Recent Theoretical and Experimental Results in Multiuser Zero Forcing Relaying

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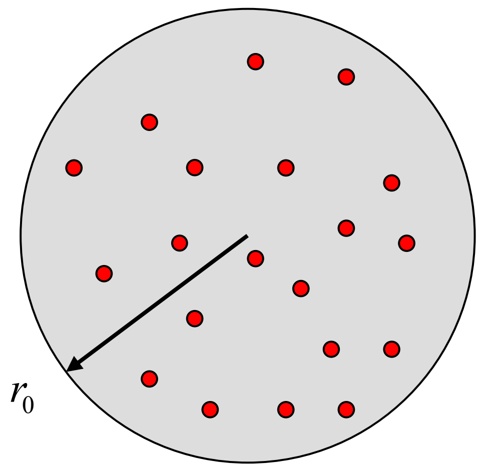
Outline

- **A simplistic consideration of the capacity of wireless ad hoc networks**
- **Zero Forcing beamforming**
  - co-located antennas
  - multiuser ZF relaying
- **Performance results**
  - improvement of sum rate in dense wireless networks
  - impact of noisy channel state information
  - impact of node mobility
- **A theoretical analysis of Multiuser ZF relaying with noisy channel state information**
  - average SINR at destination
  - tightness of approximation
  - some implications
- **Conclusions**

Wireless Ad Hoc Network



- area: $A_0 = \pi r_0^2$
- average path length:

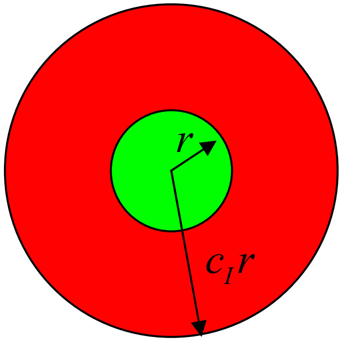
$$d_{SD} = c_{SD} \cdot r_0$$

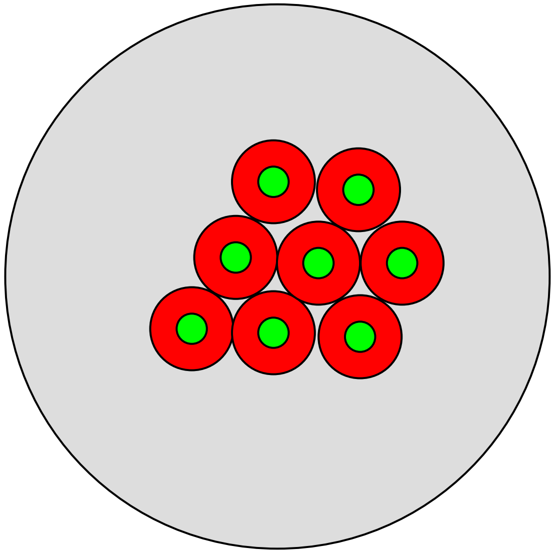
Some assumptions

- all nodes generate the same offered load
- no idle queues
- symmetric traffic pattern
- no overhead due to routing and multiple access
- no multi-access collisions
- number of nodes sufficiently large to justify the consideration of averages
- scheduling ensures minimum SINR at receiver: $SINR_r$
  - ergodic rate per hop:

$$R_a \geq \log_2\left(1 + SINR_r\right) \text{ bit/channel use}$$

Interference Model





- motivation: ensure minimum SINR at receivers
- range area:

$$A_r = \pi r^2$$

- interference area:

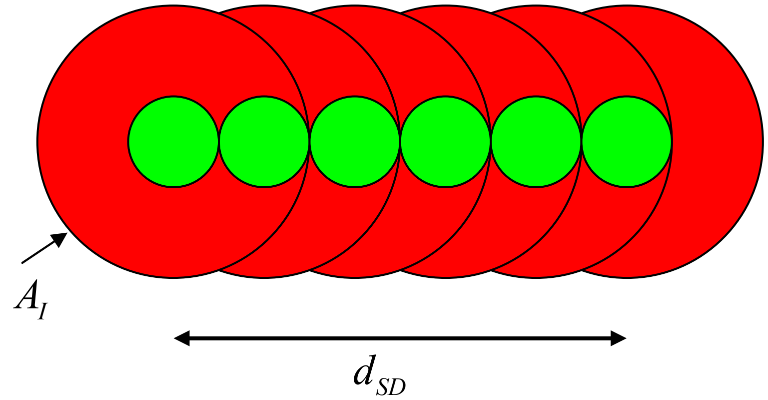
$$A_I = \pi c_I^2 r^2$$

- Note: SINR is a function of c_I

- spatial reuse of same physical resource
- distributed „spatial multiplexing“
- number of simultaneous transmissions:

$$N_{Sim} \approx A_0 / A_I$$

Multihop Paths



- the spatial resources/packet required by a multihop link are given by the sum of the interference areas of all channel uses, which are required to deliver one symbol
- average number of hops:

$$N_h = \frac{d_{SD}}{r} = c_{SD} \frac{r_0}{r}$$

- average sum interference area:

$$A_p = N_h \cdot A_I = c_I^2 \cdot c_{SD} \cdot \pi \cdot r_0 r$$

Sum Rate of Network

- sum rate of network in delivered bit/channel use:

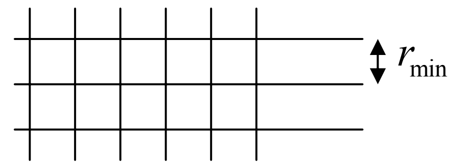
$$R_{sum} \approx \frac{A_0}{A_P} \cdot \log_2\left(1 + SINR_r\right) = \frac{\log_2\left(1 + SINR_r\right)}{c_{SD} c_I^2} \cdot \frac{r_0}{r}$$

- multihop transmission favorable (small r)

- the average minimum hop length $r_{\min}$ depends on the total number of nodes N

- for a regular 2-D network we have

$$r_{\min} = c_r \cdot \frac{r_0}{\sqrt{N}}$$



- thus the maximum average sum rate in bit/channel use follows as:

$$R_{sum}^{\max} \approx \frac{\log_2\left(1 + SINR_r\right)}{c_r c_{SD} c_I^2} \cdot \sqrt{N} \sim \sqrt{N}$$

Points of View

- Network view:

$$R_{sum}^{\max} \sim \sqrt{N}$$

- user view:
  - rate per node

$$\sim \sqrt{N} / N = 1/\sqrt{N}$$

  - number of hops (delay)

$$N_h = \frac{c_{SD} \cdot r_0}{r_{\min}} = \frac{c_{SD}}{c_r} \cdot \sqrt{N}$$

- user view: network transmit energy per delivered symbol

$$E_P = \left(\frac{r_{\min}}{d_{SD}} \right)^{\gamma} \cdot E_{SD} \cdot N_h$$

$$= E_{SD} \cdot \left(\frac{c_r}{c_{SD}} \right)^{\gamma - 1} \cdot N^{-\frac{\gamma - 1}{2}}$$

- γ : path loss exponent
- E_{SD} : reference transmit energy, which is required for 1-hop link from source to destination:

Can we trade off delay and network transmit energy?

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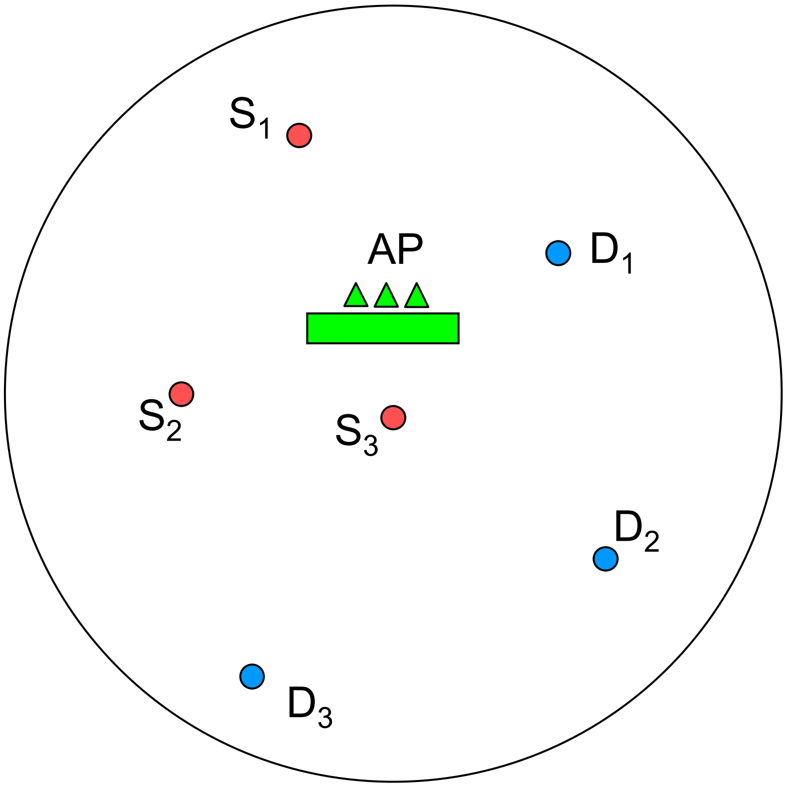
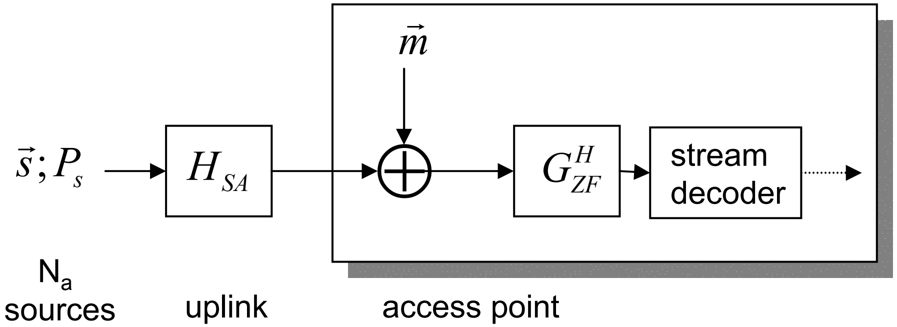
Zero Forcing Beamforming with Co-Located Antennas



uplink:



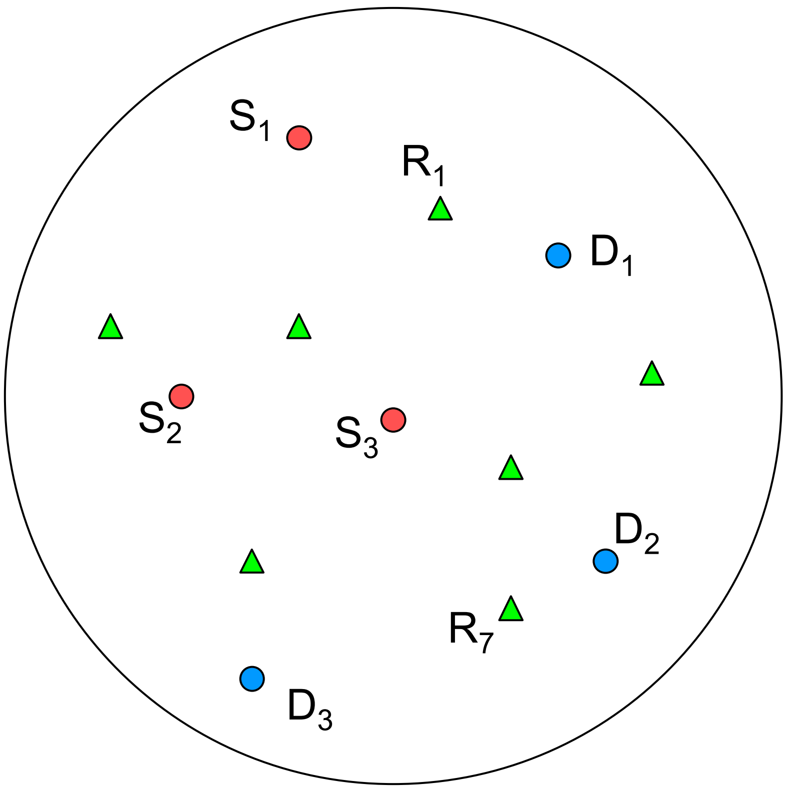
- Zero Forcing beamforming:

$$G_{ZF}^H \cdot H_{SA} = I$$

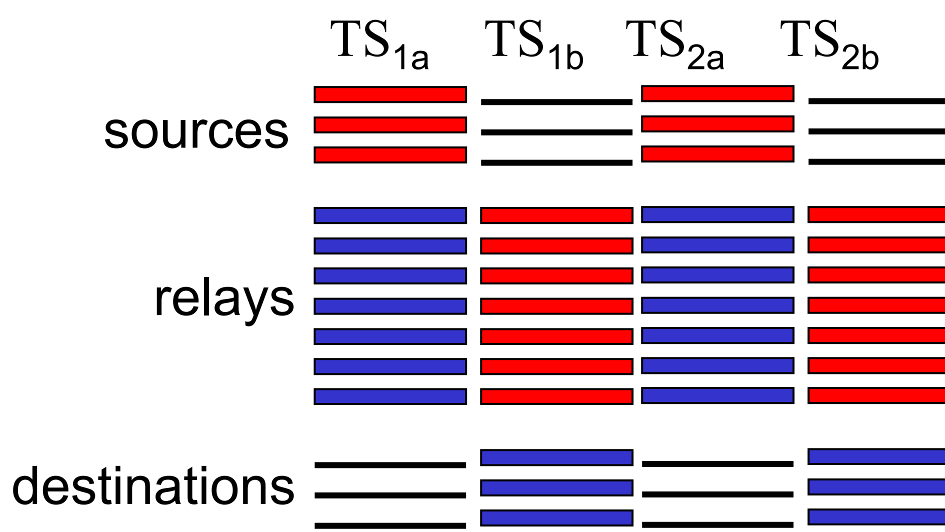
- requires cooperation between the antennas (non-diagonal gain matrix G_{ZF})
- requires at least N_a antennas, if the mobile nodes have one antenna

Multiuser Relaying in Ad Hoc Networks

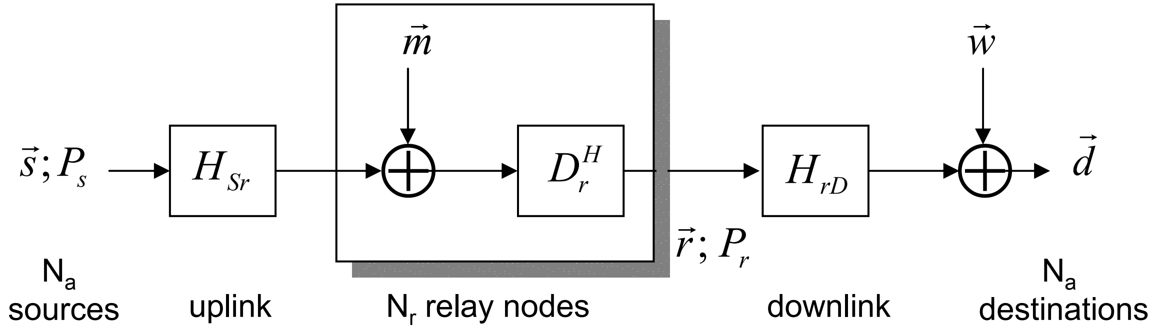
Goal: distributed beamforming in infrastructureless ad hoc network



- N_r linear amplify&forward relays
- no cooperation between relays
- N_a source/destination pairs
- all source/destination links use same physical channel
- two-hop relay traffic pattern:



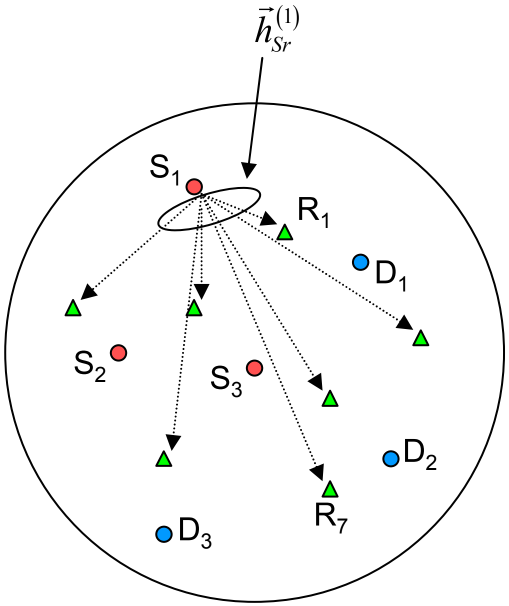
System Model

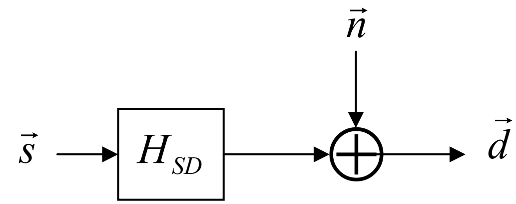


- global phase reference at relays (coherent relaying)
- no power loading across sources
- $P_s = P_r$
- total power constraint: $P_r = 1$
- link power constraint: $P_r = N_a$
- diagonal gain matrix D_r (compare to beamforming)
- received signal:

$$\vec{d} = H_{rD} \cdot D_r^H \cdot H_{Sr} \cdot \vec{s} + H_{rD} \cdot D_r \cdot \vec{m} + \vec{w} \equiv H_{SD} \cdot \vec{s} + \vec{n}$$

Multiuser Zero Forcing Relaying





$$H_{SD} = H_{rD} \cdot D_r^H \cdot H_{Sr}$$

gain vector: $\vec{d}_r = diag\left(D_r\right)$

$$H_{SD}\left[p,q\right] = \sum_{k=1}^{N_r} H_{rD}\left[p,k\right] \cdot \vec{d}_r^{\,*}\left[k\right] \cdot H_{Sr}\left[k,q\right]$$

$$H_{SD}\left[p,q\right] = \vec{d}_r^{\,H} \cdot \left(\vec{h}_{rD}^{(p)} \odot \vec{h}_{Sr}^{(q)}\right)$$

- for N_a source/destination pairs at least

  $$N_r = N_a \cdot \left(N_a - 1\right) + 1$$

  relays are required (*minimum relay configuration*)
- beamforming: N_a

ZF: $H_{SD}\left[p,q\right] = 0 \quad \forall p \neq q$

set of $N_a \cdot \left(N_a - 1\right)$ linear equ.

Excess Relay Case

- compound interference matrix

$$A_{ZF} \equiv \begin{bmatrix} \left(\vec{h}_{rD}^{(p)} \odot \vec{h}_{Sr}^{(q)}\right)^T \\ \bullet \\ \bullet \end{bmatrix} \quad \forall \; p \neq q$$

- any ZF gain vector $\vec{d}_{ZF}$ lies in the nullspace N_{ZF} of A_{ZF}, i.e.

$$\vec{d}_{ZF} = N_{ZF} \cdot \vec{y}_{ZF}$$

- for the minimum relay configuration the matrix N_{ZF} is $(N_r \times 1)$, i.e. $\vec{y}_{ZF}$ is a scalar.

- if we have more relays, we can optimize $\vec{y}_{ZF}$

- Let SNR_k be the SNR of source/destination link k for a given channel realization

<u>Optimization criteria:</u>

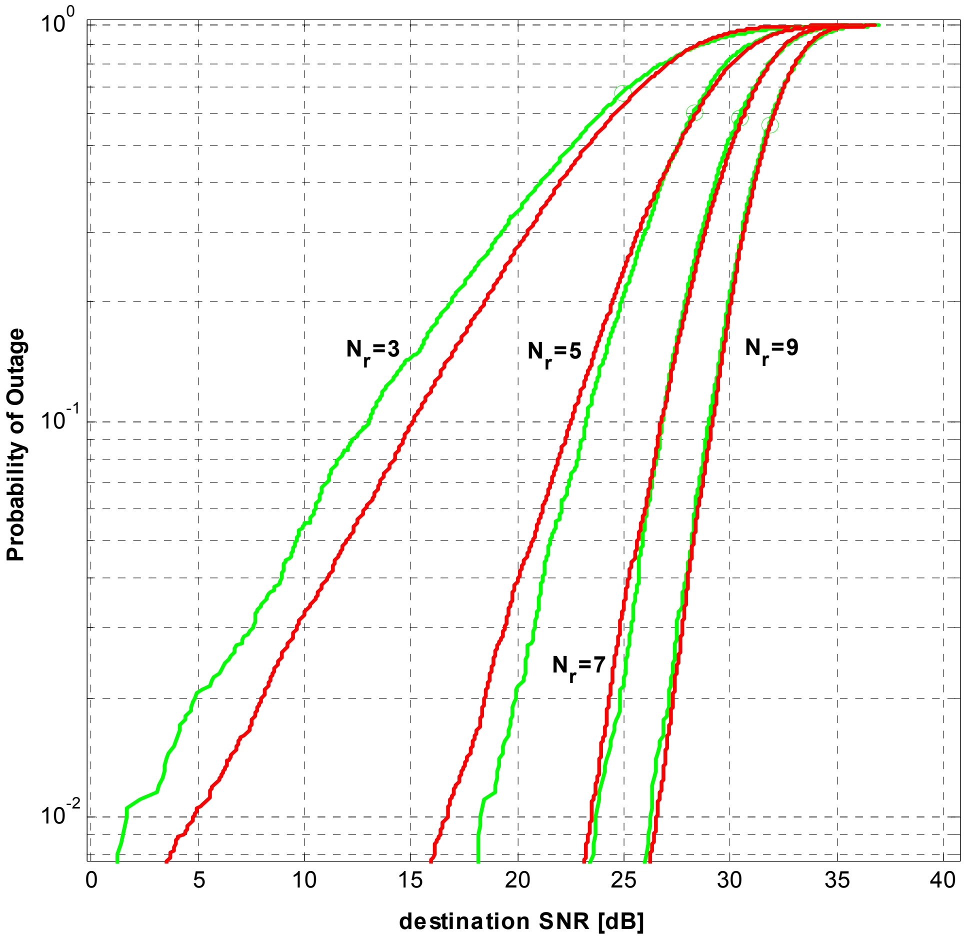
- fairness and diversity: maximize the minimum rate of all source/destination link

$$\vec{y}_{ZF} = \arg\max_{\vec{y}_{ZF}} \left[\min_k (SNR_k) \right]$$

- network performance: maximize the sum rate of all source/destination links

$$\vec{y}_{ZF} = \arg\max_{\vec{y}_{ZF}} \left[\sum_k \log_2(1 + SNR_k) \right]$$

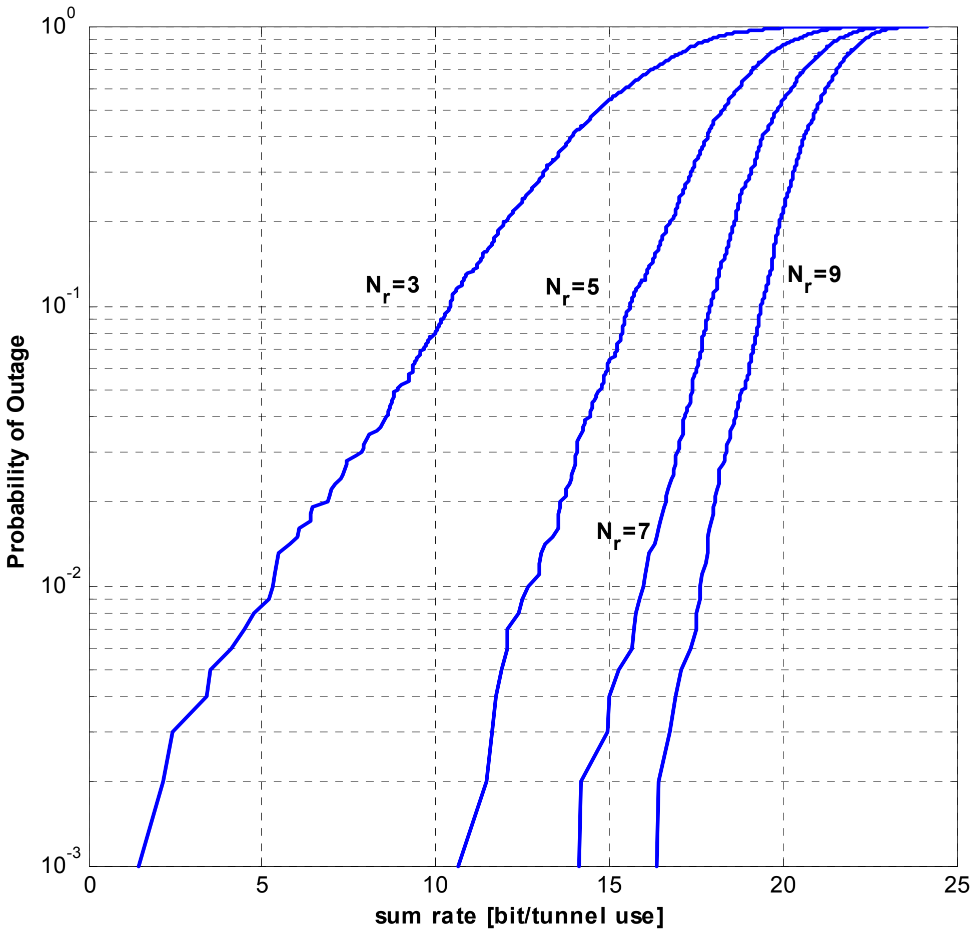
Optimization criterion fairness: diversity performance



<u>CDF of destination SNR</u>

- two source/destination pairs
- link power constraint
- i.i.d. complex normal channel coefficients
- parameter: number of relays
- green: MUZFRel
- circle: mean
- red: N_r - 2 fold diversity

Optimization criterion fairness: sum rate



CDF of sum rate

- two source/destination pairs
- link power constraint
- i.i.d. complex normal channel coefficients
- parameter: number of relays
- Note: array gain

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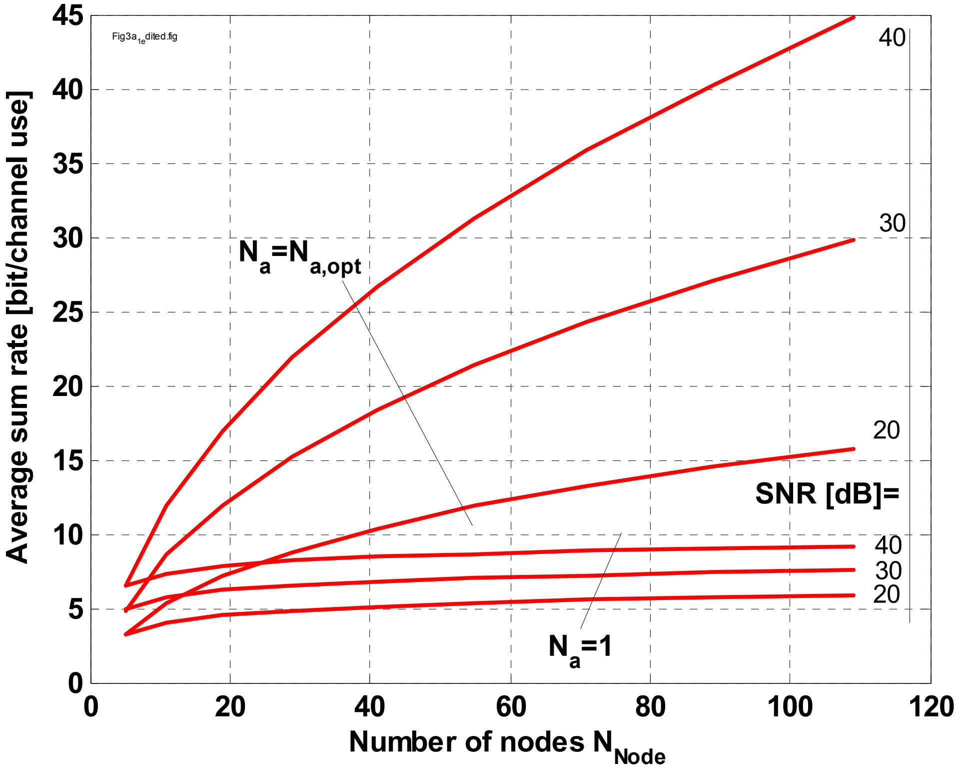
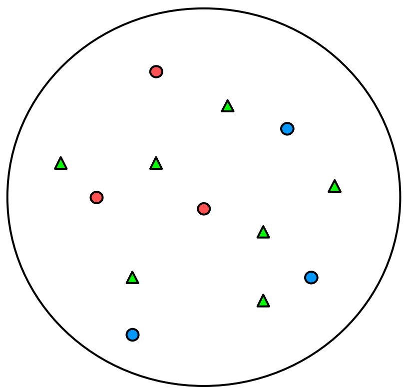
Maximum Average Sum Rate

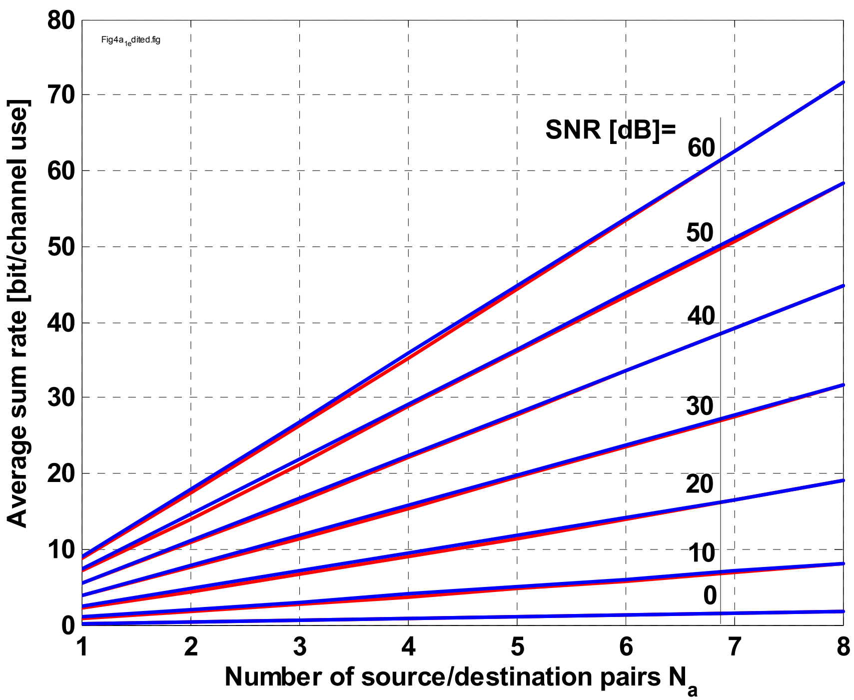


- total power constraint
- i.i.d. complex normal channel coefficient
- N_{Node} nodes in the network
- out of them $N_{a,opt}$ sources/destinations



substantial improvement of average sum rate under total power constraint

Average Sum Rate of Minimum Relay Configuration



- link power constraint
- number of nodes in the network (minimum relay configuration):

$$N_{Node} = N_a^2 + N_a + 1$$

- approximation of average sum rate:

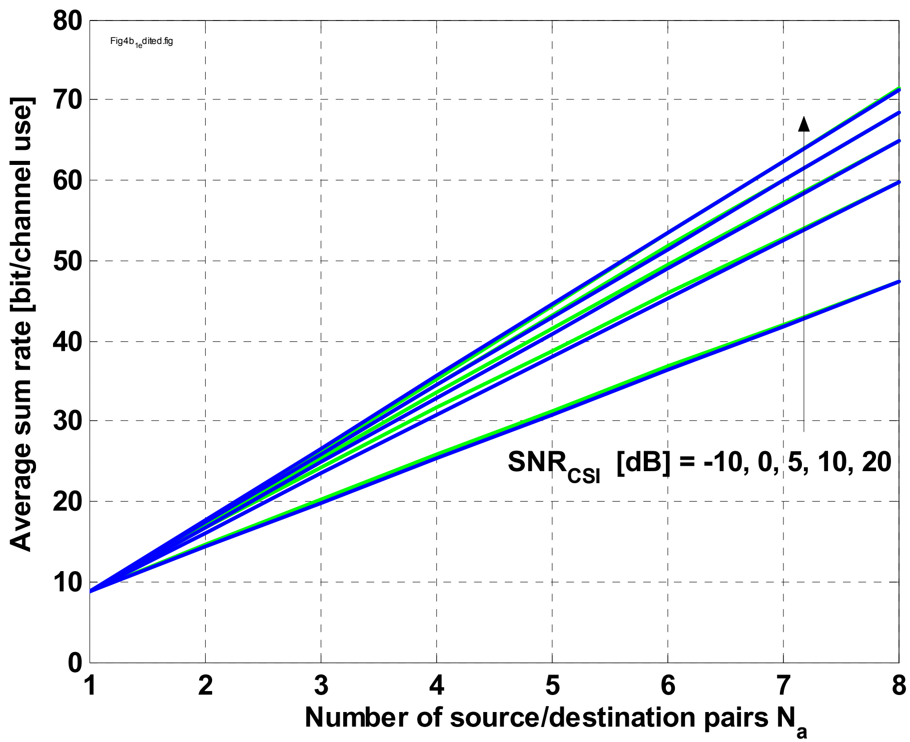
$$\bar{C}_{sum} \approx N_a \cdot 0.5 \cdot \log_2 \left(1 + 0.22 \cdot SNR\right)$$

full spatial mutiplexing gain

no distributed array gain

under link power constraint the sum rate is essentially proportional to $\sqrt{N_{Node}}$

Impact of Noisy CSI on Minimum Relay Configuration



- per link power constraint
- pilot based channel estimation

$$\hat{h} = h + w$$

- channel estimator SNR:

$$SNR_{est} = \sigma_h^2 / \sigma_w^2$$

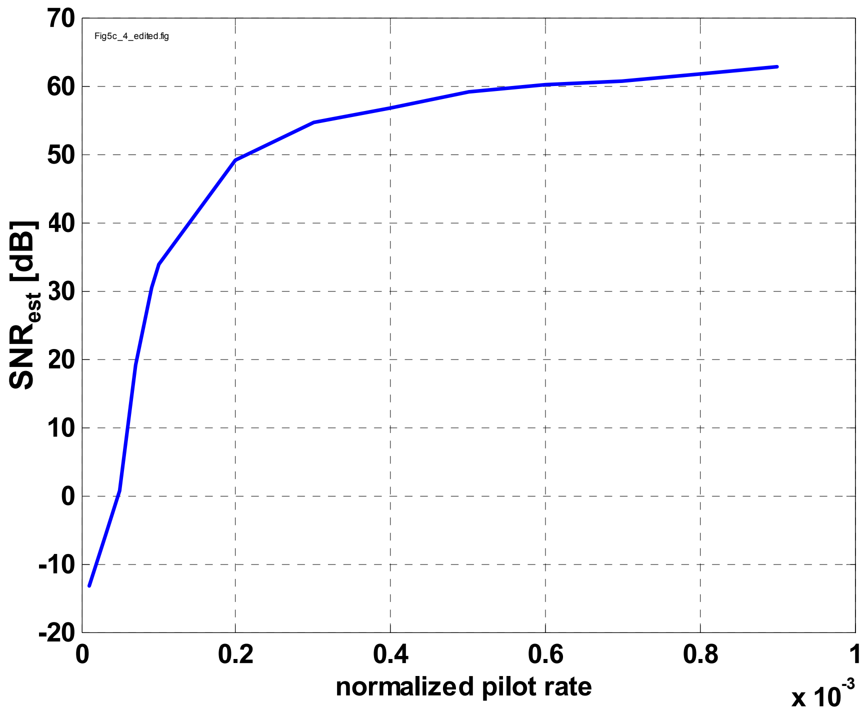
- excess SNR of the channel estimation

$$SNR_{CSI} = SNR_{est} / SNR$$

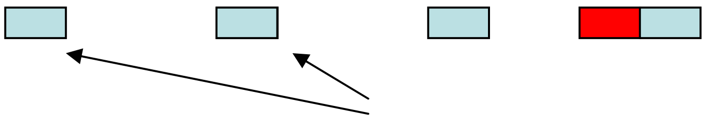
noisy CSI essentially introduces a SNR loss

$$\bar{C}_{sum} \approx \frac{(N_a - 1) \cdot \log_2\left(1 + 0.22 \cdot SNR \cdot \left\{\frac{1}{1 + 5.5 \cdot SNR_{CSI}}\right\}\right) + \log_2(1 + 0.22 \cdot SNR)}{2}$$

Impact of Node Mobility on the estimator signal to noise ratio SNR_{est}

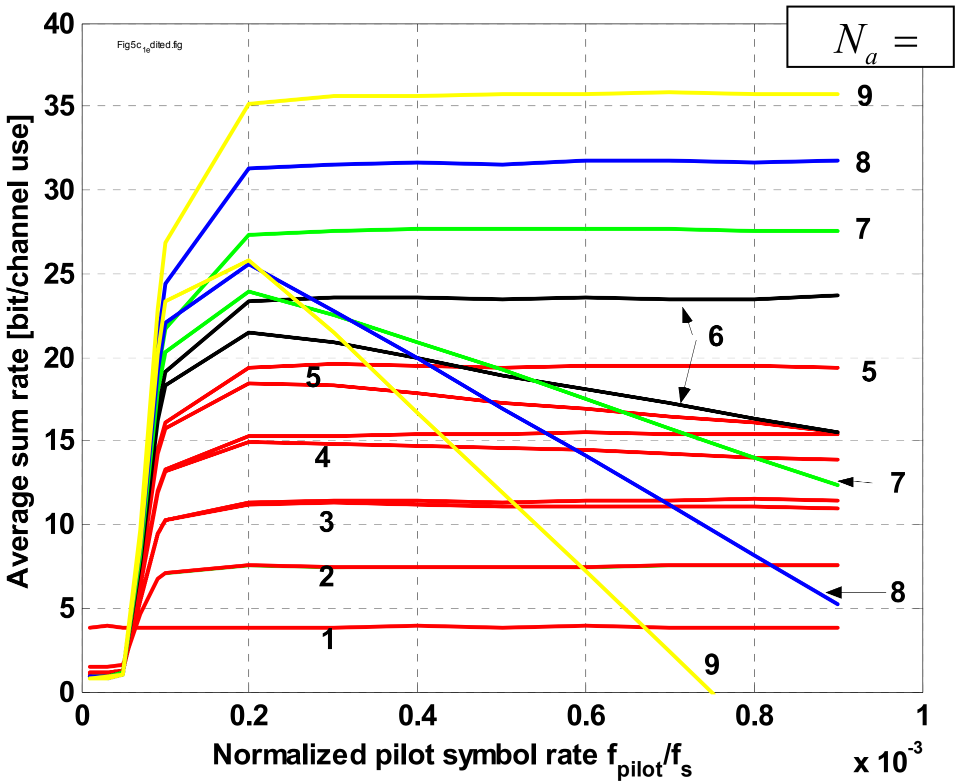


f_{pilot} / f_s



- equispaced *pilot symbols*
- Jakes doppler spectrum; f_D=20Hz
- f_s=1Mbaud symbol rate
- MMSE prediction of channel coefficients based on 10 most recent observations
- prediction error reduces estimator SNR
- Note: the estimation error due to node mobility is not inversely proportional to the SNR

Net Sum Rate under Node Mobility



- *measurement* of the local channel coefficients at each relay: one channel per source and per dest.: $2N_a$
- *dissemination* of the local CSI to all other relays requires $2N_a$ channel uses per relay: $2N_aN_r$
- *case a*: only measurement overhead
- *case b*: measurement and dissemination overhead
- SNR=30dB

- the overhead constraints the achievable spatial multiplexing gain
- however still a sixfold improvement of the sum rate in this example

Summary I

- Multiuser Zero Forcing Relaying: a novel distributed beamforming scheme for wireless ad hoc networks
  - requires a global phase reference at the relays
  - requires essentially $N_a N_a$ relays
- *Minimum relay configuration* achieves full spatial multiplexing gain but no distributed array gain
- Noisy CSI introduces *equivalent SNR loss*
- Even with moderate node mobility a *substantial increase in sum rate* is possible

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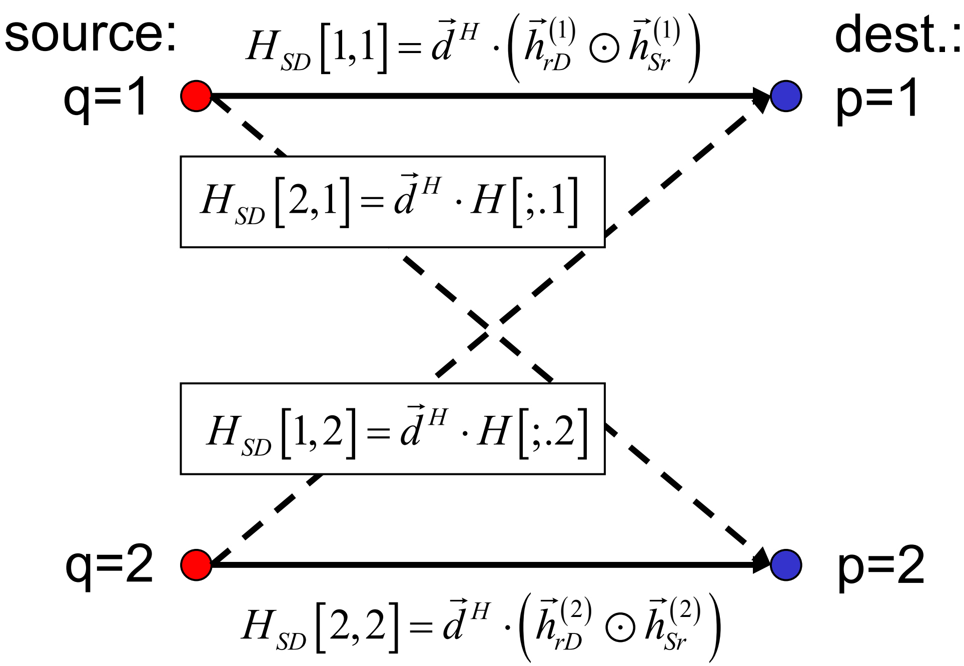
Equivalent Channel Matrix for N_a=2



Analysis of the average destination SINR with noisy channel state info

Minimum relay configuration:

$$N_r = 3$$

$$H \equiv \left[\vec{h}_{rD}^{(1)} \odot \vec{h}_{Sr}^{(2)} \quad \vec{h}_{rD}^{(2)} \odot \vec{h}_{Sr}^{(1)}\right]$$

ZF relaying:

$$\vec{d} = null(H) \equiv \vec{u}$$

Noisy channel state info:

$$H \rightarrow \hat{H} \ \text{ and } \ \vec{d} = null\left(\hat{H}\right)$$

$\Rightarrow \vec{d}^H \cdot H \neq 0$: interlink interference

Average Interference power

- estimation error

$$\boxed{\hat{H} = H + \Delta H}$$

- orthonormal basis V

$$H = V \cdot V^H \cdot H$$

- decomposition of estimation error

$$\Delta H = \Delta H_u + \Delta H_V$$

- decomposition of gain vector

$$\vec{d} = \vec{d}_u + \vec{d}_V$$

- the gain vector solves

$$\left(\vec{d}_u^H + \vec{d}_V^H\right)\left(H + \Delta H_V + \Delta H_u\right) = 0$$

$$\Rightarrow \vec{d}_u^H \cdot \left(\Delta H_u\right) + \vec{d}_V^H \cdot \left(H + \Delta H_V\right) = 0$$

- vector with interference coefficients

$$\vec{h}_{ISI}^H \equiv -\vec{d}_V^H \cdot H = -\left(\vec{d}_u^H \cdot \Delta H_u + \vec{d}_V^H \cdot \Delta H_V\right)$$

- small perturbation

$$\vec{h}_{ISI}^H \approx -\vec{d}_u^H \cdot \Delta H_u$$

and $\vec{d}_u \approx -\vec{u}$; thus

$$\boxed{\vec{h}_{ISI} \approx -\vec{u}^H \cdot \Delta H_u = -\vec{u}^H \cdot \Delta H}$$

- noisy channel state information

$$\hat{\vec{h}}_{Sr}^{(q)} = \vec{h}_{Sr}^{(q)} + \vec{x}_{Sr}^{(q)}$$

- the columns of $\hat{H} = H + \Delta H$

$$\hat{\vec{h}}_{Sr}^{(q)} \odot \hat{\vec{h}}_{rD}^{(p)} = \left(\vec{h}_{Sr}^{(q)} + \vec{x}_{Sr}^{(q)}\right) \odot \left(\vec{h}_{rD}^{(p)} + \vec{x}_{rD}^{(p)}\right)$$

- interference coefficient

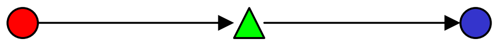
$$h_{ISI}^{(p,q)} = \tilde{H}_{SD}[p,q]$$

$$\approx -\vec{u}^H \left(\vec{h}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{h}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)}\right)$$

- average interference power

$$\boxed{\bar{P}_{ISI}^{(p)} \approx \sigma_s^2 \cdot \sigma_x^2 \cdot \left[2\left(N_a - 1\right)\left(g_r\left(N_a\right) + \sigma_x^2\right)\right]}$$

Average SINR



- Average SNR at the relay

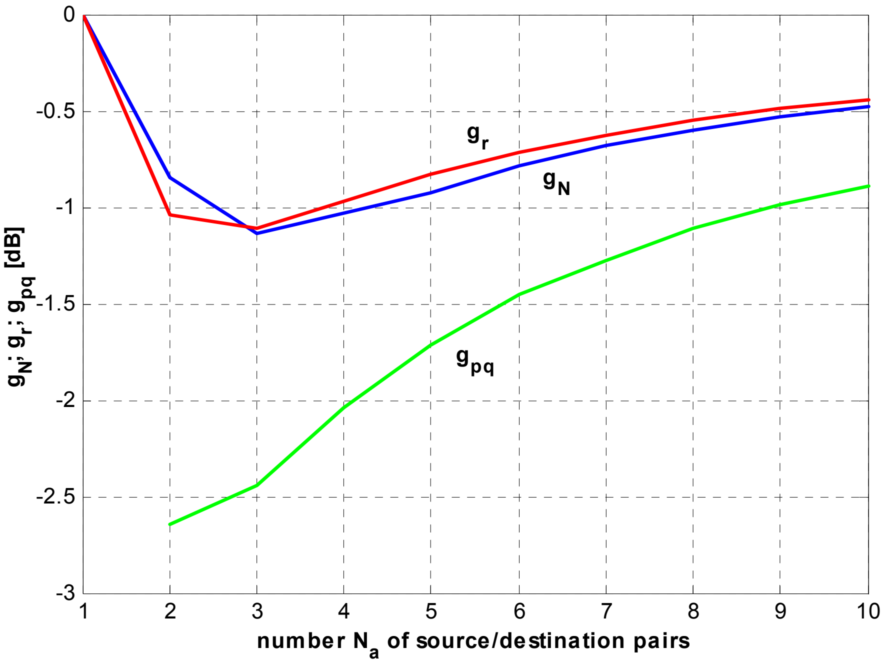
$$SNR_u = \sigma_s^2 / \sigma_m^2$$

- Average SNR at the destination with noiseless relay

$$SNR_d = \sigma_s^2 / \sigma_w^2$$

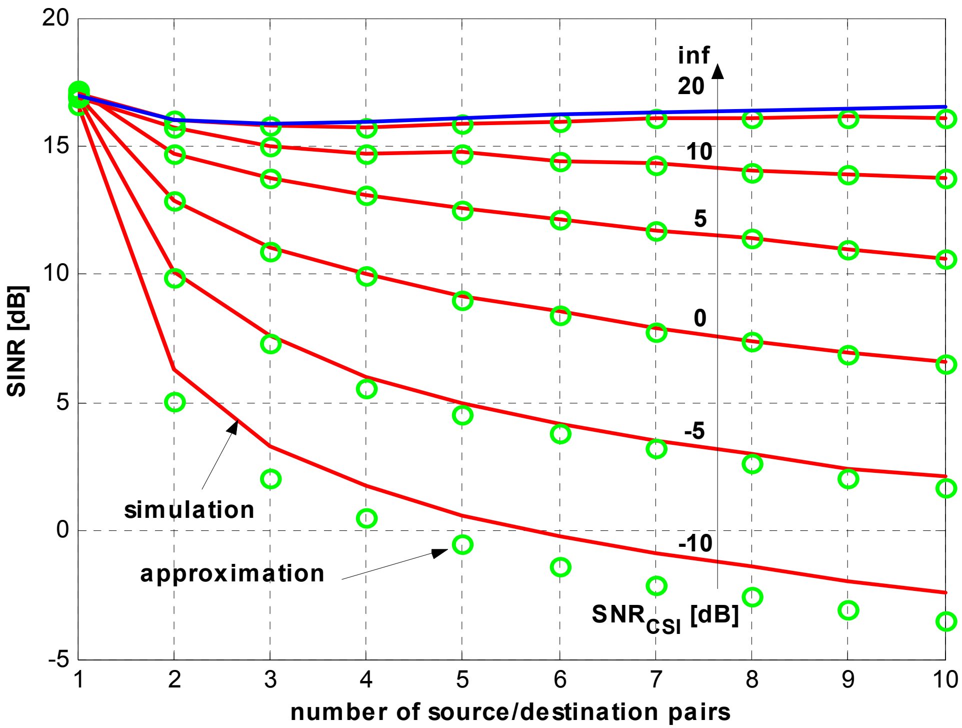
- SNR of the channel estimator

$$SNR_{est} = 1/\sigma_x^2$$



$$\overline{SINR}^{(p)} = \frac{c^2 \cdot \overline{S}^{(p)}}{c^2 \cdot \left(\overline{\sigma}_r^{(p)2} + \overline{P}_{ISI}^{(p)} \right) + \sigma_w^2} = \frac{g_N(N_a)}{\left(SNR_u^{-1} + 2SNR_{est}^{-1} \cdot (N_a - 1)\left(1 + SNR_{est}^{-1} / g_r(N_a)\right) + SNR_d^{-1} \frac{P_s}{\overline{P}_{r,0}} \right)}$$

Tightness of Small Perturbation Analysis



- mimimum relay configuration
- $SNR_u = SNR_d = SNR = 20dB$
- $SNR_{CSI} = SNR_{est} - SNR$ [dB]

small perturbation analysis is tight for parameter space of practical interest

Optimum Power Allocation

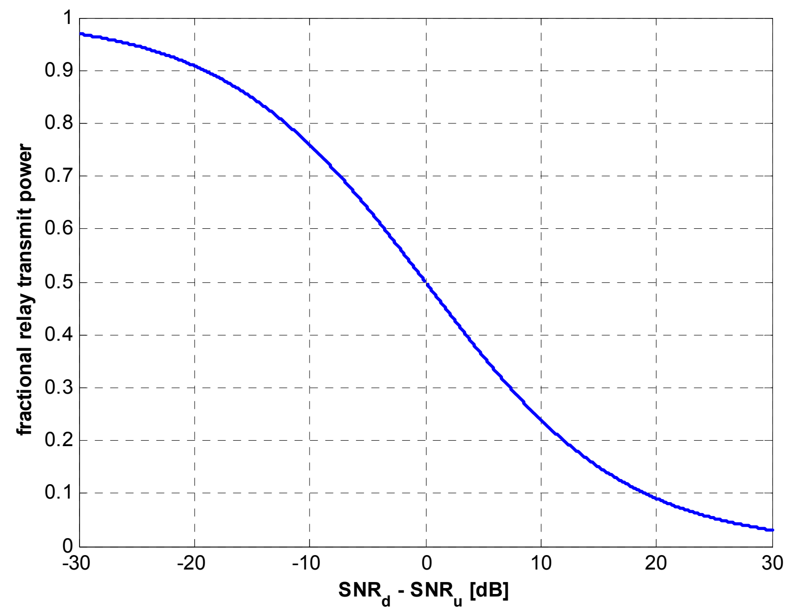
$$\overline{SINR} = \frac{g_N(N_a)}{\left(SNR_u^{-1} \cdot \frac{1}{(2-2x)} + 2SNR_{est}^{-1} \cdot (N_a - 1)\left(1 + SNR_{est}^{-1} / g_r(N_a)\right) + SNR_d^{-1} \cdot \frac{1}{2x} \right)}$$

- network transmit power per information symbol: $P = \left(P_S + \overline{P}_{r,0}\right) = 2N_a$
- fractional relay transmit power $x \equiv \overline{P}_{r,0} / P$

<u>optimal power allocation</u>

$$x_{opt} = \frac{1-\sqrt{y}}{1-y} \quad \text{with} \quad y = \frac{SNR_d}{SNR_u}$$

- independent of SNR_{est}



Large System Performance

Approximation for $SNR_{est} > 10$

$$\overline{SINR} \approx \frac{SNR \cdot g_N(N_a)}{2} \cdot \frac{1}{\left(1+\left(\frac{SNR}{SNR_{est}}\right) \cdot (N_a - 1)\right)} \quad \text{with} \quad SNR_d = SNR_u \equiv SNR$$

SNR loss due to noisy CSI

For a large number of source/destination pairs N_a the system is operated in the low SNR regime, if SNR_{est} is finite

⇒in contrast to perfect CSI the sum rate of the network is finite in the large system limit:

$$\lim_{N_a \to \infty} \overline{R}_{sum} = \frac{SNR_{est}}{4 \cdot \ln 2}$$

For $SNR_{est} = 30dB$ and $SNR = 20dB$ this corresponds to 127 source/destination pairs with perfect CSI

Summary II

- Multiuser ZF relaying achieves a distributed spatial multiplexing gain with single antenna nodes
- Analysis of average SINR based on small perturbation assumption
- Numerical verification of tightness of approximation
- Noisy CSI introduces SNR-loss, which is proportional to the number of source/destination pairs
- Optimal fractional relay transmit power is independent of quality of the CSI
- In the large system limit the sum rate of the network is proportional to $\sqrt{N_{node}}$; with noisy CSI the sum rate saturates however

Comparison of Multihop and Multiuser ZF Relaying

Multihop transmission

- Network view:

$$R_{sum}^{\max} \sim \sqrt{N}$$

- user view:
  - number of hops (delay)

$$N_h \sim \cdot\sqrt{N}$$

  - network transmit energy per delivered symbol

$$E_P \sim N^{-\frac{\gamma-1}{2}}$$

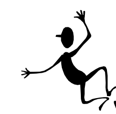
Multiuser ZF relaying

- Network view:

$$R_{sum}^{\max} \sim \sqrt{N}$$

- user view:
  - number of hops (delay)

$$N_h = 2$$



  - network transmit energy per delivered symbol

$$E_P = const.$$

Example of trade off between delay and network transmit energy

THE END