Study of MIMO channel capacity for IST METRA models

Matilde Sánchez Fernández, M[a] del Pilar Cantarero Recio and Ana García Armada
Dept. Signal Theory and Communications
University Carlos III of Madrid
{mati,agarcia}@tsc.uc3m.es

Abstract— **In this paper MIMO channel capacity is studied from a simulation point of view. The channel models used for capacity computation are the proposed in IST-I-METRA European project.**

I. Introduction

The need for increasing capacity in a mobile communication systems is a fact due to the user demands. In particular, the user requirements for high data rates are focusing the technologies alternatives towards powerful coding schemes and multiple antenna systems.

In a conventional SISO (Single Input Single Output) system the channel capacity is limited by the signal to noise ratio, however, MIMO (Multiple Input Multiple Output) systems show promising results in increasing channel capacity. MIMO systems show a capacity increase that might depend linearly with the minimum number of antenna elements in the transmitter or receiver part, assuming for this assess that the total transmitter power is independent of the number of antennas. This capacity increase is given by the exploitation the multiple elements make of multipath diversity and spatial diversity.

The paper is organized as follows. First in section II capacity in a conventional SISO systems will be presented together with a MIMO channel capacity. In section III MIMO channel models used for calculations will be presented. Then in section IV the performance obtained will be discussed and finally some conclusions will be drawn.

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II. SISO vs MIMO capacity

The channel capacity for a conventional SISO system is limited by Shannon's formula [1]:

$$C = \log_2(1 + SNR) \qquad (1)$$

Thus the capacity in bit/symbol depends exclusively on the signal to noise ratio at the receiver. At this point, for a constant noise power, increasing the capacity of the channel in 1 bit/symbol implies that the signal power should be doubled.

An alternative for increasing channel capacity without additional power consumption are MIMO systems. Here the use of multiple elements both in transmission and reception exploits channel dispersion and increases the channel capacity according to the following formula [2]:

$$C = \log_2 \det \left[\mathbf{I}_{n_R} + \frac{\rho}{n_T} \mathbf{H}^{\mathbf{H}} \mathbf{H} \right] \qquad (2)$$

where n_R and n_T are respectively the number of receivers and transmitters at each end. ρ is the total signal to noise ratio and $\mathbf{H}$ is the channel matrix. The channel matrix represents the attenuation coefficients of a flat fading channel between antenna elements. Thus the matrix dimensions are $n_T n_R$.

III. MIMO Channel models

Several channel models are being proposed for MIMO systems [3]. There is a first group based on a detailed description of the propagation environment, called deterministic models. Within this group two distinctions are made: the reproduction of recorded impulse responses based on extensive measurements campaigns and the ray-tracing techniques, based on geometric optics that allow predicting the multipath propagation in a given environment from its geometrical description.

The second group is classified as stochastic models and they do not rely on a specific site description but on reproducing observed phenomena by means of stochastic processes. This group is also subdivided into: geometrically-based stochastic models (GBSM), parametric stochastic models (PSM) and correlation-based stochastic models. Correlation-based stochastic models rely on the second order statistics of the channel coefficients to fully characterize the MIMO channel. To this last category belongs the channel models developed by IST project IST-1999-11729 METRA [4].

IST METRA model is based on a tapped delayed line where the complex gaussian coefficients are defined by means of their second order statistics. These characterize the spatial correlation at both the transmitter and receiver side together with the temporal correlation:

$$\mathbf{H}(\tau) = \sum_{l=1}^{L} \mathbf{A}_l \delta(\tau - \tau_l) \tag{3}$$

The generation of each of the matrices $\mathbf{A}_l$ follow the next procedure:

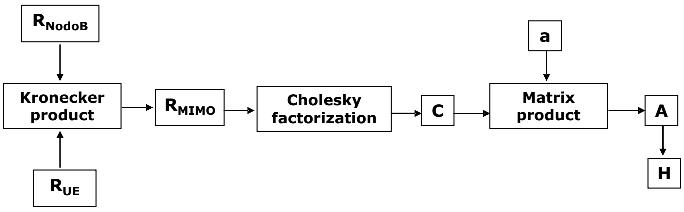

Fig. 1. Tap matrix generation.

where $\mathbf{R}_{NodoB}$ and $\mathbf{R}_{UE}$ are the spatial correlation coefficient at both sides and **a** are $n_R n_T$ complex gaussian independent coefficients that in the subsequent temporal dimension models the fading with the corresponding Doppler spectra.

IST METRA has defined four channel models following the previous approach. Temporal correlation is present in the four models by means of Doppler spectrum. The first model is a flat fading model with no spatial correlation that will be used in our study as an upper bound for the maximum channel capacity achieved. The spatial correlation present in the rest of the models is described by means of parameters such as PAS (Power Azimut Spectrum), AS (Azimut Spread), AoA (Angle of Arrival) and spacing among antennas. The three models with spatial correlation also include multipath propagation up to 6 paths.

	Case 1 Rayleigh Uncorrelated	Case 2 (pedestrian A) Rice Correlated	Case 3 (vehicular A) Correlated	Case 4 (pedestrian B) Correlated
PDP	-	ITU Pedestrian A	ITU Vehicular A	ITU Pedestrian B
Number of Paths	1	4	6	6
Doppler Spectra	Classical	Classical	Laplace	Laplace
Speed	-	3-40-120 Km/h		
UE Topology	-	0.5 λ	0.5 λ	0.5 λ
UE PAS	-	Rice Component: K=6dB Uniform 360º	Laplacian AS=35 Uniform 360º	Laplacian AS=35 Uniform 360º
UE Movement Direction	-	0	22.5	-22.5
AoA	-	22.5	67.5	-67.5 for odd paths 22.5 for even paths
Node B Topology	-	Uniform Linear Array: 0.5 λ or 4 λ		
Node B PAS	-	Laplacian AS=5	Laplacian AS=10	Laplacian AS=15
Node B AoA	-	20º,5	20º,5	2,-20,10,-8,-33,31

Fig. 2. IST METRA channel models.

For our channel capacity studies, several simplifications will be made. As it was previously said the uncorrelated channel model will be used as a reference for capacity achievement. The rest of the models will be considered to show the capacity degradation due to spatial correlation, for that reason just one of the paths will be studied at each time.

IV. Capacity Studies

The uncorrelated environment will be a reference for our capacity study. The capacity gains depending on the number of antennas and the signal to noise ratio will be shown by means of the capacity cumulative density function (CDF) and its mean. The correlated environments will be studied under the same conditions and a final comparison will be made to provide a reference for the capacity loss due to correlation.

For both studies, channel independent time samples will be taken to evaluate the cumulative density function according to equation 2.

A. Uncorrelated environment

The empirical CDF for a signal to noise ratio is shown next for different number of transmitter and receiver antennas.

The channel gains depending on the number of antennas are better shown keeping constant either the number of transmitters or receivers and showing the mean capacity.

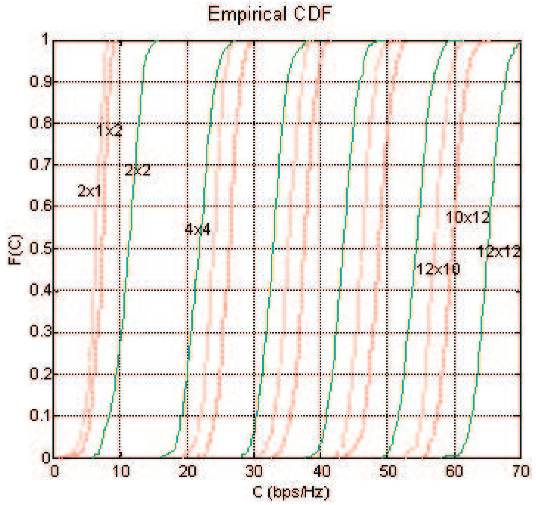

Fig. 3. Uncorrelated channel, signal to noise ratio 20 dB.

From figures IV-A and IV-A several affirmations can be made. The first one is that the capacity curves keeping constant the number of transmitters show higher slope when increasing the number of receivers that when keeping constant the number of receivers and increasing the number of transmitters, thus, the asymptotic value is reached before when keeping constant the number of receivers.

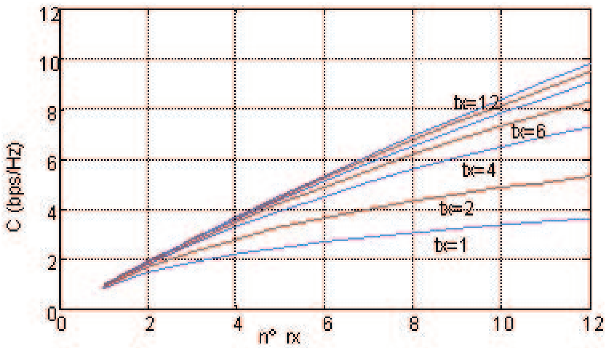

Fig. 4. Uncorrelated channel, signal to noise ratio 0 dB.

Two explanations might be issued for this behavior. The channel capacity gain is less when varying the number of transmitters and keeping constant the number of receivers since the power transmitted by each antenna decreases when increasing its number. The second explanation related to the asymptotic behavior is given by the fact that the limiting parameter for channel capacity is given by the minimum number of elements either in transmission or in reception. Consequently when one of the values is greater than the other the limiting parameter is the smaller.

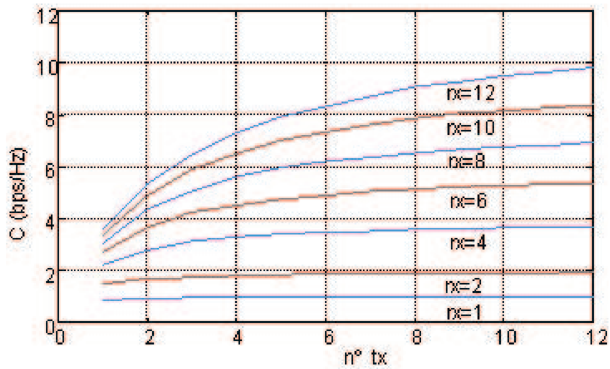

Fig. 5. Uncorrelated channel, signal to noise ratio 0 dB.

Another analysis can be made regarding the signal to noise ratio in the system. In a SISO system, increasing the channel capacity in 1 bit/symbol implies a signal to noise ratio increase of 3 dB. In a MIMO system with spatial uncorrelation, the channel gain achieved for each 3 dB of signal to noise ratio increase is the number of elements that are being used in the system. This is shown in figure IV-A.

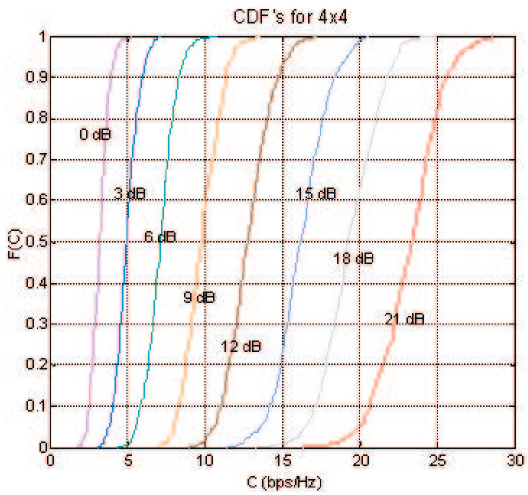

Fig. 6. Uncorrelated channel, signal to noise ratio variation with 4 transmitting and receiving antennas.

B. Spatially correlated environments

The two levels of correlation are shown in the following figures, where like in the previous section the channel capacity increase is shown for different number of transmitting and receiving elements.

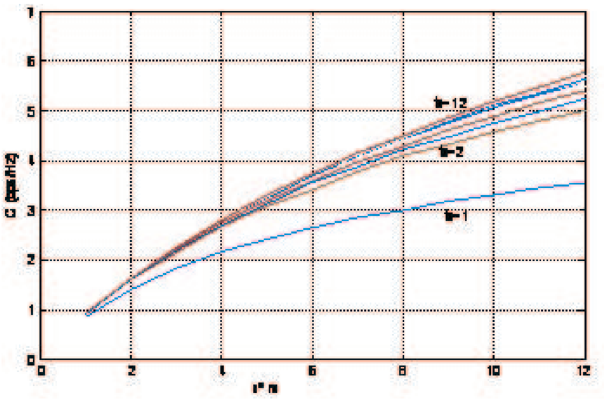

Fig. 7. Case 2 channel, signal to noise ratio 0 dB.

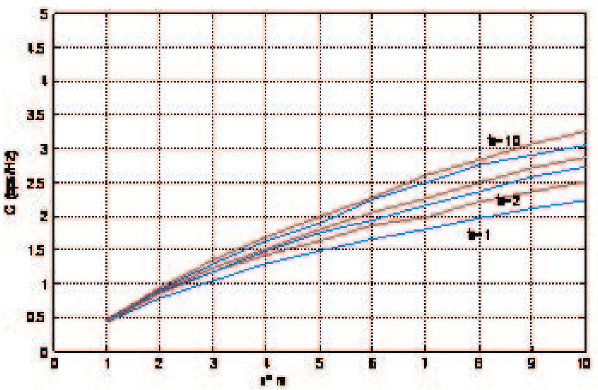

Fig. 8. Case 3 channel, signal to noise ratio 0 dB.

It can be observed that the capacity growth is bigger in Case 2 channel than in Case 3. The same way in both cases it is pretty clear that the capacity reaches a saturation level when increasing the number of transmitters, the reason for that is that in this situation the number of elements in the system is not governing any more the capacity growth, being the correlation level the leading parameter.

Similarly, the behavior related to the dependence with the signal to noise ratio shows channel capacity loss due to correlation. It should be noted that in the uncorrelated environment for each 3 dB increase in the signal to noise ratio, the increase in bit/symbol is as much as the minimum number of elements. In the correlated case it can be observed in the following figures that this increase in bit/symbol is less than the minimum number of antennas.

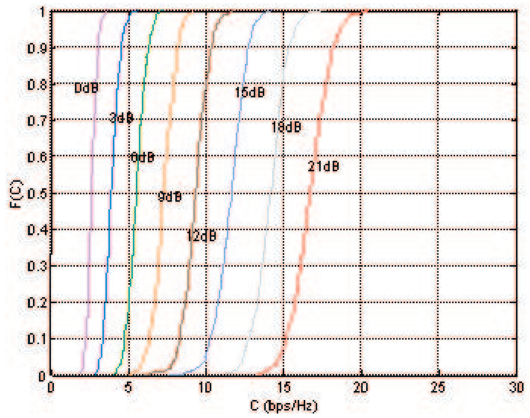

Fig. 9. Case 2 correlated channel. 4 transmitting and receiving antennas.

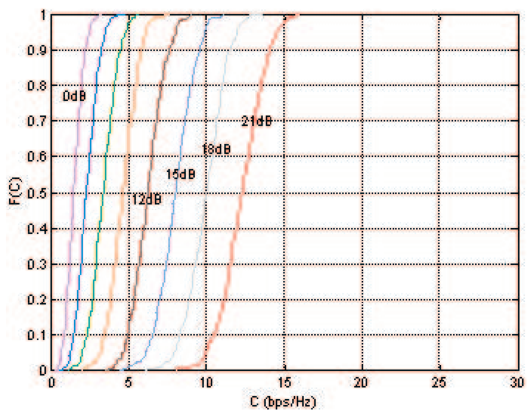

Fig. 10. Case 3 correlated channel. 4 transmitting and receiving antennas.

The results show that for 4 transmitting and receiving elements the channel capacity increase for every 3 dB in the signal to noise ratio is bellow 4 bit/symbol, and goes for 3 bit/symbol for Case 2 to 2 bit/symbol for Case 3.

Finally we will show a comparative graph to clearly illustrate what was previously stated.

The channel capacity loss can be observed in figure IV-B where the CDF degradation is evident.

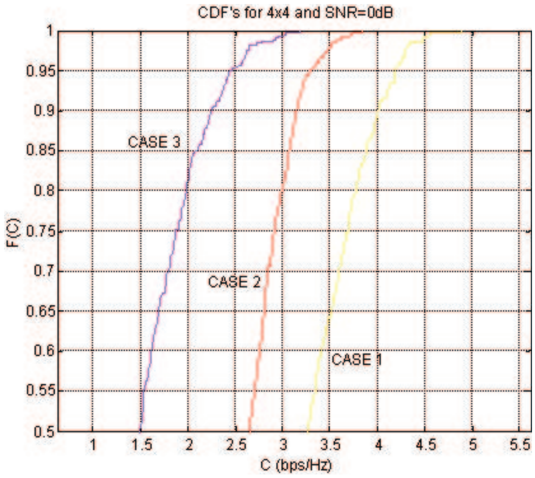

Fig. 11. Case 3 correlated channel. 4 transmitting and receiving antennas.

V. Conclusions

In this paper several channel environments have been analyzed in order to study the channel capacity behavior under different parameters, such as number of transmitters, receivers, the signal to noise ratio and the spatial correlation. It is shown that spatial correlation is a key issue that limits the channel capacity gain. The same way the convenience of using receiver diversity instead of transmitter is shown by means of the capacity gain when increasing the number of transmitters and receivers.

References

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