PERFORMANCES OF LDPC-CODED OFDM TRANSMISSIONS AND APPLICATIONS ON FIXED AND MOBILE RADIO CHANNELS

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Questions to be addressed:

I. Parameters and encoding-decoding of LDPC Codes

II. Performances of LDPC-Coded Single Carrier Transmissions

III. Mapping-Demapping on QAM-Signal Constellation and Decision of Non-coded Bits

IV. Performances of LDPC-Coded Adaptive Modulation Schemes in Multi-Carrier (OFDM) Transmissions

V. Conclusions

VI. Questions for further study

I. Parameters and encoding-decoding of LDPC Codes

1.Types of LDPC Codes Employed

- LDPC codes are block codes defined by three parameters, k, j, p, [1]:

k, j, p integers; p is a prime integer; $j < k \leq p$; (1)

- j equals the number of control equations a codeword bit is involved in
- k equals the number of bits involved in a control equation.
- Some references [2,3] define the codes by using d_v and d_c:

$$d_v = j; \quad d_c = k;$$

- The LDPC-control matrix H, is a 4-cycle free sparse matrix that might take three forms, defining three types of LDPC codes:

a)randomly generated by computer search [2], [3],
b)complete array-code control matrices [3]
c)triangular-shaped array-code control matrices [4].

Types of LDPC Codes Employed

a) Randomly generated control matrix

- The rate and the code word length can not be easily modified in order to match the application requirements

b) Complete array-code control matrices

- Will not be dealt with in this presentation

Triangular-shaped control matrices

- Denoting by I the unity p×p matrix and by α a p×p matrix obtained from the I matrix by a single column-shift, (for p = 5 a type of α matrix will look like 2)), the general matrix H is expressed in (2).
- This matrix has a zero determinative;

$$H = \begin{bmatrix} I & I & I & \cdots & I \\ I & \alpha & \alpha^{2} & \cdots & \alpha^{k-1} \\ I & \alpha^{2} & \alpha^{4} & \cdots & \alpha^{2(k-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & \alpha^{j-1} & \alpha^{2(j-1)} & \cdots & \alpha^{(j-1)(k-1)} \end{bmatrix} \qquad \alpha_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{b.} \qquad (2)$$

- By different processing the triangular–shaped H_T or the complete control H_C matrices are obtained [4].

Triangular-shaped control matrices

- the (jp x kp) H_T control matrix is expressed by (3):

$$H_T = \begin{bmatrix} I & I & I & . & I & I & . & I \\ 0 & I & \alpha & . & \alpha^{j-2} & \alpha^{j-1} & . & \alpha^{k-2} \\ 0 & 0 & I & . & \alpha^{2(j-3)} & \alpha^{2(j-2)} & . & \alpha^{2(k-3)} \\ : & : & : & : & : & : & : & : \\ 0 & 0 & 0 & . & I & \alpha^{j-1} & . & \alpha^{(j-1)(k-1)} \end{bmatrix} \quad (3)$$

2. Comparison of LDPC Codes generated by array-code control matrices

Triangular-shaped H_T matrices

- word length N, numbers of control bits C, information bits J,ratio R_T:
- $N_T = k \cdot p$; $C_T = j \cdot p$; $J_T = (k-j) \cdot p$; $R_T = (k-j)/k$;   (4)

Complete H_C matrices

the rate increase is negligible, considering the implementation issues.

- performances of H_C codes are not significantly better than the ones of H_T codes.

3. Shortening the LDPC Codes

- for J' < J information bits, the H_s matrix of the shortened code is obtained by deleting the rightmost J-J' columns from the parent-code matrix H_T;
- the shortened code rate is smaller than the parent-code rate

$$R' = J'/(J'+C) < R = J/(J+C) \qquad (5)$$

3. Generation of the Control Matrix H_T

- The regular structure of the triangular-shaped matrix allows for a systematic generation, starting from the code parameters j, k, p.
- Using the property of the matrix α, $\alpha^p = I$ (pxp), and the rule that gives the power of α inside the H_T (3), an algorithm to compute the indexes (row, column) of the positions that take the logical value "1", in terms of j, k, p, can be determined.
- This regular structure simplifies the implementation of the decoding algorithm

4. Encoding the LDPC Codes

- *a. Classical approach*

- For $v = [c_0,\ldots,c_{jp-1}, i_0,\ldots,i_{(k-j)p-1}]$, the control bits c_m require solving the C equations system:

$$Hv^t = 0 \tag{6}$$

- Two major shortcomings:

- for great values of parameters j and/or p, C becomes large implying a significant computational load that increases the processing time and/or the hardware required by the implementation;
- it requires all information bits i_l at the same time inducing a one-codeword additional latency in the system.

Encoding the LDPC Codes

•*b. Proposed encoding method*

The (jp x kp) H_T matrix is split into two matrices D (jp x jp) and E [(kp-jp) x jp]. The D matrix is square and has a non-zero determinative, see (3).

$$[H]\cdot[v]^t = [0] \Leftrightarrow [D]\cdot\begin{bmatrix} c_0 \\ \vdots \\ c_{jp-1} \end{bmatrix} + [E]\cdot\begin{bmatrix} i_0 \\ \vdots \\ i_{(k-j)p-1} \end{bmatrix} = [0] \Leftrightarrow \begin{bmatrix} c_0 \\ \vdots \\ c_{jp-1} \end{bmatrix} = [F]\cdot\begin{bmatrix} i_0 \\ \vdots \\ i_{(k-j)p-1} \end{bmatrix} \qquad (7)$$

- Expanding the encoding matrix F (jp x (k-j)p) and denoting by $[f_l]$ the columns of F, each of them a jp-bit vector:

$$[f_0]\cdot i_0 + \ldots [f_{(k-j)p-1}]\cdot i_{(k-j)p-1} = [C] \qquad (8)$$

Encoding the LDPC Codes

$$[f_0]\cdot i_0 + \ldots [f_{(k-j)p-1}]\cdot i_{(k-j)p-1} = [C] \qquad (9)$$

- The off-line computed encoding matrix F is stored column-by-column in the implementing device.
- Each information bit multiplies the column with the same index, and then the results are accumulated, giving the control bits vector [C].
- The encoding matrix of the shortened code F_s may be obtained by deleting the rightmost J-J' columns of matrix F.
- This algorithm is faster and computes the control bits, [C], in a parallel way by using one information bit for every step;
- This approach allows a serial employment of the information bits and, consequently, does not insert any additional latency in the system.
- It allows an adaptive use of different codes, according to the channel's actual state

5.Decoding the LDPC Codes

- Employs the message-passing algorithm (MP) [2], not described here.
- Based on the Bayes criterion, it requires the previous computation of the a posteriori probabilities, $F_n^0(r/0)$ and $F_n^1(r/1)$, for every bit of a codeword, where r denotes the received vector, n is the bit index and 0/1 denote the bit logical value.
- It extracts an estimated group of N bits, v', that is checked by means of syndrome-computation;
- If the syndrome equals zero, the algorithm considers v' to be the correct codeword;
- Otherwise it performs another iteration adjusting the values of the *a posteriori* probabilities by using some internal values computed in the previous iteration.

Decoding the LDPC Codes

- The maximum number of iterations allowed, B, is a parameter of the algorithm.
- This algorithm does not search for the closest codeword compared to the received sequence, but tries to correct every bit $\Rightarrow$
- The number of error bits after the decoding is always smaller than the one of error bits prior the decoding (confirmed by extensive simulation.
- This property might lead to the decrease of error-packet length that should be corrected by the outer RS (or BCH) code that follows the inner LDPC applications employing concatenated codes.

II. Performances of LDPC-Coded Single Carrier Transmissions

- Performances of the LDPC codes, would be evaluated by BER vs. SNR performances of a 2–PSK modulation coded with the proposed code on an AWGN channel.
- BER performances are determined by computer simulation.
- code rates chosen were approximately R = 0.75, 0.5, 0.33 and 0.2

1. Changing a LDPC code rate

a) by changing the code parameter j, for given values of parameters k and p (see family F1 below);

b) by shortening the same code, namely decreasing the number of information bits (see family F2 below).

2. Performances of LDPC codes on an AWGN channel

F1	j	N	C	R	F2	j	N	C	R
C_{11}	Non-coded			1	C_{21}	Non-coded			1
C_{12}	3	434	93	0.78	C_{22}	3	434	93	0.78
C_{13}	7	434	217	0.50	C_{23}	3	186	93	0.5
C_{14}	9	434	279	0.35	C_{24}	3	143	93	0.35
C_{15}	11	434	341	0.21	C_{25}	3	118	93	0.21

Table 1 LDPC- codes parameters employed in simulations

- The two families of codes are based on the same values of k = 14, p = 31.
- They fulfil two requirements:
  - to have a good "correction capability"
  - to have short code words, for multi-user transmissions.
- Family F1 was obtained by changing parameter j (method a.)
- Family F2 was obtained by shortening a "parent code", j = 3 (method b.).

Performances of LDPC codes on an AWGN channel

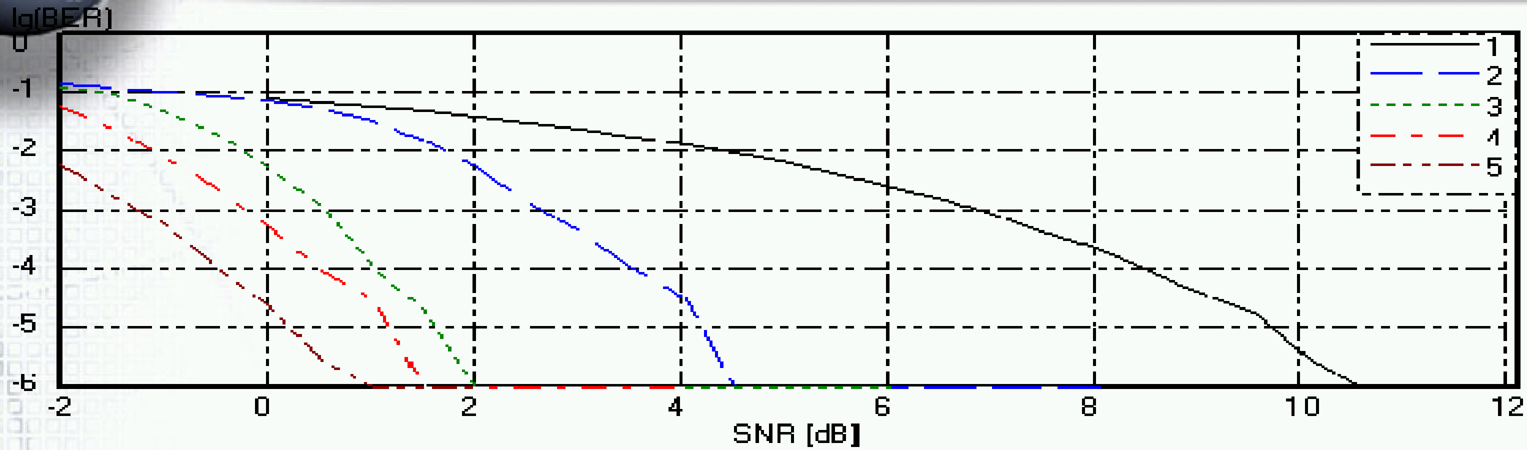

Fig. 1. BER vs. SNR of 2-PSK coded with the LDPC codes of family F1

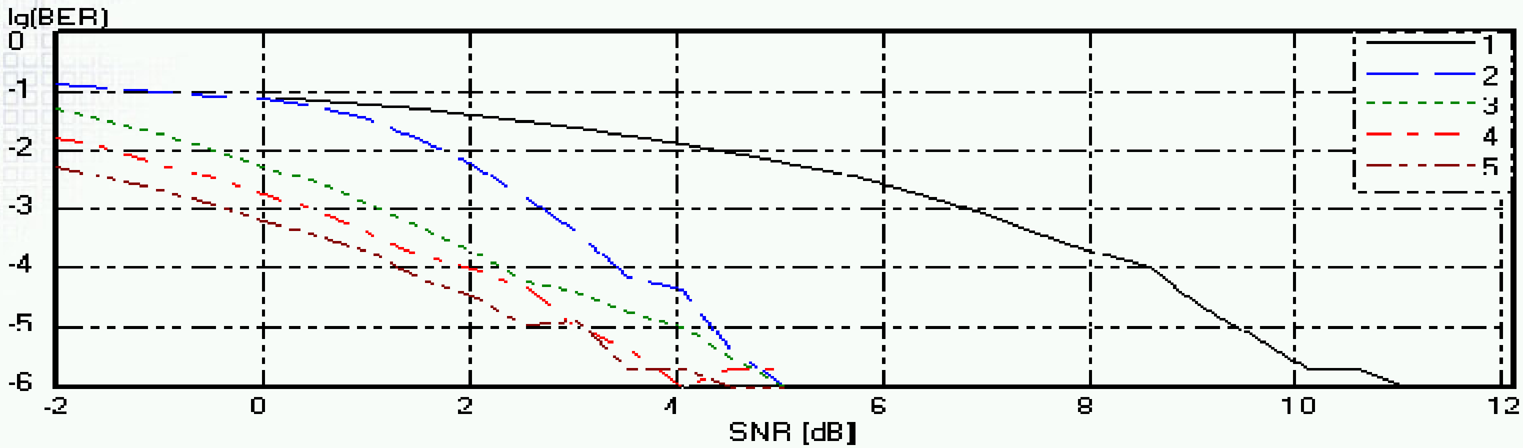

Fig. 2. BER vs. SNR of 2-PSK coded with the LDPC codes of family F2

Performances of LDPC codes on an AWGN channel

- Performances of family F2 codes are poorer at the same rate, due to the fact that the number of control equations in which a bit is involved is always j = 3, compared to j = 7, 9 or 11 for the codes of family F1 (see codes C13 – C23).
- Due to the fact that j is constant, regardless the rate, the coding gain is practically the same.

Code	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	Code	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}
C_G -dB	-	6	8.5	9	10	C_G -dB	-	6.5	6.5	6.5	6.5
R	1	0.78	0.5	0.35	0.21	R	1	0.78	0.5	0.35	0.21

Table 2 Coding gains of LDPC code families of figs 1 and 2 (at BER = 10^{-6})

Performances of LDPC codes on an AWGN channel

- The coding gain provided by the “parent code (R = 0.78) is about 6.5 dB, comparable to ones of convolutional codes of R = ½ and K = 5 – 7,
- The R = ½ LDPC code provides a coding gain of 8.5 dB, larger than the one the convolutional codes.
- The MP decoder has about the same implementation complexity as the 64-state Viterbi decoder ⇒ the LDPC codes provide about the same coding gain as the convolutional code, at a higher rate.
- Compared to the turbo codes, the R = ½ LDPC code ensures BER =10^{-6} at a SNR = 2 dB, about 1 dB higher than the turbo-codes [5], but it requires only one MAP decoder instead of two decoders (MAP + MLD) and a de-interleaver.
- Implementation complexity of a LDPC-MAP decoder requires about 50% less processing than the one of turbo-codes decoder [Flarion].
- Decreasing the code rate by changing the code parameter j ⇒ additional coding gains of up to 3.5dB, code C15 requiring a SNR= 1dB for BER=10^{-6}.

Comparison to Shannon limit

- Maximum spectral efficiency, β_{wMi}, that could be provided by a code of rate R_{ci} in an error-free transmission across an f_s bandwidth is:

$$\beta_{wMi} = \frac{1}{2}\log_2(1 + R_{ci} \cdot \frac{S}{N})\,[\text{bps}/\text{Hz}]; \qquad (10)$$

- Error-free transmission is accomplished for $SNR_s > SNR_{0i}$ $BER(SNR_{0i}) = 1 \cdot 10^{-5}$.
- Maximum spectral efficiency of C_i, β_{wM0i}, is computed using (12) and SNR_{0i}.
- Minimum SNR for which β_{wi} could be obtained, SNR_{mi}, is computed by replacing β_{wi} in (12).
- The $\Delta SNR_i = |SNR_{mi} - SNR_{0i}|$ is also listed in table 3.

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Comparison to Shannon limit

Code	R_{ci}	β_{wi} [bps/Hz]	β_{wMi} [bps/Hz]	SNR_{0i} [dB]	SNR_{mi} [dB]	ΔSNR_i [dB]
C_{12}	0.785	0.785	1.59	4.1	-0.35	4.45
C_{13}	0.5	0.5	0.80	1.7	-0.82	2.52
C_{14}	0.357	0.357	0.546	1.1	-1.04	2.14
C_{15}	0.214	0.214	0.29	0.2	-1.26	1.46

Table 3. Ideal and actual performances of codes of family 1

- codes of a certain length come closer to the theoretical limits as their rate decreases.
- rather short codes, easy to implement, are close enough to the theoretical maximum performances.

3. Increasing the performances of the LDPC codes

a. For performances closer to the theoretical limits the codeword length should be increased, i.e. the parameter p should take larger values (23 – 73), but parameters k and j should be kept constant, $\Rightarrow$ code rate would not be changed.

Using a code with greater value of p; $\Rightarrow$ longer code words $\Rightarrow$ complicates implementation and not be suitable for multi-user transmissions. Simulations showed that for p = 73 (k = 14, j = 3; N_c = 1022 bits) the coding gain is about 11 dB.

b. increasing the maximum number of iterations/codeword of the MP decoder; simulations showed that increasing B = 25, $\Rightarrow$ extra coding gain of 0.5-1 dB, at the expense of a longer processing time required.

Increasing the performances of the LDPC codes

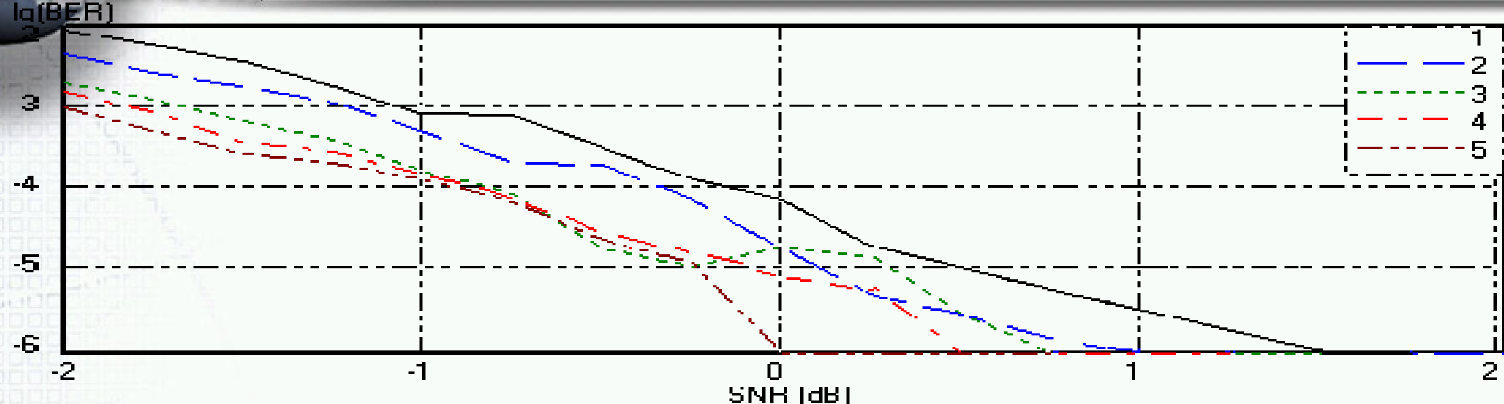

Figure 3 BER performances of 2-PSK LDPC-coded with codes of list 1

C141– k=14; j=11; p=23; N=322 b;

C142– k=14; j=11; p=31; N=434 b;

C143– k=14; j=11; p=43; N=602 b;

C144– k=14; j=11; p=53; N=742 b;

C145– k=14; j=11; p=73; N=1022b

List 1 Codes of rate R_{c14} = 0.214 and different codeword length N

Code	R_{ci}	B_{wi} [bps/Hz]	β_{wMi} [bps/Hz]	SNR_{0i} [dB]	SNR_{mi} [dB]	ΔSNR_i [dB]
C_{141}	0.214	0.214	0.352	0.5	-1.26	1.76
C_{142}	0.214	0.214	0.291	0.2	-1.26	1.46
C_{143}	0.214	0.214	0.280	0.0	-1.26	1.26
C_{144}	0.214	0.214	0.274	-0.1	-1.26	1.16
C_{145}	0.214	0.214	0.268	-0.2	-1.26	1.06

Table 4. Ideal and actual performances of codes from list 1

III. Mapping-Demapping on the QAM-Signal Constellation and Decision of the Non-coded Bits

1. Bit Mapping

Performed in two ways: one-dimensional and bi-dimensional

One-dimensional mapping

- The multibit assigned to an axis of the QAM constellation, coded and non-coded bits, is mapped according to a 2-level Gray encoding.
- The b-tuple is mapped by splitting it into two sets of b/2 bits, each set being assigned to one of the I and Q coordinates of the QAM vector.

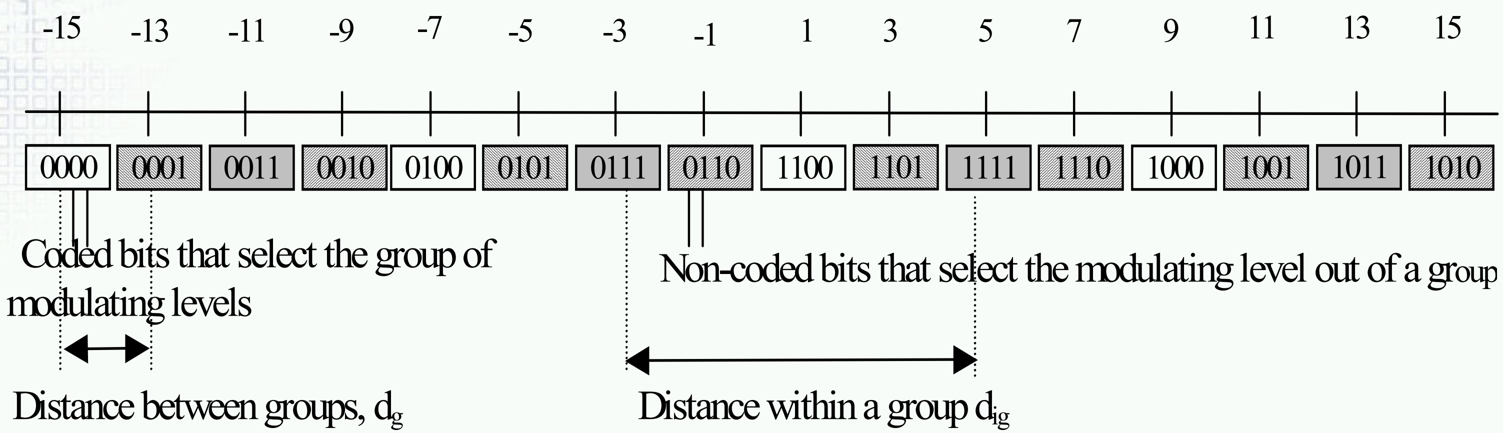

Figure 4 Bit mapping on a constellation axis for b = 8

Bit Mapping

- The amplitude levels employed on each axis belong to the set A:

$$A = \left\{ L(l) = 2l - \left(L_b - 1\right), l = 0, 1, \cdots L_b - 1 \right\}; \quad L_b = 2^{\frac{b}{2}} \qquad (11)$$

- This bit-mapping generates only square QAM constellations; it is simpler because the mapping is identical on both coordinates.

Bi-dimensional mapping

- The coded bits are mapped according to a set partitioning method (similar to TCM)
- The non-coded bits (that define the vectors inside a subset) are mapped according a Gray mapping, as much as it is possible.
- This method can also be applied to “cross” constellations that carry odd numbers of bits/symbol

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Bit Mapping

- A "cross" constellation is obtained by rotating with 45º and scaling a "square" constellation with $N' = 2^{2p} \cdot 3^2$ and removing the 2^{2p} points placed in the four corners;
- Thus we get:

$$N = 2^{2p} \cdot 3^2 - 2^{2p} = 2^{2p+3}; \qquad (12)$$

- For p =1 and for p =2 the 32-QAM and the 128-QAM are obtained;
- The amplitude levels on each axis of the "parent" constellation are obtained in a similar manner to (11) .
- This method does not generate an 8-point constellation.
- The 8-QAM constellation is generated by table reading.

Soft-Demapping

2. Soft-Demapping

- For multibit/symbol modulations, the *a posteriori* probabilities $F_n^0(0/r)$ and $F_n^1(1/r)$ of each bit are extracted from the received level on the I or Q branches (for unidimensional mapping by (13)), [1]:

$$F_j^1 = \frac{\sum_{l=1}^{2^{b/2}} \exp\left(-\frac{(r-L(l))^2}{2\sigma^2}\right) \cdot b_{lj}}{\sum_{l=1}^{2^{b/2}} \exp\left(-\frac{(r-L(l))^2}{2\sigma^2}\right)}; \; j = 0,\ldots,b/2-1; \quad (13)$$

- b_{lj} - logical value of j-th bit of the l-th modulating level of the I or Q of the demapped vector. A similar expression is derived for F_j^0 and the two values are normalized to their sum.
- For bi-dimensional mapping an 2-dimension form of (13) is used.
- Soft-demapping requires a previous estimation of the noise variance σ with estimation errors smaller than 2 dB, so that the performances of the MP-decoder would not be decreased

Decision of the Non-Coded Information Bits

3. Decision of the Non-Coded Information Bits

- The information non-coded bits mapped on a QAM vector can be decided by two methods:

  a) Hard-decision - by applying the Bayes criterion to the probabilities provided by the soft-demapping or by selecting the level that is closest to the received level; this method does not employ the information provided by de-coding the coded bits placed on the same tone during the same symbol period.

  b) Soft decision - it considers the information provided by the decoding of the coded bits.

Decision of the Non-Coded Information Bits

- For one-dimensional mapping, the b-approach employs the decoded bits of the current symbol, from MP decoder, in order to select the group of levels (PL), and the received level r, in order to select the closest level from the group, as shown in figure 5 for a “10” decoded combination.

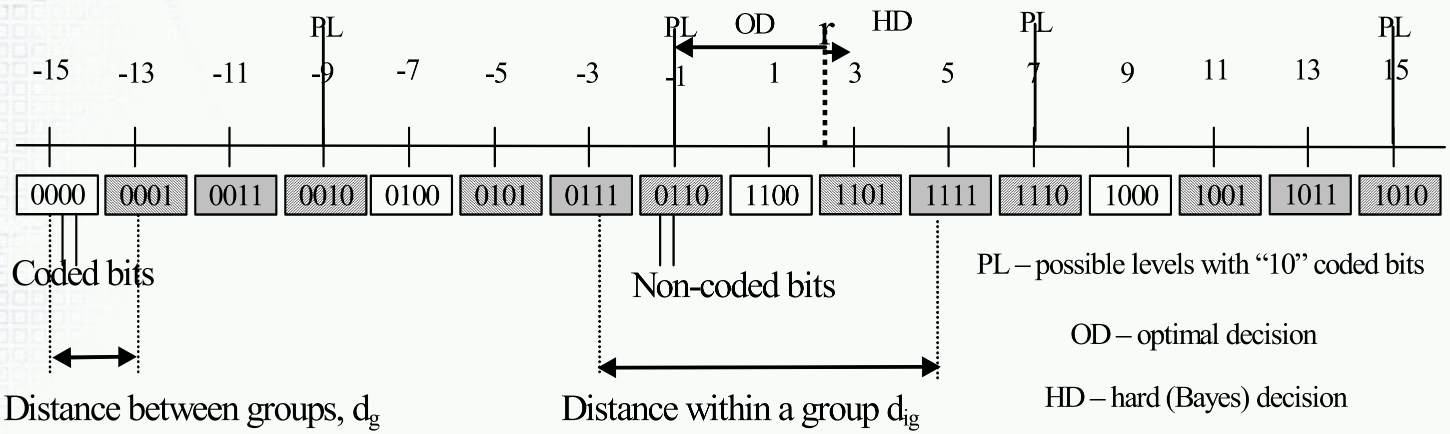

Figure 5. Optimal decision for “10” MP-decoded bits

- soft decision provides lower BER of the non-coded bits, as confirmed by performed simulations

Bit-loading on a Multi-carrier Transmission

- **4. Bit-loading on a Multi-carrier Transmission**
- Denoting by T_i the number of tones in group i, by G the number of tone-groups, by N_{ci} and N_{ni} the numbers of coded and non-coded bits on the i-th tone-group, the payload of the transmission would be:

$$D = \sum_{i=1}^{G} T_i (N_{ci} R_{ci} + N_{ni}); \qquad (14)$$

- (14) may be applied for DMT as well; for OFDM all tones carry the same number of bit/symbol;
- the third possibility of adapting the coded modulation rate: employment of different numbers of coded and non-coded bits on the same QAM-symbol.
- The rate of the coded modulation is computed by:

$$R_{CM} = \frac{N_{ci} \cdot R_{ci} + N_{ni}}{N_{ci} + N_{ni}} \qquad (15)$$

BER Performances of LDPC-Coded QAM Configurations Employing Non-Coded Bits

- 256-QAM (N_{ci} + N_{ni} = 8 bits) coded configurations using non-coded bits are presented in table 5 and their BER vs. SNR performances in figure 6.
- LDPC code employed is (k = 14, j = 3, p = 31; R_c = 0.78), C_G = 7 dB on a 2-PSK modulation (code C_{12} figure 1).

Table 5.

No	N_{ci}	N_{ni}	R_{CM}	C_G	No	N_{ci}	N_{ni}	R_{CM}	C_G	No	N_{ci}	N_{ni}	R_{CM}	C_G
1	0	8	1	-	3	4	4	0.890	6	5	8	0	0.780	7.5
2	2	6	0.945	5	4	6	2	0.835	6.5					

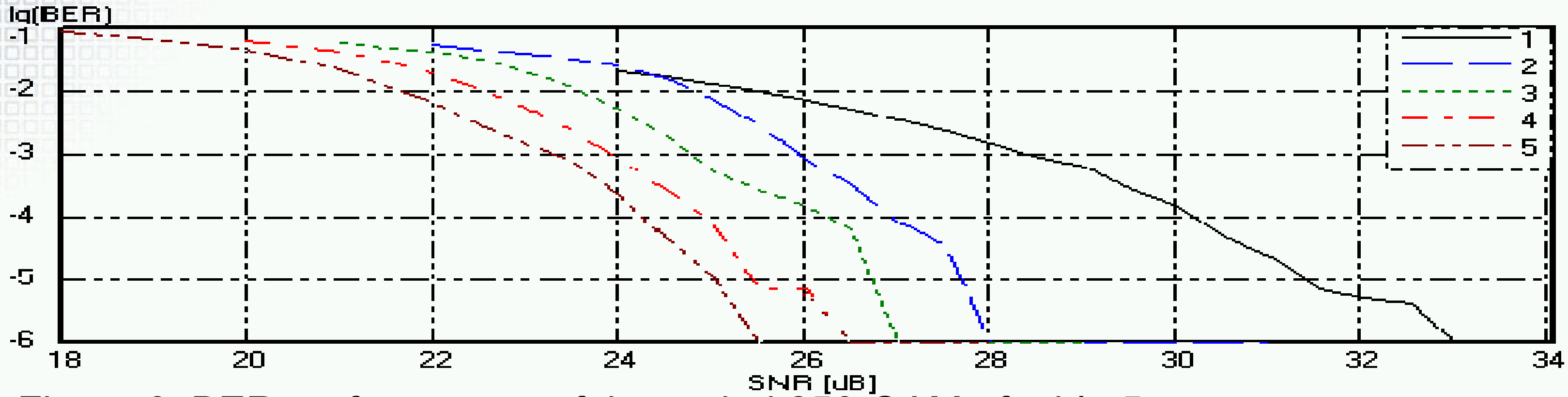

Figure 6. BER performances of the coded 256-QAM of table 5

Throughput Performances of LDPC-Coded QAM Configurations Employing Non-Coded Bits

- Employment of non-coded bits leads to a significantly increase of the coding rate, e.g. from 0.78 to 0.945, and to a increased throughput, at the expense of a coding gain decrease of about 1-2.5 dB.

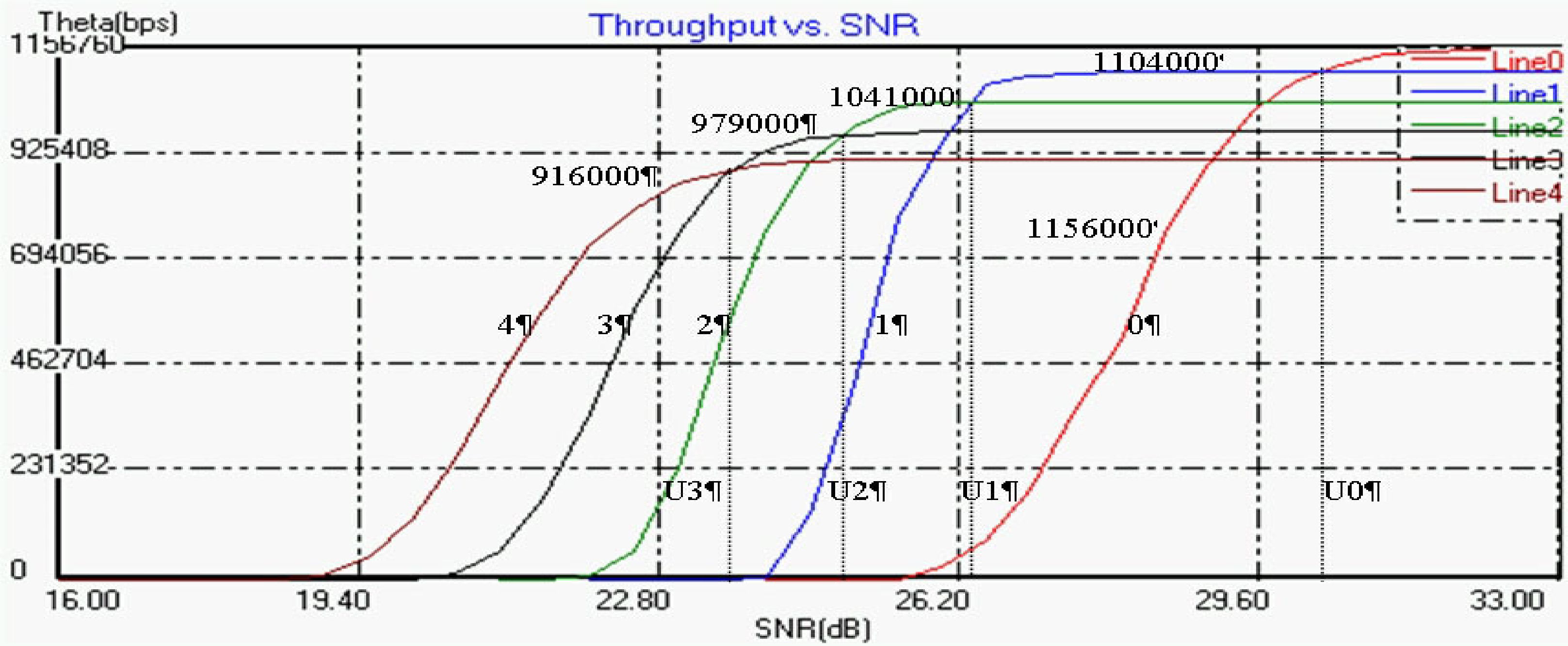

Fig. 7. Throughput vs. SNR of coded configurations from table 5

BER Performances of LDPC-Coded QAM Configurations Employing Non-Coded Bits

- Performances of the configurations of table 5, compared to the Shannon limit (12), are listed in table 6.

No.	R_{ci}	β_{wi} [bps/Hz]	β_{wMi} [bps/Hz]	SNR_{0i} [dB]	SNR_{mi} [dB]	ΔSNR_i [dB]
1	1	8	10.46	31.5	24	7.5
2	0.945	7.52	9.05	27.5	22.9	4.6
3	0.890	7.12	8.64	26.5	21.9	4.6
4	0.835	6.68	8.21	25.5	20.85	4.65
5	0.780	6.24	8.31	25.0	19.8	5.2

Table 6. Ideal and actual performances of coded configurations from table 5

- The rather small decrease in performances could be explained by the “protection” of the non-coded provided by the 2-level (Gray) mapping and by their soft decoding

BER Performances of LDPC-Coded QAM Configurations Employing Non-Coded Bits

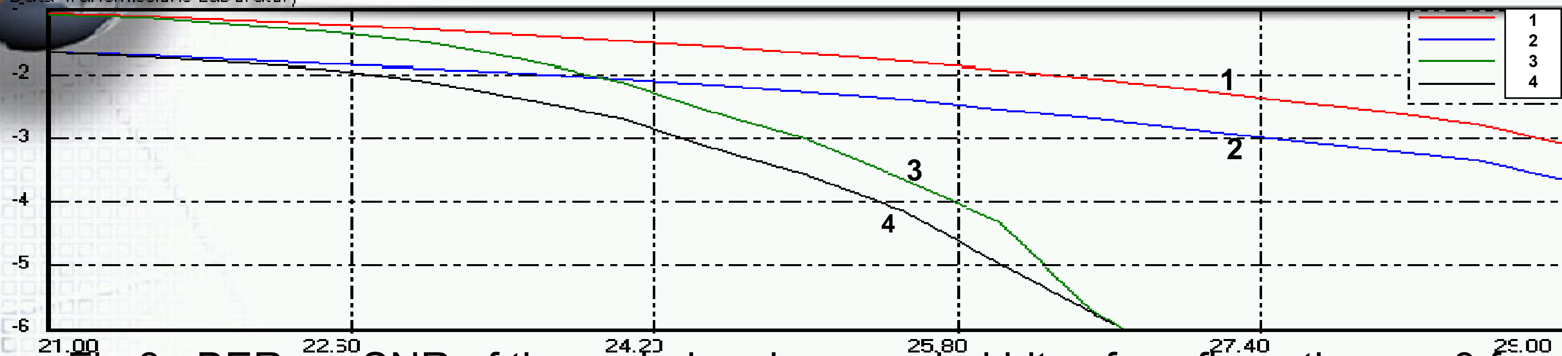

Fig.8 BER vs. SNR of the coded and non-coded bits of configuration no. 3 from table 3 (line1 – coded bits Bayes decision; line 2 – non-coded bits Bayes decision; line 3 – coded bits MP decoding; line 4 – non-coded bits soft decoding)

- Non-coded bits have lower BER than the coded bits, both before the decoding and after it, due to 2-level Gray mapping of the coded and non-coded bits.
- The number of error bits is always smaller after the decoding process, than before it.
- This, combined with additional investigations, indicate that the two decoders might require some smaller outer codes (small RS or even BCH – [Hughes]) in FEC schemes employing concatenated codes.

IV. Performances of LDPC-Coded Adaptive Modulation Schemes in Multi-Carrier Transmissions

1. **Simulation Environment and Parameters [7]**

The simulation program that implements the LDPC-coded MC transmissions allows the following parameters to be set:

- LDPC code parameters (k, j, p),
- number of tone-groups G, number of tones within a group Ti, bit-loading for each group (N_{ci}, N_{ni}), allowing the definition of an user bin
- the bin rate
- maximum number B of iterations/codeword of the LDPC decoder, range and step of SNR, test length.

Simulation Environment and Parameters

It displays:

- The BER values and the BER vs. SNR characteristic for the selected SNR range, the Frame error rate and its characteristic vs. SNR, and the Throughput vs. SNR characteristic.
- The number of coded bits error after the decoding of each codeword, and the number of non-coded bits decided by soft-demapping.
- The simulations were performed on a test of 10^6 information bits and the maximum number of B = 15 iterations/codeword for MP algorithm.

2. Performances of Adaptive LDPC-Coded OFDM Transmissions

a. Non-Coded Adaptive Transmission Scheme for Mobile Channels[6]

- Employs adaptively 256, 128, 64, 32, 16, 8, 4-QAM and 2-PSK

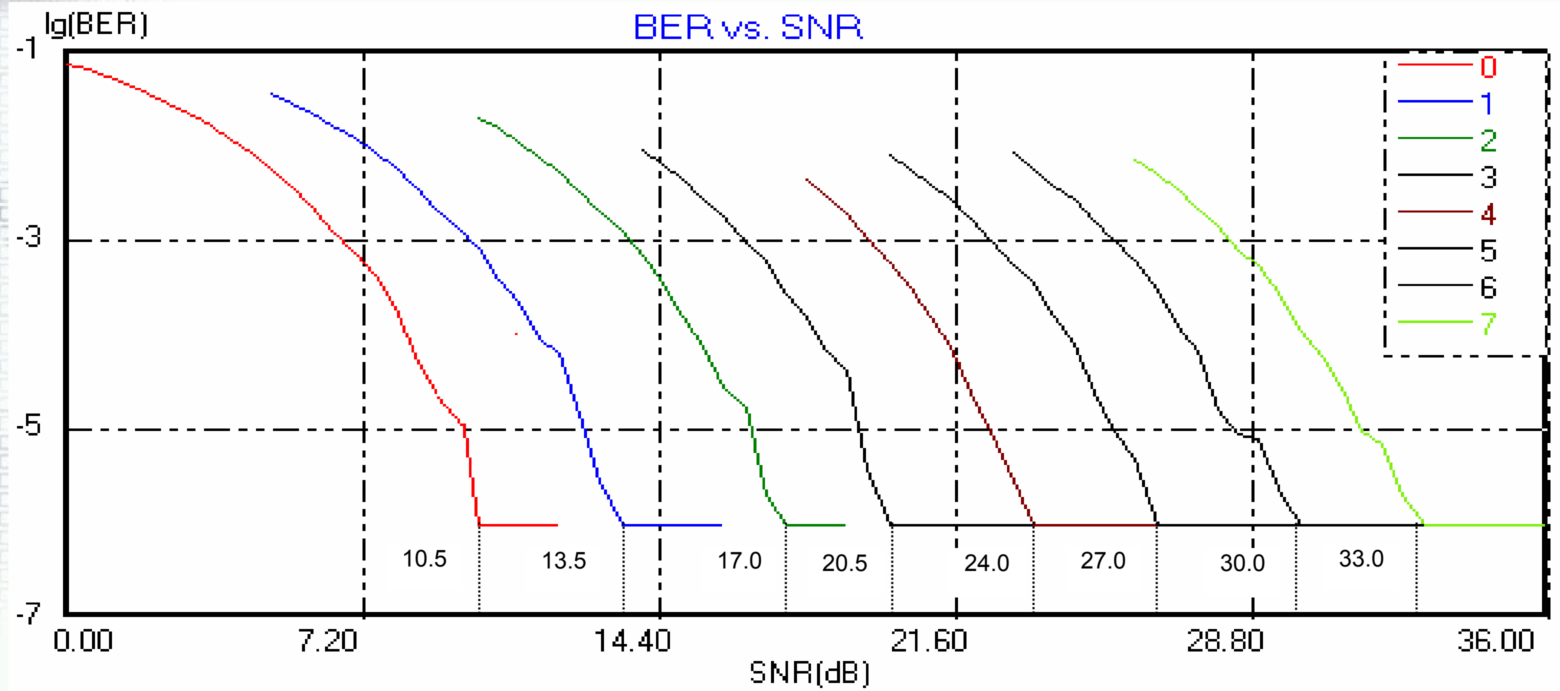

Figure 9. BER vs. SNR of 256, 128, 64, 32, 16, 8, 4-QAM and 2-PSK

The SNR was modified with a step of 0.5 dB, so an error of +/- 0.5 dB might occur.

Performances of Adaptive LDPC-Coded OFDM Transmissions

- **Adaptive Scheme Definition**
- An OFDM transmission based on a bin (allocated to one user) of T_i tones and F OFDM-symbol periods, $\Rightarrow T_s = T_i \times F$ QAM symbols, out of which only A_s are "active" symbols being used for the payload.
- The guard interval (cyclic prefix) is denoted by G and represents a fraction of the symbol period,
- The number of bits/QAM symbol is n_i (defines the QAM constellation employed),
- The bin rate is D_b and the CRC (required for channel estimation-prediction) is t bits long.
- The nominal payload for constellation i (n_i) is:

$$D_{ni} = \frac{1}{1+G} \cdot \frac{A_s}{T_s} \cdot D_b \cdot T_s \cdot n_i \cdot \left(1 - \frac{t}{A_s \cdot n_i}\right) \quad (16)$$

Performances of Adaptive LDPC-Coded OFDM Transmissions

- For D_b = 1500 bins/sec, T_s = 120 symbols, A_s = 108 symbols, G = 0.11, t = 8 bits and n_i =1, 2, 4, 5, 6, 7, 8, [6].

n_i	1	2	3	4
D_{ni}(kbps)	135.135	281.086	427.027	572.927
n_i	5	6	7	8
D_{ni}(kbps)	718.909	864.864	1010.810	1156.756

Table 7 Nominal payloads for non-coded 256, 128, 64, 32, 16, 4-QAM and 2-PSK

- The throughput is computed considering only correctly received bins (17); $BinER_{ni}$ - bin error probability of the n-c configuration - n_i bits/symbol.

$$\theta_{ni} = D_{ni}(1 - BinER_{ni}) \qquad (17)$$

Performances of Adaptive LDPC-Coded OFDM Transmissions

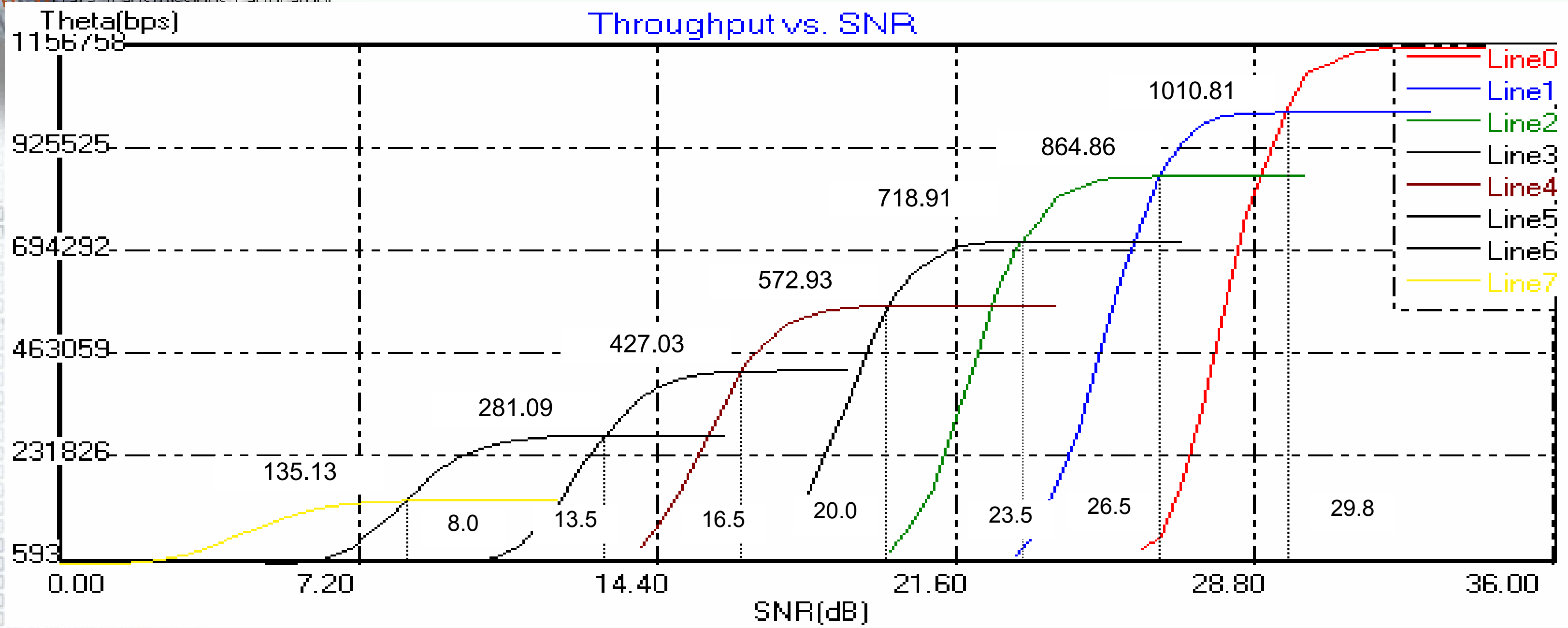

Fig.10 Θni vs. SNR for non-coded 256 (0), 128(1), 64 (2), 32(3),16 (4), 4 (5) QAM and 2-PSK (6)

- The threshold values are determined by graphical calculations, due to the fact that the SNR variation step was set to 0.5 dB.
- They are closer than 1 dB to those computed theoretically in [6]
- The separation between thresholds is about 3-3.5 dB
- The average throughput is 525.92 kbps for SNR > 0 dB, 578.07 kbps for SNR > 3dB or 655.09 kbps for SNR > 7dB.

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b. Coded Transmission Scheme for Mobile Channels

- Since the OFDM transmissions are reported to have poor performances without coding, the BER_{ci} vs. SNR and Θ_{ci} vs. SNR performances of the coded configurations should be evaluated as well.
- The coded configurations employ one of 6 codes, see table 8, with coding rates ranging from 0.78 to 0.5. Flarion and DoCoMo employ codes with significantly smaller rates.
- To keep the coding rate high, coded and non-coded bits were mapped on a QAM symbol.
- For flexibility and a simpler implementation, each bin contains a codeword ⇒ only codes with short code words were considered.

Table 8.

C1	14,3,31; R= 0.785	C2	10,3,31; R= 0.7	C3	10,3,23; R= 0.7
C4	10,5,23; R= 0.5	C5	14,7,31; R= 0.5	C6	10,5,13; R= 0.5

Performances of Adaptive LDPC-Coded OFDM Transmissions

0. $n=8$; $n_c=0$; $n_n=8$; $N_c=0$; $N_n=868$; $R_{CM}=1$; C0; 256-QAM; $D_{ni}=1156.75$
1. $n=8$; $n_c=4$; $n_n=4$; $N_c=432$; $N_n=432$; $R_{CM}=0.892$; C1; 256-QAM; $D_{ci}=1041.89$
2. $n=7$; $n_c=2$; $n_n=5$; $N_c=216$; $N_n=540$; $R_{CM}=0.91$; C2; 128-QAM; $D_{ci}=895.94$
3. $n=6$; $n_c=4$; $n_n=2$; $N_c=432$; $N_n=216$; $R_{CM}=0.856$; C1; 64-QAM; $D_{ci}=750.00$
4. $n=5$; $n_c=2$; $n_n=3$; $N_c=216$; $N_n=324$; $R_{CM}=0.88$; C3; 32-QAM; $D_{ci}=636.48$
5. $n=4$; $n_c=2$; $n_n=2$; $N_c=216$; $N_n=216$; $R_{CM}=0.85$; C3; 16-QAM; $D_{ci}=490.54$
6. $n=4$; $n_c=2$; $n_n=2$; $N_c=216$; $N_n=216$; $R_{CM}=0.75$; C4; 16-QAM; $D_{ci}=441.89$
7. $n=3$; $n_c=2$; $n_n=1$; $N_c=432$; $N_n=0$; $R_{CM}=0.784$; C3; 16-QAQM; $D_{ci}=382.50$
8. $n=2$; $n_c=2$; $n_n=0$; $N_c=216$; $N_n=0$; $R_{CM}=0.70$; C3; 4-QAM; $D_{ci}=194.64$
9. $n=2$; $n_c=2$; $n_n=0$; $N_c=216$; $N_n=0$; $R_{CM}=0.50$; C4; 4-QAM; $D_{ci}=136.48$
10. $n=1$; $n_c=1$; $n_n=0$; $N_c=108$; $N_n=0$; $R_{CM}=0.50$; C6; 2-PSK; $D_{ci}=58.10$

List 2. Coded configurations employed and their nominal payloads

Performances of Adaptive LDPC-Coded OFDM Transmissions

- Considering (15), (17) and (4), ideal payload of the coded configuration D_{ci} is:

$$D_{ci} = \frac{1}{1+G} \cdot \frac{A_s}{T_s} \cdot D_b \cdot T_s \cdot n_i \cdot (1 - \frac{jp}{A_s \cdot n_i}); \qquad (18)$$

- The throughput of the coded configurations is (19); $BinER_{ci}$ is the bin error probability of the coded-QAM constellation with index i in list 2.

$$\theta_{ci} = D_{ci}(1 - BinER_{ci}) \qquad (19)$$

- The bin-error rate of a configuration employing LDPC-coded bits and non-coded bits is difficult to compute theoretically
- The values of the bin error probability provided by our simulation program, using a large number of bins as samples.

Performances of Adaptive LDPC-Coded OFDM Transmissions

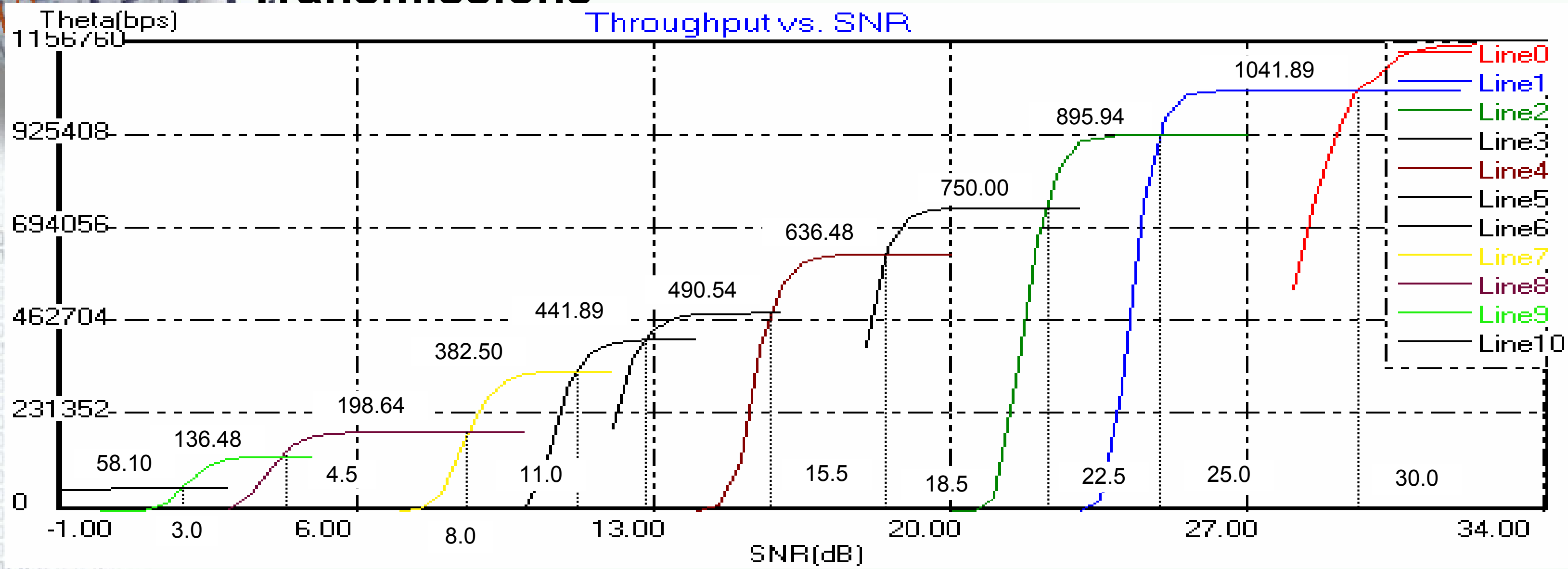


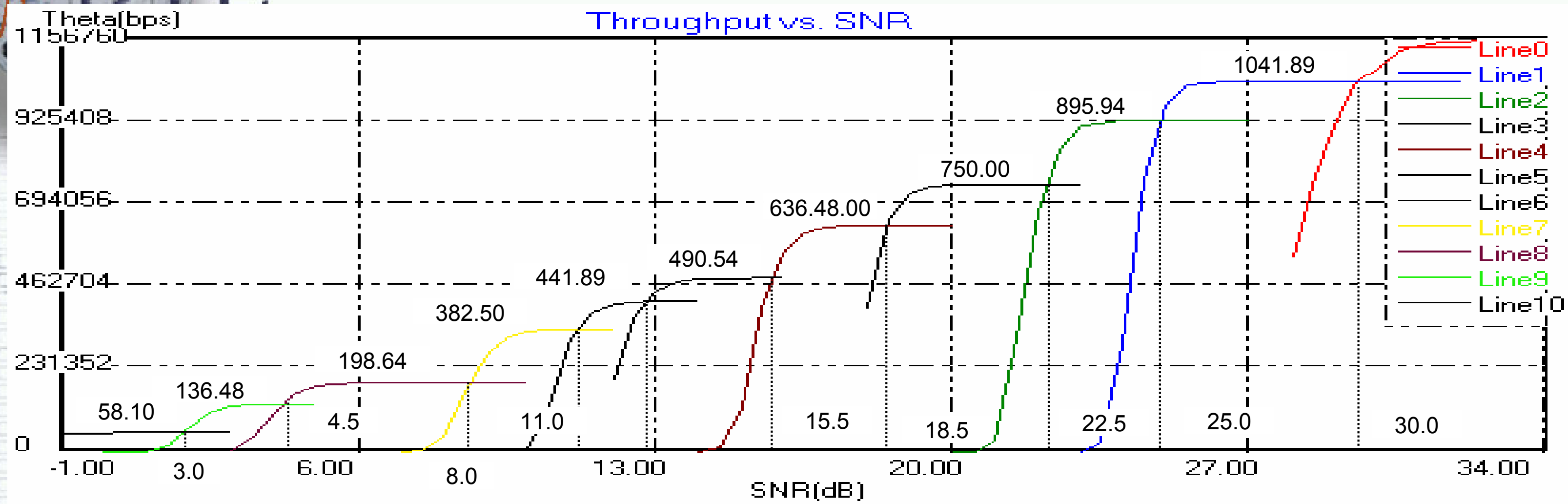
Fig.11 Θ_l vs. SNR for the coded configurations of list 2

- The SNR variation step was set to 0.5 dB.
- The separation between thresholds ranges between 2-5 dB
- The average throughput is:

- 610.93 kbps for SNR > 0dB; 525.92 kbps – non-coded – 1.16
- 666.22 kbps for SNR > 3dB; 578.07 kbps – non-coded – 1.15
- 741.09 kbps for SNR > 7dB; 655.09 kbps – non-coded – 1.13

- Configurations with relatively equally separated thresholds may be built.
- The effects of codes with lower rates upon throughput should be studied.

Fig.11 Θ_l vs. SNR for the coded configurations of list 2



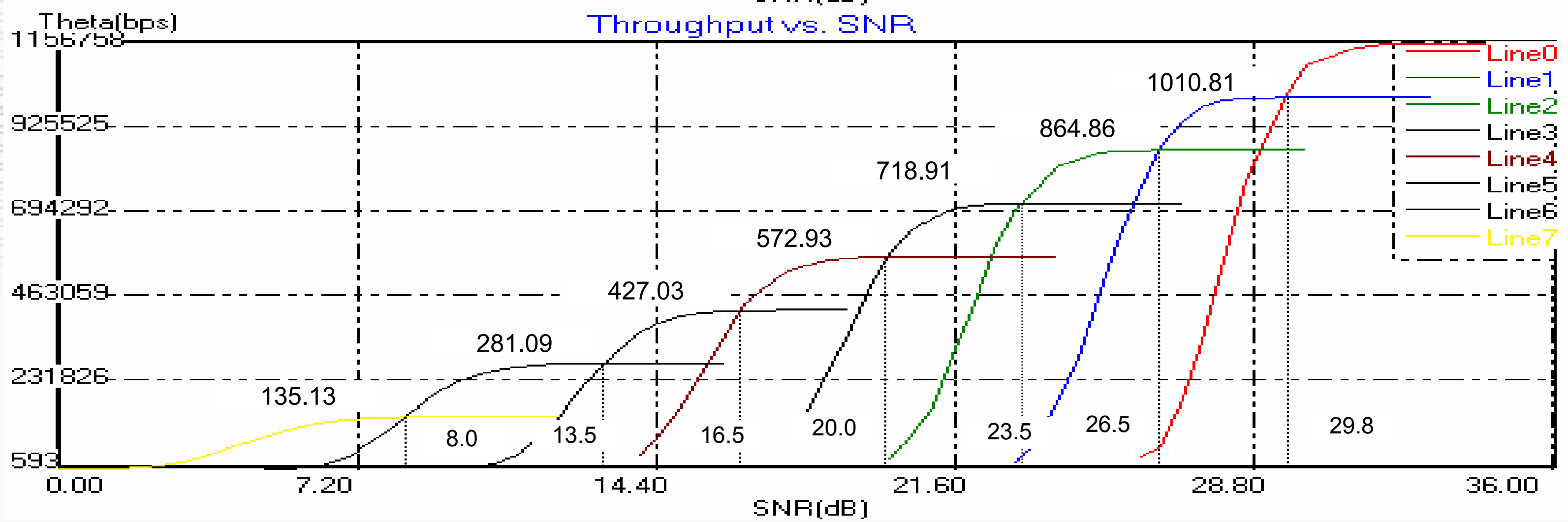


Fig.10 Θni vs. SNR for non-coded 256 (0), 128(1), 64 (2), 32(3),16 (4), 4 (5) QAM and 2-PSK (6)

Performances of Adaptive LDPC-Coded OFDM Transmissions

Considering some SNRs that belong to the same regions defined by the non-coded and coded schemes, we get the values of table 9.

SNR (dB)	32	28	26	24	22	20	18
Θ_{nc}-kbps	1157	1010	865	865	719	573	573
Θ_c -kbps	1157	1042	1042	896	750	750	636
Θ_c / Θ_{nc}	1	1.03	1.20	1.03	1.04	1.30	1.11
SNR (dB)	15	13	11	8	6	3	0
Θ_{nc}-kbps	427	282	282	135	100	~20	0
Θ_c -kbps	490	442	382	194	136	58	~40
Θ_c / Θ_{nc}	1.14	1.56	1.35	1.43	1.36	2.9	Large

Table 9. Comparison between throughputs of non-coded and coded schemes

- Coded configurations require no CRC for the channel state estimation-prediction, since the MP decoding performs the syndrome check only as a control and this syndrome check can be employed as a CRC.
- Schemes with only square constellations employing only one LDPC code ensure a decreased throughput, but require a significantly simpler implementation, [8].

Performances of Adaptive LDPC-Coded OFDM Transmissions

- Spectral efficiency of average user for K=50 users for the coded and non-coded configurations for uniformly distributed phases of the received multipath signals received by each user

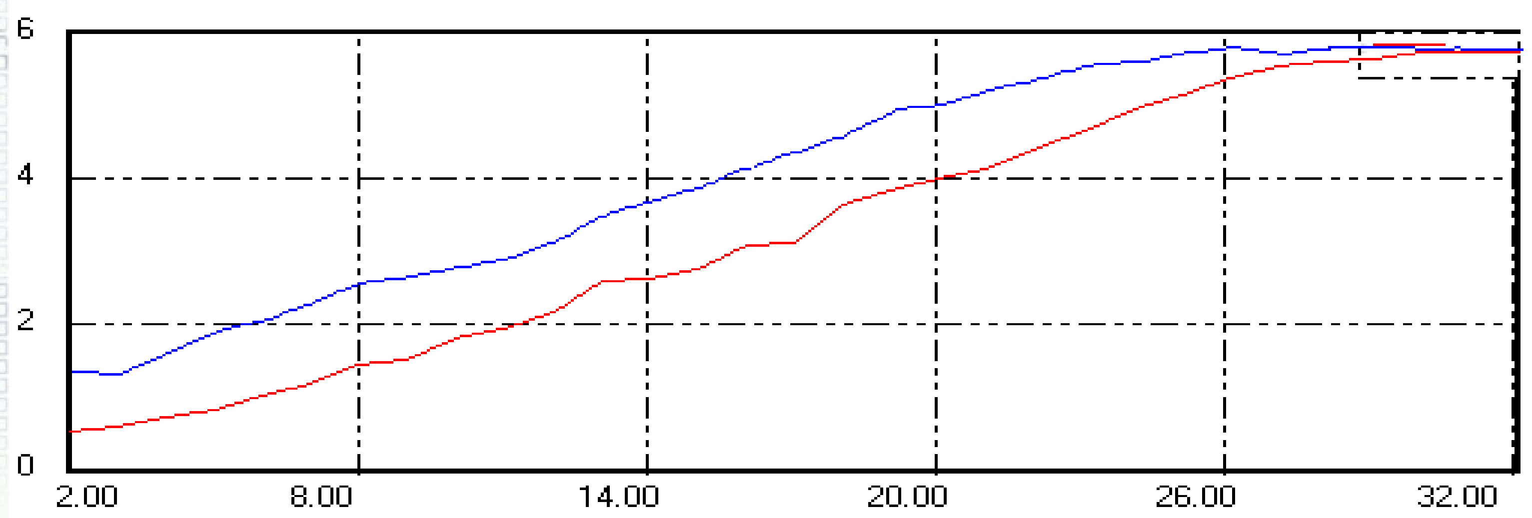
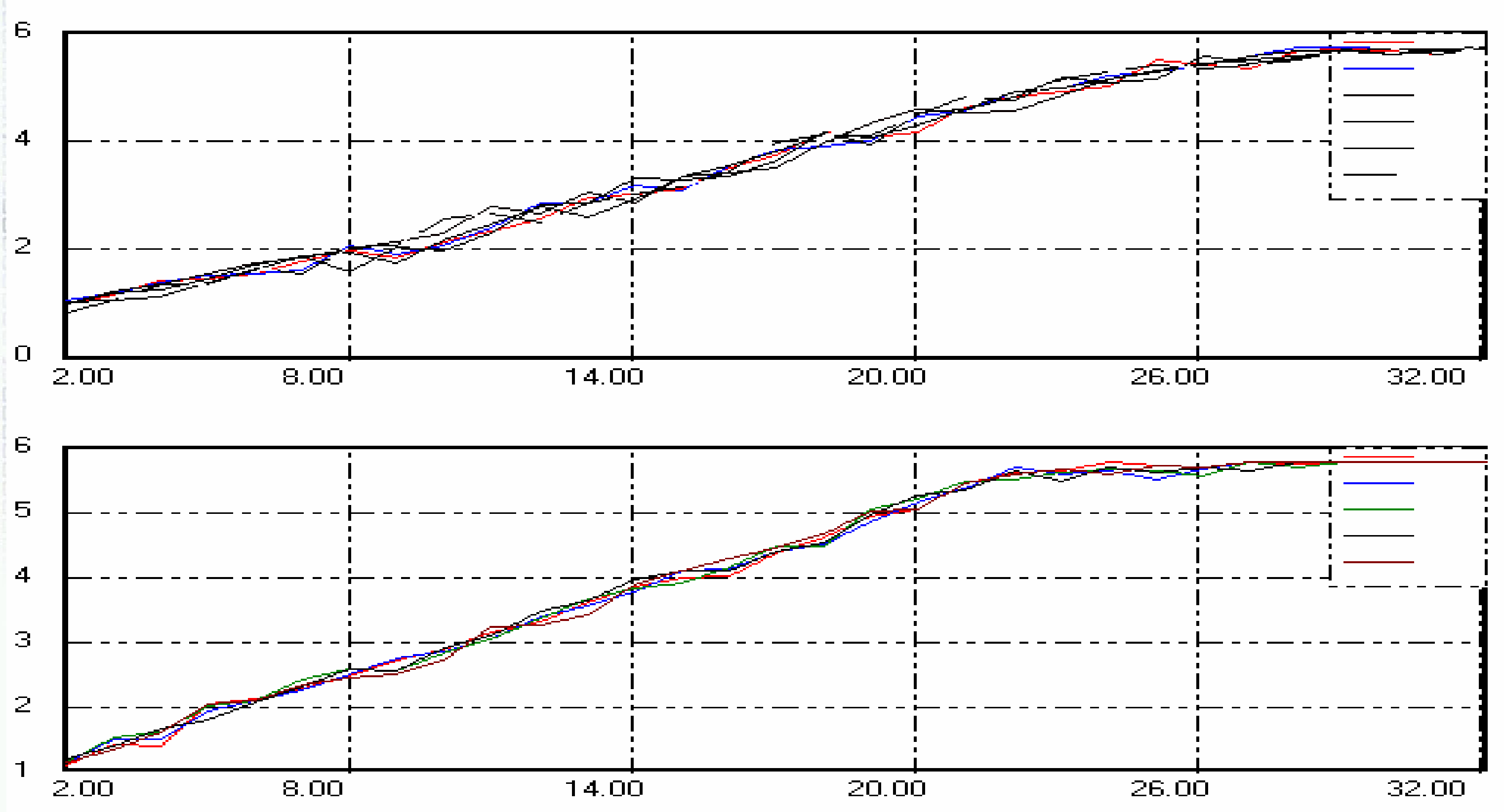


Fig.12 Spectral efficiency vs. SNR for the coded and non-coded configurations

- For the same phase of the received signals an improvement of about 0.5 bps/Hz occurs when the number of users varies from K = 5 to K = 50 at the SNR of 12-20 dB.

Performances of Adaptive LDPC-Coded OFDM Transmissions

- For the same phase of the received signals an improvement of about 0.5 bps/Hz occurs when the number of users varies from K = 5 to K = 50 at the SNR of 12-20 dB. Fig. 13 -Up – 5 users; Down – 50 users.



Performances of Adaptive LDPC-Coded OFDM Transmissions

- **c. Coded Scheme for Fixed Terrestrial Channels**
- Employs adaptively 256, 64, 16, 4-QAM and is intended for DVB-T, variant 2k, transmissions.
- The number of useful tones employed is A_s = 1704, the bin rate is D_{bin} = 4.4614 kbin/s, equaling the OFDM-symbol rate, and G = 0.25.
- The employed LDPC code is (71,15,97; R_c = 0.788) for 256, 64, 16-QAM and (47, 16, 73; R_c = 0.66) for 4-QAM
- Each OFDM symbol contains a codeword.
- For a simpler implementation only square QAM constellations and only two LDPC codes were considered.

1. non-coded: n1=8; nc1=0; nn1=8; Nc=0; Nn=13632; RCM=1;256-QAM;

2.coded: n2=8; nc2=4; nn2=4; Nc=6816; Nn=6816; RCM=0.894; 256-QAM

3.coded: n3 = 6; nc3 = 4; nn3 = 2; Nc = 6816; Nn = 3408; RCM = 0.859; 64-QAM;

4.coded: n4 = 4; nc4=4; nn4=0; Nc=6816; Nn=0; RCM=0.788; 16-QAQM;

5.coded n5=2; nc5=2; nn5=0; Nc=3408; Nn=0; RCM=0.66; QPSK

List 3. Coded configurations employed

Performances of Adaptive LDPC-Coded OFDM

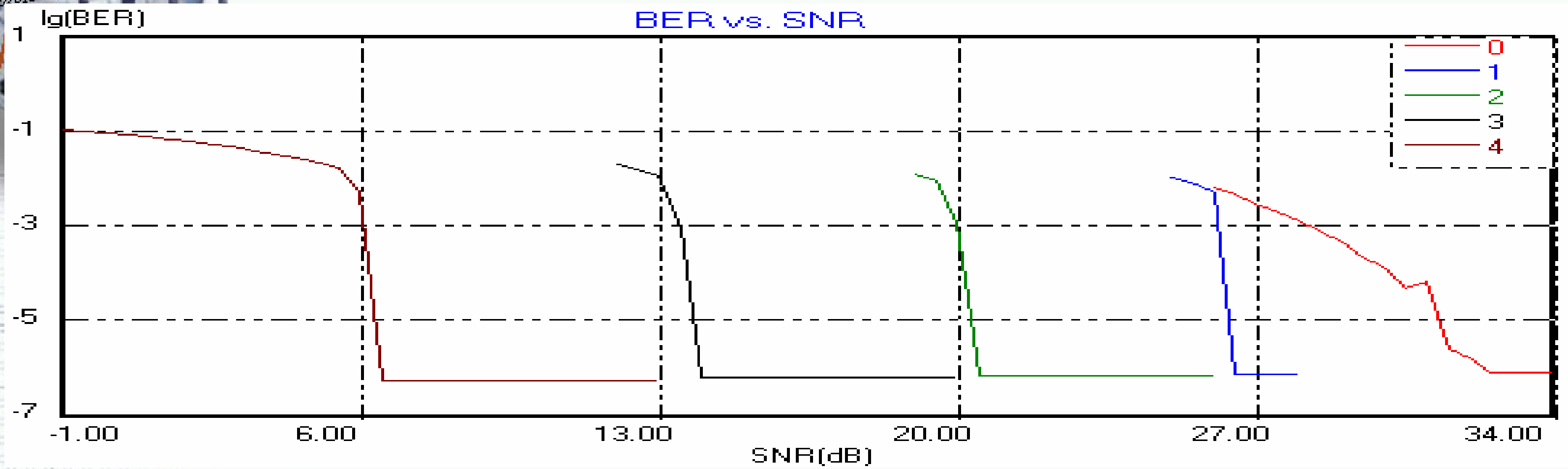


Fig. 14 BER vs. SNR for 256–nc (1), 256-c (2), 64-c (3), 16-c (4) and 4-c QAM (5)

•**Comparison with the simulated figures of EN 300744 standard (C/N ratio for BER=2·10^{-4} gaussian noise, about same rates of coded configurations)**

n_i	8- nc	8 –c; R=0.89	6 –c; R= 0.86	4 –c; R=0.75	2 –c; R=0.66
CNR(E) dB			19.3	12.5	3.1
CNR (L) dB		25.5	19.5	13	5
D_{ci}(kbps)(E)			24.88	14.93	4.98
D_{ci} (kbps)(L)	48,65	43,49	31,34	19,17	6,970

Table 10. Nominal payloads for non-coded 256, coded 256, 64, 16, 4-QAM – see list 3

Performances of Adaptive LDPC-Coded OFDM Transmissions

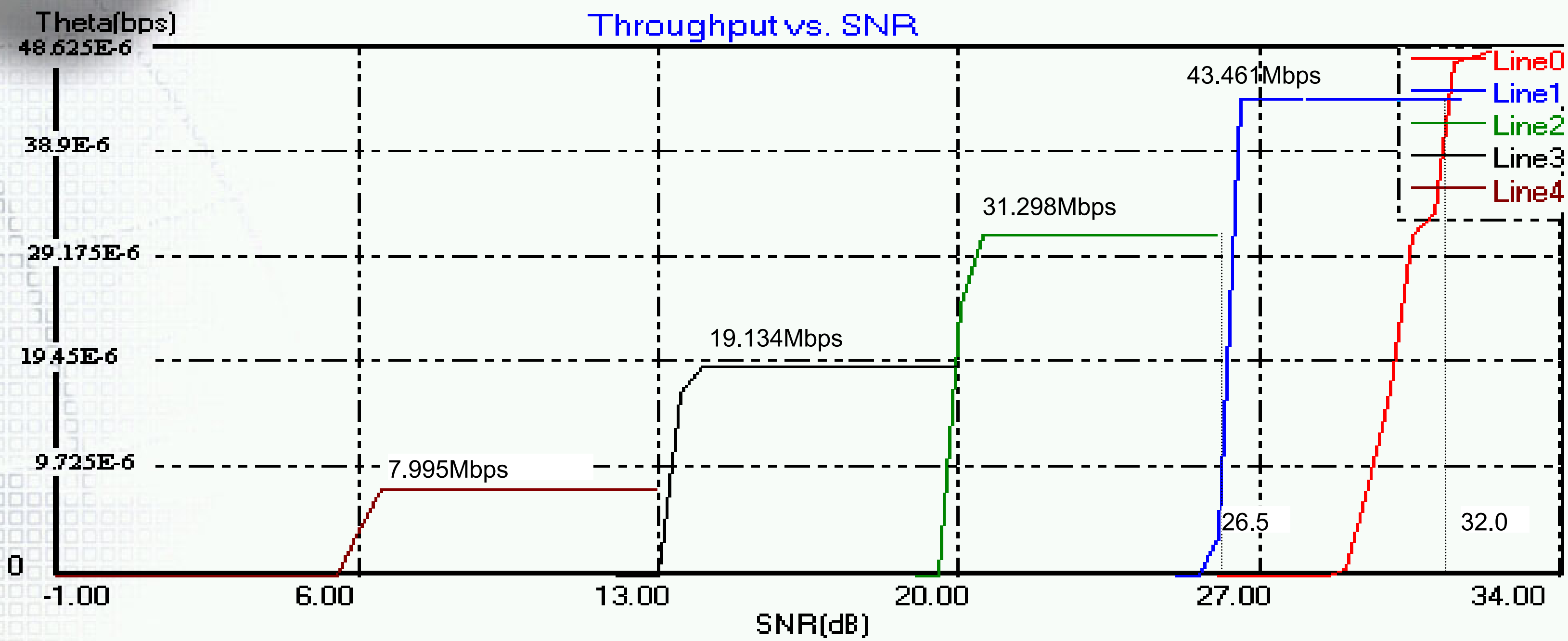


Fig.15 Θni vs. SNR for 256–nc (1), 256-c (2), 64-c (3), 16-c (4) and 4-c QAM (5) – see list 3

Conclusions

- The array-based LDPC codes generated by triangular-shaped matrices allow for a simple encoding and a moderate complexity decoding, compared to the turbo codes.
- The BER performances of the single carrier LDPC-code QAM modulations are close to the ones provided by the similar modulations coded with turbo codes at the same rate and the same number of iterations per code word.
- The LDPC Message-Passing decoder requires the *a priori* knowledge of the channel's noise variance with an approximation smaller than 2 dB.
- Due to the behavior of the LDPC-decoding algorithm and to the soft-decision of the non-coded bits, the authors estimate that small and high rate RS outer codes should be employed in FEC schemes based on concatenated codes.

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Conclusions

- A very flexible rate changing LDPC-coded scheme can be obtained by using a bit-loading that combines coded and non-coded bits and different LDPC codes.
- The rate of the coded modulation can be increased significantly by using non-coded bits at the expense of a rather small coding gain loss, but providing a significant increase of the throughput.
- The LDPC-coded adaptive modulation schemes as the one proposed might ensure significant coding and throughput gains, compared to the correspondent non-coded scheme, at the expense of a rather low redundancy inserted (high coding rate), considering the range of SNR across which it operates.
- Longer codes spread over more bins, might improve the performances
- Employing a single LDPC code and changing only the QAM constellation and ratio of coded/non-coded bits provides smaller throughput gains, but involves a simpler implementation.

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Questions for further study

- Generation and performances of the short LDPC codes generated by randomly generated control matrices [2]
- On-line evaluation of the channel's noise variance
- Study of post LDPC decoding error distribution
- Selection of the outer code for a FEC scheme employing concatenated codes
- Study of the GS and BM decoding algorithm for the RS codes
- Performance evaluation of a FEC scheme employing concatenated codes
- Channel estimation by using the syndromes of the block codes (for fixed channels)
- Performances of LDPC-coded QAM configurations on a channel affected by flat and selective fading and by the Doppler frequency shifts and in a mobile multi-user environment
- Study of effects of user-bin allocation by frequency-hopping (Flarion)
- Evaluation of throughput considering the retransmissions of the H-ARQ protocols

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