Performance Analysis of Spatial Multiplexing and Maximal Ratio Combining Systems in the Presence of Polarization Diversity

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Abstract

This paper examines the performances of spatial multiplexing (SM) and maximal ratio combining (MRC) systems in the presence of the polarization diversity.

It is assumed that dual-polarized antennas are used at both the transmitter and the receiver. The symbol error rate (SER) for SM and MRC systems is obtained as a function of cross polarization discrimination (XPD) and cross correlation coefficients. In this paper, in contrast with the general approach, XPD is chosen as a random variable and its effect on the system performance is analysed.

1. Introduction

Spatial diversity is a technique that improves performance in fading channels and is achieved by using multiple antennas. Significant gains can be realized from the use of spatial diversity if the separation between the antennas is sufficiently large; then, there is a significant degree of statistical independence between the received fading signals in each of diversity branches. The polarization diversity has an advantage over spatial antenna diversity in that the two antennas can be co-located. This minimizes the need for extra space at the base stations. The co-location of the antennas implies that this technique can also be employed at the mobile station with no size constraints.

Polarization diversity provides the receiver two signals on orthogonal polarizations. Polarization diversity relies on the ability of scatterers in the propagation path to depolarise and decorrelate the transmitted signal, thus giving rise to some coupling of energy into the orthogonal polarization. The performance of a polarization diversity system therefore depends strongly on the environment and can be formulated in terms of the XPD and signal cross correlations. *XPD* is defined as the ratio of the power received in the wanted (transmitted) polarization to that received in the unwanted (orthogonal) polarization expressed in dB. The studies reported in the literature have focused mainly on quantifying the XPD and the envelope cross correlation between the branches. Results show that the XPD is typically between 5-20 dB, and is much higher in suburban than in urban environments [1], [3], [4], [5].

The cross correlation coefficient is a measure of the correlation between the signals arriving at the two antennas. The signals can be in the same or in different polarizations. Signals that are totally uncorrelated will have a cross correlation of zero. System performance will improve as the cross correlation between the signals decreases.

Section 2 introduces a SM system. In Section 3 the channel model for a dual polarized single-input single-output link is defined. In Section 3.2 the performance of the SM system for this channel is analysed. Section 3.3 will introduce the performance the performance analyses of a MRC system for the same channel. Numerical results are given in Section 3.4. Finally, Section 4 presents the conclusions.

2. Spatial Multiplexing

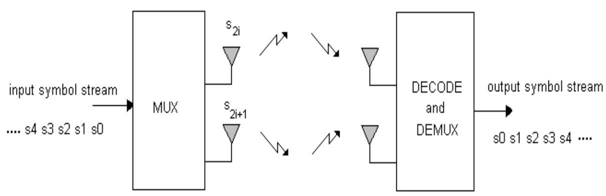

Figure 1 SM system configuration

SM exploits multiple antennas at both the transmitter and the receiver to increase the bit rate in a wireless radio link with no additional power or bandwidth consumption. The performance of this technique is highly dependent on channel statistics.

To explain the working principle of spatial multiplexing system, assume that the number of antennas at transmitter and receiver is 3 (N=3). At the transmitter the stream of (possibly coded) information symbol {b1, b2, b3, b4, b5, b6...} is split into N=3 independent substreams {b1, b4,...}, {b2, b5,...}, { b3, b6,...}. These substreams are applied separately to the N transmit antennas. At the receiver, each of the N substreams is recovered and are multiplexed together. A linear SM receiver can be viewed as a bank of superposed spatial weighting filters where every filter aims at extracting one of the multiplexed substreams by spatially nulling the remaining ones. This evidently assumes that the substreams have different signatures.

A SM system thus allows a transmitter-receiver pair to communicate through multiple parallel spatial channels hence allowing for a possible N-fold improvement in the link speed. More improvement is actually obtained by taking into account the diversity gain offered by the multiple antennas Such performance factors are derived, ideally, under the assumption that the substreams are truly independent from each other, spatially. In the reality the level of independence between substreams will determine the actual link performance.

3. Channel Model

Figure 2 shows a schematic description of the link between the transmitter and the receiver. Transmitter has antennas with vertical and horizontal polarizations and receiver gets the symbols with vertically and horizontally polarized antennas.

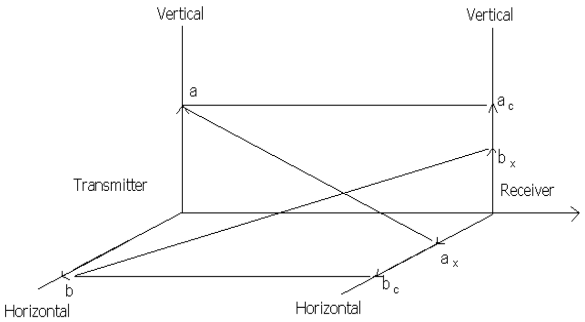

Figure 2 Schematic description of the channel

Cross polarization discrimination, XPD, is the ratio of the power received in wanted polarization (transmitted) to that received in the unwanted (orthogonal) polarization expressed in dB.

$$XPD_v = 20\,log_{10}\,|a_c / b_x|\ \text{dB} \tag{1}$$

Cross polarization isolation, XPI, is the ratio of the energy received in wanted (transmitted) polarization to that unwanted signal with wanted polarization.

$$XPI_v = 20\,log_{10}\,|a_c / a_x|\ \text{dB} \tag{2}$$

Under certain conditions, XPI and XPI values assumed to be same but this is not realistic. In this paper, we will consider a channel model, which is based on XPD values.

Cross correlation coefficient is defined as

$$\rho_e = \frac{E[(r_1 - \bar{r}_1)(r_2 - \bar{r}_2)]}{\sqrt{E\,[(r_1-\bar{r}_1)^2]E[(r_2-\bar{r}_2)^2]}} \tag{3}$$

where r_1 and r_2 are the envelopes of the signals on the diversity branches [1].

The channel is assumed to be flat over the frequency-band of interest. The input-output relationship is given by

$$\boldsymbol{r} = \sqrt{E_s}\,\boldsymbol{H}\,\boldsymbol{x} + \boldsymbol{n} \tag{4}$$

where $\boldsymbol{x} = [x_o \quad x_1]^t$ is the transmit signal vector whose elements are taken from a finite (complex) constellation chosen such that the average energy of the constellation elements is 1, $\sqrt{E_s}$ denotes the average symbol energy. $\boldsymbol{r} = [r_o \quad r_1]^t$ is the receive signal vector, and **n** is complex valued Gaussian noise with $E\left(\mathrm{n}\,\mathrm{n}^H\right) = \sigma_n^2\,\mathrm{I}_2$.

$$\boldsymbol{H} = \begin{bmatrix} h_{o,o} & h_{o,1} \\ h_{1,o} & h_{1,1} \end{bmatrix} \tag{5}$$

H is the channel transfer matrix, which is also the polarization matrix. The polarization matrix describes the degree of suppression of individual co-polarized and cross-polarized components, cross correlation, and cross coupling of energy from one polarization state to the other polarization state. The correlation between the elements of the channel transfer matrix **H** depends on the propagation conditions. The signals x_0 and x_1 are transmitted on the two different polarizations, r_0 and r_1 are the signals received on the corresponding polarizations. The channel is hence a 2-input 2-output channel, since each polarization mode is treated as an independent physical channel.

Assume that the channel is Rayleigh fading, i.e., the elements of the matrix **H** are complex Gaussian random variables with zero mean.

$$E\left(|\,h_{o,o}\,|^2\right) = E\left(|\,h_{1,1}\,|^2\right) = 1 \tag{6}$$

$$E\left(|h_{o,1}|^2\right) = E\left(|h_{1,o}|^2\right) = \alpha \tag{7}$$

where α depends on the XPD ($XPD = 10\,log(1/\alpha)$) and $0 < \alpha \le 1$. The case of $\alpha = 1$ means that two physical antennas on each side of the link employing the same polarization. Transmit and receive correlation coefficients can be defined as

$$t = \frac{E\left(h_{o,o}\,h_{o,1}^*\right)}{\sqrt{\alpha}} = \frac{E\left(h_{1,o}h_{1,1}^*\right)}{\sqrt{\alpha}} \tag{8}$$

$$r = \frac{E\left(h_{o,o}\,h_{1,o}^*\right)}{\sqrt{\alpha}} = \frac{E\left(h_{o,1}\,h_{1,1}^*\right)}{\sqrt{\alpha}} \tag{9}$$

It is also assumed that $E\left(h_{o,o}\,h_{1,1}^*\right) = E\left(h_{1,o}h_{o,1}^*\right) = 0$. This implies that the path gains for co-polar and cross-polar channels are uncorrelated and the path gains from co-polar to cross-polar and from cross-polar to co-polar channels are uncorrelated.

3.1. Statistical Modelling of XPD

Measured values of XPD and correlation coefficients have been reported in [2]-[6]. In [6] the distribution of the XPD is given for the different measurements for vertically and horizontally polarized (VP-HP) antennas. In this paper it is assumed that XPD is a lognormal random variable and XPD (dB) is Gaussian. Figure 3 shows the distribution of cross polarization discrimination.

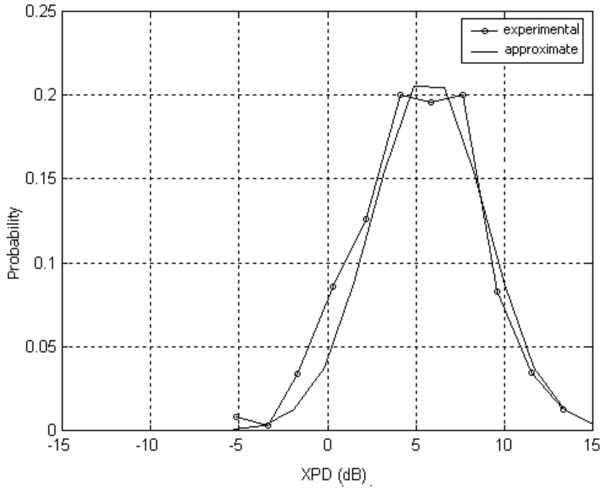

Figure 3 Experimental [7] and approximate distribution of cross polarization discrimination XPD (dB)

Channel fading is assumed Rayleigh and flat. It is also assumed that the channel is unknown to the transmitter, but is perfectly known to the receiver where the maximum likelihood (ML) decoding is performed. The receiver computes the ML estimate according to

$$\hat{x} = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{r} - \sqrt{E_s}\, \boldsymbol{H}\boldsymbol{x} \right\|^2 \tag{10}$$

The minimization is performed over the set of all possible codevectors, $\boldsymbol{x}$.

$\boldsymbol{c}$ and $\boldsymbol{e}$ are the codevectors of size 2×1 and assume that $\boldsymbol{c}$ was transmitted. For a given realization of the channel transfer matrix **H**, the probability that the receiver decides erroneously in favor of the vector $\boldsymbol{e}$ is given by [8]

$$P(\boldsymbol{c} \to \boldsymbol{e} | \boldsymbol{H}) = Q\left(\sqrt{\frac{E_s}{2\sigma_n^2} d^2(\boldsymbol{c},\boldsymbol{e}|\boldsymbol{H})} \right) \tag{11}$$

where $d^2(\boldsymbol{c},\boldsymbol{e}|\boldsymbol{H}) = \left\| \boldsymbol{H}(\boldsymbol{c}-\boldsymbol{e}) \right\|^2$

By defining $\boldsymbol{y} = \boldsymbol{H}(\boldsymbol{c}-\boldsymbol{e})$, $d^2(\boldsymbol{c},\boldsymbol{e}|\boldsymbol{H}) = \left\| \boldsymbol{y} \right\|^2$. Using the Chernoff bound $Q(x) \le e^{-x^2/2}$, equation 11 becomes

$$P(\boldsymbol{c} \to \boldsymbol{e} | \boldsymbol{H}) \le e^{-\frac{E_s}{4\sigma_n^2}\left\| \boldsymbol{y} \right\|^2} \tag{12}$$

The 2×1 vector $\boldsymbol{y}$ is Gaussian because $\boldsymbol{H}$ was assumed to be Gaussian. The pairwise error probability averaged over all channel realizations is [8].

$$P(\boldsymbol{c} \to \boldsymbol{e}) = E_{\boldsymbol{H}}(P(\boldsymbol{c} \to \boldsymbol{e}|\boldsymbol{H})) \le \frac{1}{1+\lambda_1(\boldsymbol{C_y})\frac{E_s}{4\sigma_n^2}} \frac{1}{1+\lambda_2(\boldsymbol{C_y})\frac{E_s}{4\sigma_n^2}} \tag{13}$$

where $\lambda_i(\boldsymbol{C_y})$ denotes the eigenvalues of the 2×2 covariance matrix of $\boldsymbol{y}$, defined as $\boldsymbol{C_y} = E\left(\boldsymbol{y}\boldsymbol{y}^{\boldsymbol{H}}\right)$.

3.2. Performance of the MRC System

For the system given in Figure 2, the performance of a MRC system is derived as follows.

The received signals are given in Equation 4, when the transmitted signal is assumed as $\boldsymbol{x} = [x_o \quad x_o]^t$. The output of the two-branch MRC system is

$$\tilde{x}_o = (h_{00} + h_{01})^* r_o + (h_{10} + h_{11})^* r_1 \tag{14}$$

For PSK signals, the ML estimator at the output of the maximal ratio combiner estimates according to the following rule:

The ML estimator at the output of MRC produces an estimate $\tilde{x}_o$ from the received signal. Using this estimation of $\tilde{x}_o$, the ML estimator decides that the symbol x_i was transmitted if $d^2(\tilde{x}_o, x_i) \le d^2(\tilde{x}_o, x_k) \quad \forall i \ne k$.

3.3. Numerical Results

In the simulations, first the elements of the polarization matrix **H** are realized. A complex Gaussian random vector **W** with zero mean and unit variance (whose element are independent) is subsequently obtained. Then, by using the linear transformation $\boldsymbol{h} = \boldsymbol{T}\boldsymbol{h_w}$, the channel matrix with the wanted properties is obtained. Then, the vector $\boldsymbol{h_w}$ is formed by using $\boldsymbol{W}$. The covariance matrix of **h** is known and the covariance matrix of $\boldsymbol{h_w}$ is a unitary matrix and the relationship between the covariance matrices of these two vectors is $E\left(\boldsymbol{h}\boldsymbol{h}^*\right) = E\left(\boldsymbol{T}\boldsymbol{h_w}\boldsymbol{h_w^*}\boldsymbol{T}^*\right) = \boldsymbol{T}\boldsymbol{T}^*$ where

$$E\left(\boldsymbol{h}\boldsymbol{h}^*\right) = \begin{bmatrix} 1 & t\sqrt{\alpha} & r\sqrt{\alpha} & 0 \\ t\sqrt{\alpha} & 1 & 0 & r\sqrt{\alpha} \\ r\sqrt{\alpha} & 0 & 1 & t\sqrt{\alpha} \\ 0 & r\sqrt{\alpha} & t\sqrt{\alpha} & 1 \end{bmatrix} \tag{15}$$

By using Cholesky decomposition, the transformation matrix $\boldsymbol{T}$ is evaluated from Equation 15. Then using the equation $\boldsymbol{h} = \boldsymbol{T}\boldsymbol{h_w}$, the coefficients $h_{o,o}$, $h_{1,1}$, $h_{o,1}$, $h_{1,o}$ are obtained.

The simulated system, which consists of 1 dual-polarized transmit and 1 dual-polarized receive antenna, uses 4-PSK and employs a ML receiver. The signal-to-noise ratio SNR is

$$SNR = 10\log\left(\frac{2E_s}{\sigma_n^2}\right)$$

Figure 4 shows the SER of a SM and a MRC system in the presence of polarization diversity, obtained by using Monte Carlo simulation for $t = 0.5$, $r = 0.3$, $\alpha = 0.4$ (XPD = 4 dB).

Performance of a SM system is not as good as the performance of the MRC system, because ML estimators used for these systems are different. Note that, in a SM system, two different symbols are simultaneously transmitted from the two orthogonally polarized antennas while the same symbol is transmitted from the two antennas in a MRC system.

Figure 5 shows the performance of the MRC system with dual polarized antennas, both at the transmitter and the receiver, when the parameter α is chosen as a constant (deterministic) and a lognormal random variable. Random variable XPD (dB) is a Gaussian random variable with 4 dB mean and variances between 5-20 dB. The effect of variance on the system performance becomes more important at higher SNRs.

Figure 6 shows the effect of the random variable XPD when its variance is constant and its mean changes.

4. Conclusions

The polarization diversity has an advantage over spatial antenna diversity in that the two antennas can be co-located. It is assumed that dual-polarized antennas are used at both the transmitter and the receiver. The SER of SM and MRC systems is obtained as a function of XPD and cross correlation coefficients. XPD is modelled as a random variable. MRC system is better than SM system since the same symbol is transmitted from both antennas. As α increases, the system performance improves. The effect of XPD on the performance of the system is analysed by choosing α as a random variable.

Acknowledgement

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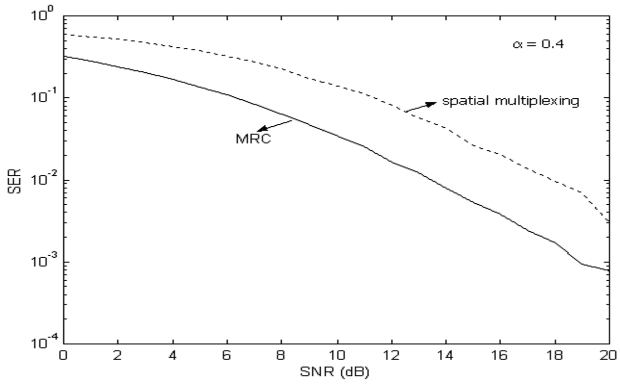

Figure 4 Performance of MRC system with dual polarized antennas both a receiver and transmitter when t = 0.5, r = 0.3.

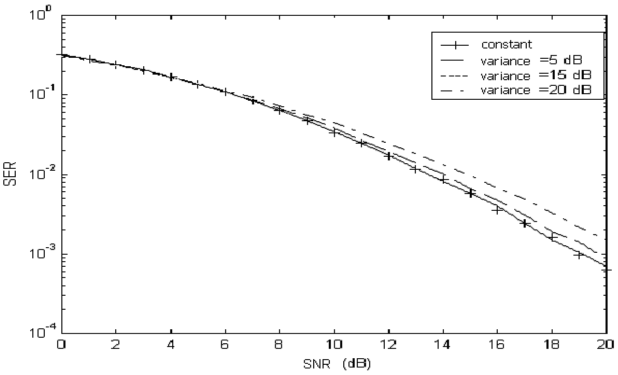

Figure 5 Performance of the MRC system with dual polarized antennas both at receiver and transmitter, for t = 0.5, r = 0.3 and when the XPD is a lognormal random variable with mean=4 dB.

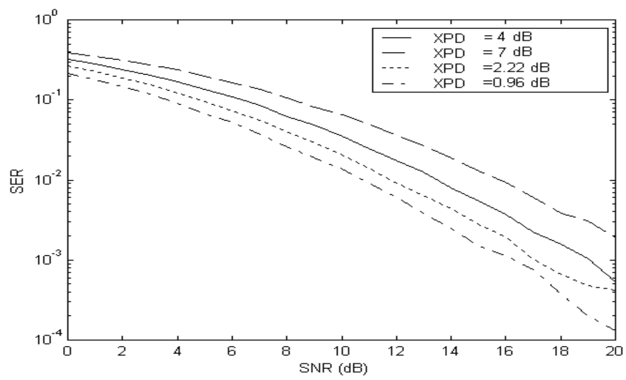

Figure 6 Performance of the MRC system with dual polarized antennas both a receiver and transmitter when XPD is constant (deterministic) and t = 0.5, r = 0.3.