches under the hypothesis of uncorrelated input and noise sequences. Such a comparison has a sense if and only if detectors and decoders used in both receivers have the same complexity.

A possible complexity estimate of the shortening prefilter approach is  $O(R^3(l_p+1)^2)+O(T^3(M_s+1)^2)$  operations for each  $\Delta$  while the MS-WMF approach requires mainly  $O(T^3l_p^2 + T^3)$  operations. As we can see, the shortening approach is far more complex.

## VI. SIMULATION RESULTS AND DISCUSSION

We consider a ST-BICM using 4-PSK modulation (Gray labeling) and rate-1/2 64-state non-recursive convolutional (NRC) code. Code word length is N = 1024 bits yielding L = 256 c.u. per channel block. The spectral efficiency is  $\eta = 2$  bits p.c.u. The channel fading matrix coefficients are equally distributed (EQ). In the turbo-equalizer, the prefilter order is set to  $L_F = 15$ . DFSE (1 survivor per state) is considered with a MS-WMF front-end. With the shortening approach, the channel memory is reduced to  $M_s = 1$  and the output number is  $R_s = 4$ . 4 iterations are performed. The MIMO channel is assumed block-static (P = 1) and comprises T = 2 transmit and R = 2 receive antennas. The Matched Filter Bound is used to evaluate the loss inherent to the suboptimality of both techniques.

As we can see, the shortening approach seems to be unable to deal with high selectivity. For the EQ5 channel, the gap between both approaches at  $BLER = 10^{-2}$  is reduced to 0.6 db. The shortening approach becomes better from  $Eb/N_0 = 7db$ . The same phenomena occurs with the EQ 3 channel where the shortening approach outperforms the MS-WMF approach from  $Eb/N_0=2db$ . The gain (with shortening) at  $BLER=10^{-2}$  is thus 0.6db. We also observe that the shortening is always better at the first iteration. However, the gain between the second and the fourth iteration is more important with MS-WMF approach and increases with the channel selectivity. Moreover, it seems that, at high SNR, the shortening approach slope is better allowing him to recover a higher diversity order.

## VII. CONCLUSION

In this paper, we investigate to decode a ST-BICM over a MIMO ISI Channel using a trellis based post-detector. The MS-WMF does not exist for more receive than transmit antennas whereas the shortening approach copes with any antennas configuration. Simulation results tend to show that the MS-WMF followed by an iterative reduced-state trellis based post detector outperforms the channel shortening approach at fixed complexity and high channel selectivity. However, it seems that the channel shortening approach is an interesting alternative for low frequency selective channel or non iterative schemes.

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Fig. 1. Block-static MIMO system 2×2 EQ-10



Fig. 2. Block-static MIMO system 2×2 EQ-3