Turbo-Equalization for Single-Carrier Transmission over MIMO Broadband Wireless Channel: Performance/complexity trade-offs

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***Abstract*— This paper addresses the issue of advanced equalization methods for space-time communications over MIMO broadband wireless channel. Instead of resorting to conventional multiuser detection techniques (based on the straightforward analogy between antennas and users), we adopt a different point of view and separate time equalization from space equalization, thus introducing a higher degree of freedom in the overall space-time equalizer design. The linear part of the equalizer is derived in both time and frequency domains.**

I. Introduction

We propose a class of powerful iterative (turbo) equalizers for interleaved Space-Time Codes (STC) transmitted over MIMO broadband wireless channel and apply them to the decoding of STBICM. Our concern is to solve the problem of InterSymbol Interference (ISI) and Co-Antenna Interference (CAI) cancellation with polynomial (at most cubic) complexity in all system parameters, while performing as close as possible from the theoretical available benchmarks, namely the (coded) Matched-Filter Bound (MFB) and the channel outage capacity. Our equalizing strategy is inspired from the seminal papers by Wang and Poor [1] and Tüchler et al. [2] and motivated by the observation that, far from being trivial, the space-time generalization offers interesting degrees of freedom in the receiver design. Indeed, in contrast with many others capitalizing on the analogy between MultiUser Interference (MUI) and CAI (e.g., [3]), the original approach adopted here views the MIMO signaling as nothing but the transmission of a multidimensional space modulation on a SIMO channel. This shift in viewpoint naturally calls for decoupling ISI cancellation, performed in Minimum Mean Square Error (MMSE) multidimensional sense, from CAI resolution, performed in Maximum A Posteriori (MAP) sense. Such a functional split is the core contribution of the paper. We derive the turbo-equalizer in a quite generic way in order to highlight its suitability to many interleaved space-time coding schemes, including among others Space-Time Bit-Interleaved Coded Modulation (STBICM). The paper is organized as follows. Section II introduces the communication model. Section III is the core of our contribution, where the proposed multidimensional turbo-equalization strategy is derived in both time [4] and frequency domains [5] [6]. Section IV is devoted to numerical results. Section V concludes the paper.

Notation

- The superscripts $^{\mathsf{T}}$ and † indicate conjugate, transpose and Hermitian transpose, respectively.
- diag$\{.\}$, tr$\{.\}$ and det$\{.\}$ denote diagonal, trace, and determinant operators on square matrices, respectively.
- We use the proportionality symbol $\propto$ in order to indicate that the quantity in the RHS is defined up to a multiplicative factor chosen to produce a true probability mass function (pmf) or probability density function (pdf).
- $\mathbf{U}_L$ denotes the DFT unitary matrix of dimension $L \times L$ with entries $[\mathbf{U}_L]_{n,k} = \frac{1}{\sqrt{L}} e^{-j\frac{2\pi nk}{L}}$.
- The block DFT matrix of dimension $NL \times NL$ is simply defined as $\mathbf{U}_{L,N} = \mathbf{U}_L \otimes \mathbf{I}_N$ where $\otimes$ is the Kronecker product.

II. Communication model

We consider per block Single-Carrier Transmission (SCT) over a MIMO frequency-selective channel with N_T inputs, N_R outputs and memory M. No information about the channel is available at the transmitter and perfect channel state information is available at the receiver. A space-time code is employed whose codewords take the form of complex matrices $\mathbf{X}$ of dimension $N_T \times L$ (L being the length in channel uses). On each transmit antenna dimension, a complex constellation $\mathcal{X}_t$ is applied. As already pointed out in the introduction, an alternative and fruitful shift in viewpoint consists in seeing codewords $\mathbf{X}$ of length L as produced by a coded N_T-dimensional modulation with alphabet $\mathcal{X} = \mathcal{X}_0 \times \ldots \times \mathcal{X}_{N_T-1}$. The (possibly non-linear) overall encoding process Ψ which maps binary data vectors $\mathbf{m}$ of dimension k_o into space-time codewords $\mathbf{X}$ comprises coding, space-time interleaving (at bit or symbol level) and modulation. The spectral efficiency is given by $\eta = k_o/L$ (in bits/c.u). Without any knowledge about the channel at the transmitter, having $\mathbb{E}\{\mathbf{x}_k \mathbf{x}_k^{\dagger}\} = \mathbf{I}_{N_T}$, $\forall k$, is the most natural power allocation strategy. The block transmission context allows us to assume that a cyclic prefix is inserted turning the complex matrix $\mathbf{X}$ into $\mathbf{X}' = \mathbf{X}\mathbf{A}_{CP}$ of dimension $N_T \times (L+M)$ where $\mathbf{A}_{CP}$ is the matrix of dimension $L \times (L+M)$ given by

$$\mathbf{A}_{CP} = \begin{bmatrix} \mathbf{0}_{(L-M)\times M} & \mathbf{I}_{L-M} & \mathbf{0}_{(L-M)\times M} \\ \mathbf{I}_M & \mathbf{0}_{M\times(L-M)} & \mathbf{I}_M \end{bmatrix}. \quad (1)$$

We also assume that the channel is quasi-static, i.e., constant for $L+M$ channel uses. The channel is fully characterized by its Finite Impulse Response (FIR) made of $M+1$ non-zero symbol-spaced matrix taps $\mathbf{H}_0, ..., \mathbf{H}_M$ of dimension $N_R \times N_T$ with zero-mean circularly symmetric complex Gaussian entries satisfying the normalization mean power constraint

$$\mathbb{E}\left[\text{diag}\left\{\sum_{m=0}^{M} \mathbf{H}_m \mathbf{H}_m^\dagger\right\}\right] = N_T \mathbf{I}_{N_R}. \quad (2)$$

Let $\mathbf{Y}^{'}$ denote the receive matrix of dimension $N_R \times (L+M)$. After cyclic prefix deletion, we obtain a new matrix $\mathbf{Y} = \mathbf{Y}^{'}\mathbf{B}_{CP}$ of dimension $N_R \times L$ where

$$\mathbf{B}_{CP} = \begin{bmatrix} \mathbf{0}_{M\times L} \\ \mathbf{I}_L \end{bmatrix} \quad (3)$$

whose columns $\mathbf{y}_k \in \mathbb{C}^{N_R},\ k = 0, \cdots, L-1$, verify

$$\mathbf{y}_k = \sum_{i=0}^{M} \mathbf{H}_i \mathbf{x}_{(k-i) \bmod L} + \mathbf{w}_k \quad (4)$$

The noise vectors $\mathbf{w}_k \in \mathbb{C}^{N_R}$ are assumed zero-mean independent identically distributed (i.i.d) circularly symmetric complex Gaussian with covariance matrix $\mathbb{E}\{\mathbf{w}_k\mathbf{w}_k^\dagger\} = \sigma^2 \mathbf{I}_{N_R}$. They form the matrix noise $\mathbf{W}$ of dimension $N_R \times L$. Note that model (4) is also achieved with the so-called Zero Padding (ZP) with OverLap-Add (OLA) technique [7] (noise slightly colored) or with the SCT with IFDMA access technique [8]. Model (4) can be rewritten in time domain under the matrix form

$$\underline{\mathbf{y}} = \mathcal{H}_c \underline{\mathbf{x}} + \underline{\mathbf{w}} \quad (5)$$

where $\underline{\mathbf{x}} = vec(\mathbf{X})$, $\underline{\mathbf{y}} = vec(\mathbf{Y})$ and $\underline{\mathbf{w}} = vec(\mathbf{W})$ and where $\mathcal{H}_c$ is the block circulant matrix of dimension $N_R L \times N_T L$ whose first column is made of the channel impulse response appended by $(L-M-1)$ zeros. As a result, $\mathcal{H}_c$ can be block diagonalized in the Fourier basis as $\mathcal{H}_c = \mathbf{U}_{L,N_R}^\dagger \mathbf{\Lambda} \mathbf{U}_{L,N_T}$ with $\mathbf{\Lambda}$ =diag$\{\mathbf{\Lambda}_0, \cdots, \mathbf{\Lambda}_{L-1}\}$ where

$$\mathbf{\Lambda}_n \triangleq \sum_{k=0}^{M} \mathbf{H}_k e^{-j\frac{2\pi k n}{L}} \quad (6)$$

Hence, a frequency domain equivalent of (5) is

$$\underline{\mathbf{y}}_f = \mathbf{\Lambda} \underline{\mathbf{x}}_f + \underline{\mathbf{w}}_f \quad (7)$$

III. Multidimensional turbo-equalization

A. Brief review of the concept

As well-known, optimal Maximum Likelihood (ML) decoding of any layered interleaved space-time coding scheme is an intractable issue. To the best of the authors' knowledge, the only option to recover the ML ensemble performance is to scatter the problem into two distinct tasks, namely joint probabilistic space-time equalization together with symbol detection and channel decoding, and to resort to the turbo-principle [9]. As a specific feature, our proposed turbo-equalizer scatters the joint probabilistic space-time equalization and symbol detection into two separate stages. Suppose that, at current iteration i, a set of pmfs $\{\mathbf{P}_{t,k}^{(i-1)} \triangleq \{P_{t,k}^{(i-1)}(a) : a \in \mathcal{X}_t\}\}$ on symbols $\{x_{t,k}\}$ in $\mathbf{X}$ be available (coming from some external decision device at iteration $i-1$). At first iteration, all pmfs are uniform. The turbo-equalizer starts delivering an MMSE estimate $\mathbf{z}_k^{(i)}$ on vector symbol $\mathbf{x}_k$ given the matrix received symbol $\mathbf{Y}$ and a set of pmfs $\{\mathbf{P}_{k'}^{(i-1)} \triangleq \{P_{k'}^{(i-1)}(\mathbf{a}) : \mathbf{a} \in \mathcal{X}\}\}$ on all vector symbols $\mathbf{x}_{k'}$, $k' \neq k$ ($\mathbf{x}_k$ is premised uniformly distributed). Each pmf $P_{k'}^{(i-1)}(\mathbf{a})$ plays the role of the prior probability that $\mathbf{x}_{k'} = \mathbf{a}$ at iteration $i-1$ and, assuming independence between components in vector symbol $\mathbf{x}_{k'}$, can be evaluated as

$$P_{k'}^{(i-1)}(\mathbf{a}) = \prod_{t=0}^{N_T-1} P_{t,k'}^{(i-1)}(a_t) \quad (8)$$

Independence property is ensured in practice by some space-time interleaving at symbol or bit level. The MMSE estimate $\mathbf{z}_k^{(i)}$ of vector symbol $\mathbf{x}_k$ takes the form

$$\mathbf{z}_k^{(i)} = \mathbf{G}_k^{(i)} \mathbf{x}_k + \zeta_k^{(i)} \quad (9)$$

where $\mathbf{G}_k^{(i)}$ is a square matrix of dimension N_T and where $\zeta_k^{(i)} \in \mathbb{C}^{N_T}$ is a zero-mean random vector with conditional covariance matrix $\mathbf{\Theta}_{\zeta_k}^{(i)}$. Processing each estimate $\mathbf{z}_k^{(i)}$ separately, a MAP detector is applied for CAI resolution, which computes the EXTrinsic (EXT) pmf $\mathbf{Q}_{t,k}^{(i)} \triangleq \{Q_{t,k}^{(i)}(a) : a \in \mathcal{X}_t\}$ on each symbol $x_{t,k}$, given the set of prior pmfs $\{\mathbf{P}_{t',k}^{(i-1)}\}$ on all other components $x_{t',k}$, $t' \neq t$ ($x_{t,k}$ is postulated uniformly distributed) as

$$Q_{t,k}^{(i)}(a) \propto \sum_{\mathbf{a}\in\mathcal{X}: a_t = a} p(\mathbf{z}_k^{(i)} | \mathbf{a}) \prod_{t'\neq t} P_{t',k}^{(i-1)}(a_{t'}) \quad (10)$$

This computation is carried out assuming $\mathbf{z}_k^{(i)} | \mathbf{a} \sim \mathcal{CN}(\mathbf{G}_k^{(i)}\mathbf{a}, \mathbf{\Theta}_{\zeta_k}^{(i)})$ (Gaussian approximation). Space-time equalization is followed by a decoding pass. Using the pmfs $\mathbf{Q}_{t,k}^{(i)}$'s as observation and taking into account the code constraints, the EXT pmfs $\mathbf{P}_{t,k}^{(i)}$'s are refined as

$$P_{t,k}^{(i)}(a) \propto \sum_{\mathbf{A}=\mathbf{\Psi}(\mathbf{m})\in\mathcal{S}: a_{t,k}=a} \prod_{(t',k')\neq(t,k)} Q_{t',k'}^{(i)}(a_{t',k'}) \quad (11)$$

Finally, the calculation of the A Posteriori pmfs (APP) on symbol $x_{t,k}$

$$APP_{t,k}^{(i)}(a) \propto P_{t,k}^{(i)}(a) Q_{t,k}^{(i)}(a) \quad (12)$$

completes an iteration. While evaluating the MMSE estimate $\mathbf{z}_k^{(i)}$, a possible variant of this algorithm consists in replacing the set of pmfs $\{\mathbf{P}_{k'}^{(i-1)}\}$, $k' \neq k$ by the set of APPs $\{\mathbf{APP}_{k'}^{(i-1)} \triangleq \{APP_{k'}^{(i-1)}(\mathbf{a}) : \mathbf{a} \in \mathcal{X}\}\}$, $k' \neq k$ defined as

$$APP_{k'}^{(i-1)}(\mathbf{a}) \propto \prod_{t=0}^{N_T-1} APP_{t,k'}^{(i-1)}(a_t) \quad (13)$$

Note that, in the STBICM setting (favored in the simulation Section), pmfs $\mathbf{P}_{t,k}^{(i)}$'s are constructed from the EXT pmfs on coded bits fed back by the outer decoder. Marginal EXT pmfs on symbol digits have to be extracted (by simple marginalization) from the $\mathbf{Q}_{t,k}^{(i)}$'s and serve (after space-time de-interleaving) as intrinsic pmfs on coded bits for the outer decoder. In the rest of the paper, iteration index i is omitted for the sake of notation simplicity.

B. Derivation in time domain

Given the $\mathbf{P}_k$'s (resp. the $\mathbf{APP}_k$'s), we can compute the conditional vector mean $\widetilde{\mathbf{x}}_k \triangleq \mathbb{E}\{\mathbf{x}_k\,|\mathbf{P}_k\}$ and the conditional covariance $\mathbf{\Theta}_{\mathbf{x}_k} \triangleq \mathbb{E}\{(\mathbf{x}_k - \widetilde{\mathbf{x}}_k)(\mathbf{x}_k - \widetilde{\mathbf{x}}_k)^\dagger\,|\mathbf{P}_k\}$ of each vector symbol $\mathbf{x}_k$. In compliance with model (5), let us introduce the stacked vectors $\underline{\widetilde{\mathbf{x}}} \triangleq \mathbb{E}\{\underline{\mathbf{x}}\,|\{\mathbf{P}_k\}\}$ and $\underline{\widetilde{\mathbf{x}}}_{|k} \triangleq \mathbb{E}\{\underline{\mathbf{x}}\,|\{\mathbf{P}_{k'} : k' \neq k\}\}$ and let $\mathbf{E}_k$ be the matrix of dimension $N_T L \times N_T$ defined as

$$\mathbf{E}_k = \left[\begin{array}{ccc} \mathbf{0}_{N_T \times N_T k} & \mathbf{I}_{N_T} & \mathbf{0}_{N_T \times N_T(L-k-1)} \end{array}\right]^{\mathsf{T}} \tag{14}$$

The achieving of the biased MMSE estimate $\mathbf{z}_k$ results from the following canonical decomposition.

First step: The stacked observation vector $\underline{\mathbf{y}}$ is rendered zero-mean by subtracting the mean $\underline{\widetilde{\mathbf{y}}}_{|k}$ conditioned to $\{\mathbf{P}_{k'} : k' \neq k\}$, expressed as

$$\underline{\widetilde{\mathbf{y}}}_{|k} \triangleq \mathcal{H}_c \underline{\widetilde{\mathbf{x}}}_{|k} = \mathcal{H}_c\left(\underline{\widetilde{\mathbf{x}}} - \mathbf{E}_k \mathbf{E}_k^\dagger \underline{\widetilde{\mathbf{x}}}\right) \tag{15}$$

Second step: A biased estimate $\mathbf{z}'_k$ of the vector $\mathbf{x}_k$ is simply given by the output of the N_T-dimensional Wiener filter $\mathbf{F}'_k$ applied to the zero-mean observation $\underline{\mathbf{y}} - \underline{\widetilde{\mathbf{y}}}_{|k}$ and minimizing the conditional Mean Square Error (MSE)

$$\mathbb{E}\left\{\left[\mathbf{x}_k - \mathbf{F}'_k\left(\underline{\mathbf{y}} - \underline{\widetilde{\mathbf{y}}}_{|k}\right)\right]\left[\mathbf{x}_k - \mathbf{F}'_k\left(\underline{\mathbf{y}} - \underline{\widetilde{\mathbf{y}}}_{|k}\right)\right]^\dagger \,|\{\mathbf{P}_{k'} : k' \neq k\}\right\} \tag{16}$$

in the sense of the stochastic matrix innerproduct $\langle \mathbf{x}, \mathbf{y} \rangle \triangleq \mathbb{E}\left\{\mathbf{x}\mathbf{y}^\dagger\,|\{\mathbf{P}\}\right\}$[1]. The projection theorem together with the inversion lemma yield the following expression

$$\mathbf{F}'_k = \mathbf{C}_k^{-1}\mathbf{E}_k^\dagger \mathbf{H}_c^\dagger \mathbf{A}^{-1} \tag{17}$$

in which we have introduced the following matrices

$$\mathbf{A} = \mathcal{H}_c \mathbf{\Theta}_{\underline{\mathbf{x}}} \mathcal{H}_c^\dagger + \sigma^2 \mathbf{I}_{N_R L} \tag{18}$$

$$\mathbf{G}_k = \mathbf{E}_k^\dagger \mathcal{H}_c^\dagger \mathbf{A}^{-1} \mathcal{H}_c \mathbf{E}_k \tag{19}$$

$$\mathbf{C}_k = \mathbf{I}_{N_T} + \mathbf{G}_k\left(\mathbf{I}_{N_T} - \mathbf{\Theta}_{\mathbf{x}_k}\right) \tag{20}$$

with

$$\begin{aligned}\mathbf{\Theta}_{\underline{\mathbf{x}}} &= \mathbb{E}\left\{\left(\underline{\mathbf{x}} - \underline{\widetilde{\mathbf{x}}}\right)\left(\underline{\mathbf{x}} - \underline{\widetilde{\mathbf{x}}}\right)^\dagger |\{\mathbf{P}_k\}\right\} \\ &= \text{diag}\left\{\mathbf{\Theta}_{\mathbf{x}_0}, \ldots, \mathbf{\Theta}_{\mathbf{x}_{L-1}}\right\}\end{aligned} \tag{21}$$

Indeed, if infinitely deep space-time interleaving is assumed, both time independence between vector symbol holds and space independence between vector components hold and the covariance matrix $\mathbf{\Theta}_{\underline{\mathbf{x}}}$ is block diagonal. The output of the Wiener filter $\mathbf{F}'_k$ is given by

$$\mathbf{z}'_k = \mathbf{F}'_k\left(\underline{\mathbf{y}} - \underline{\widetilde{\mathbf{y}}}_{|k}\right) = \mathbf{C}_k^{-1}\mathbf{G}_k \mathbf{x}_k + \zeta'_k \tag{22}$$

Third step: Since the Signal-to-interference plus Noise Ratio (SINR) per antenna at the output of the equalizer is invariant by left multiplication by any square constant invertible matrix of dimension N_T, we can rather consider the output

$$\mathbf{z}_k = \mathbf{C}_k \mathbf{z}'_k = \mathbf{G}_k \mathbf{x}_k + \zeta_k \tag{23}$$

[1] Note that such a MSE implies the minimization of the MSE associated with the conventional stochastic innerproduct $\langle \mathbf{x}, \mathbf{y} \rangle \triangleq \mathbb{E}\left\{\mathbf{x}^\dagger \mathbf{y}\,|\{P(\cdot)\}\right\} = \mathbb{E}\left\{\text{tr}\left\{\mathbf{x}\mathbf{y}^\dagger\,|\{P(\cdot)\}\right\}\right\}$.

of the filter $\mathbf{F}_k = \mathbf{C}_k \mathbf{F}'_k = \mathbf{E}_k^\dagger \mathcal{H}_c^\dagger \mathbf{A}^{-1}$. In (23), $\zeta_k \in \mathbb{C}^{N_T}$ is a zero-mean random vector of (residual ISI+thermal) noise with conditional covariance matrix

$$\mathbf{\Theta}_{\zeta_k} = \mathbb{E}\left\{\zeta_k \zeta_k^\dagger\,|\{\mathbf{P}_{k'} : k' \neq k\}\right\} = \mathbf{G}_k\left(\mathbf{I}_{N_T} - \mathbf{\Theta}_{\mathbf{x}_k}\mathbf{G}_k\right) \tag{24}$$

While neglecting the time correlation between the ζ_k's is common in approaches based on Wiener filtering, the spatial correlation $\mathbf{\Theta}_{\zeta_k}$ must be taken into account and actually plays an essential role for CAI detection, as demonstrated in the simulation section. Under Genie-Aided Decoding (GAD) assumption (i.e., $\underline{\widetilde{\mathbf{x}}} = \underline{\mathbf{x}}$), the conditional filter $\mathbf{F}_k$ and the corresponding conditional correlation error matrix $\mathbf{\Theta}_{\zeta_k}$ become

$$\mathbf{F}_k^{GAD} = \mathbf{E}_k^\dagger \mathcal{H}_c^\dagger \tag{25}$$

and

$$\mathbf{\Theta}_\zeta^{GAD} = \frac{1}{\sigma^2}\sum_{m=0}^{M} \mathbf{H}_m^\dagger \mathbf{H}_m. \tag{26}$$

Hence, by further extending the Genie-Aided Decoding (GAD) assumption to CAI detection, it is straightforward to demonstrate that the maximum per-antenna SNRs, usually referred to as Matched Filter (MF) SNRs,

$$\gamma_t^{MF} = \left[\frac{1}{\sigma^2}\sum_{m=0}^{M} \mathbf{H}_m^\dagger \mathbf{H}_m\right]_{t,t}, \ \forall t \tag{27}$$

are reached. Finally, considering $\underline{\mathbf{z}} = vec(\mathbf{Z})$, we obtain the block output expression

$$\underline{\mathbf{z}} = \mathbf{F}\underline{\mathbf{y}} - \mathbf{B}\underline{\widetilde{\mathbf{x}}} \tag{28}$$

whose interest is to give prominence to a block forward and a block backward filters, respectively defined as

$$\mathbf{F} = \mathcal{H}_c^\dagger \mathbf{A}^{-1} \tag{29}$$

$$\mathbf{B} = \mathcal{H}_c^\dagger \mathbf{A}^{-1}\mathcal{H}_c - \text{diag}\left\{\mathbf{G}_0, ..., \mathbf{G}_{L-1}\right\} \tag{30}$$

Estimate $\mathbf{z}_k$ is simply extracted from $\underline{\mathbf{z}}$ as $\mathbf{z}_k = \mathbf{E}_k^\dagger \underline{\mathbf{z}}$. The complexity of this block approach is largely dominated by the inversion of the square matrix $\mathbf{A}$ of dimension $N_R L$ involved in $\mathbf{F}_k$. Indeed, due to the presence of $\mathbf{\Theta}_{\underline{\mathbf{x}}}$, matrix $\mathbf{A}$ is not block circulant despite the CP operations. Moreover, the time dependency in $\mathbf{\Theta}_{\zeta_k}$ increases the complexity of the post detector: if the MAP criterion is retained for CAI resolution, one has to compute $\mathbf{\Theta}_{\zeta_k}^{-1}$ for each k.

C. Transposition in frequency domain

In order to remedy the two aforementioned complexity impairments, the so-called unconditional approximation [10] is the key, which basically consists in minimizing the ensemble (or block) Mean Square Error (MSE) instead of the conditional instantaneous MSE. As a consequence, the conditional covariance matrix $\mathbf{\Theta}_{\underline{\mathbf{x}}}$ in $\mathbf{A}$ is replaced by its ensemble average

$$\mathbf{\Xi}_{\underline{\mathbf{x}}} = \mathbb{E}\left\{\mathbf{\Theta}_{\underline{\mathbf{x}}}\right\} = \mathbf{I}_L \otimes \mathbf{\Xi}_{\mathbf{x}} \tag{31}$$

where $\mathbf{\Xi}_{\mathbf{x}} = \mathbb{E}\left\{\mathbf{\Theta}_{\mathbf{x}_k}\right\}$. In practice, $\mathbf{\Xi}_{\mathbf{x}}$ is computed using the consistent estimator

$$\mathbf{\Xi}_{\mathbf{x}} \simeq \frac{1}{L}\sum_{k=0}^{L-1} \mathbf{\Theta}_{\mathbf{x}_k} \tag{32}$$

When we substitute $\Xi_{\underline{\mathbf{x}}}$ for $\Theta_{\underline{\mathbf{x}}}$ in matrix $\mathbf{A}$, the latter, now expressed as $\mathcal{H}_c\Xi_{\underline{\mathbf{x}}}\mathcal{H}_c^{\dagger}+\sigma^2\mathbf{I}_{N_RL}$, becomes block circulant and can be block diagonalized in the Fourier basis. In other words, the unconditional approximation turns out to be essential to yield an efficient way of computing $\mathbf{A}^{-1}$ in the Fourier domain. Moreover, since the matrix $\mathcal{H}_c^{\dagger}\mathbf{A}^{-1}\mathcal{H}_c$ is also block circulant, the coefficient $\mathbf{G}_k$ does not depend on time index k any more and can be nicely expressed in the frequency domain as

$$\mathbf{G}_k = \mathbf{G} = \frac{1}{L}\sum_{n=0}^{L-1}\mathbf{\Lambda}_n^{\dagger}\left(\mathbf{\Lambda}_n\Xi_{\mathbf{x}}\mathbf{\Lambda}_n^{\dagger}+\sigma^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{\Lambda}_n,\ \forall k \quad (33)$$

Finally, the time dependency in Θ_{ζ_k} is solved considering its ensemble average as well, which yields

$$\Xi_{\zeta} = \mathbf{G}\left(\mathbf{I}_{N_T}-\Xi_{\mathbf{x}}\mathbf{G}\right) \quad (34)$$

Model (28) can be transposed into the frequency domain as

$$\underline{\mathbf{z}}_f = \mathbf{\Phi}\underline{\mathbf{y}}_f - \mathbf{\Psi}\underline{\widetilde{\mathbf{x}}}_f \quad (35)$$

where $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are the frequency domain expressions of $\mathbf{F}$ and $\mathbf{B}$, respectively, given by

$$\mathbf{\Phi} = \mathbf{\Lambda}^{\dagger}\left(\mathbf{\Lambda}\Xi_{\underline{\mathbf{x}}}\mathbf{\Lambda}^{\dagger}+\sigma^2\mathbf{I}_{N_RL}\right)^{-1} \quad (36)$$

$$\mathbf{\Psi} = \mathbf{\Lambda}^{\dagger}\left(\mathbf{\Lambda}\Xi_{\underline{\mathbf{x}}}\mathbf{\Lambda}^{\dagger}+\sigma^2\mathbf{I}_{N_RL}\right)^{-1}\mathbf{\Lambda} - \mathbf{I}_L\otimes\mathbf{G} \quad (37)$$

Such an approach obviously requires the inversions of L squares matrices of dimension N_R. The final computational complexity is linear in the space-time code length L and insensitive to the channel memory M.

D. Concurrent sliding-window approach in time domain

Model (5) can always be locally approximated by a sliding-window model of length $L_w = L_1 + L_2 + 1 << L$ defined as

$$\underline{\mathbf{y}}_k = \mathcal{H}_w\underline{\mathbf{x}}_k + \underline{\mathbf{w}}_k \quad (38)$$

where

$$\begin{array}{r}\underline{\mathbf{y}}_k = \left[\underline{\mathbf{y}}_{k-L_2}^{\mathsf{T}} \ \cdots \ \underline{\mathbf{y}}_{k+L_1}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{N_RL_w} \\ \underline{\mathbf{w}}_k = \left[\underline{\mathbf{w}}_{k-L_2}^{\mathsf{T}} \ \cdots \ \underline{\mathbf{w}}_{k+L_1}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{N_RL_w} \\ \underline{\mathbf{x}}_k = \left[\underline{\mathbf{x}}_{k-L_2-M}^{\mathsf{T}} \ \cdots \ \underline{\mathbf{x}}_{k+L_1}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{N_T(L_w+M)}\end{array} \quad (39)$$

and where $\mathcal{H}_w$ is the $N_RL_w \times N_T\left(L_w+M\right)$ expressed as

$$\mathcal{H}_w = \begin{bmatrix} \mathbf{H}_M & \cdots & \mathbf{H}_0 & & \\ & \ddots & & \ddots & \\ & & \mathbf{H}_M & \cdots & \mathbf{H}_0 \end{bmatrix} \quad (40)$$

Introducing

$$\begin{array}{r}\underline{\widetilde{\mathbf{x}}}_k \triangleq \mathbb{E}\{\underline{\mathbf{x}}_k\,|\{\mathbf{P}_{k'} : k-L_2-M \leq k' \leq k+L_1\}\} \\ \underline{\widetilde{\mathbf{x}}}_{k|\triangle} \triangleq \mathbb{E}\{\underline{\mathbf{x}}_k\,|\{\mathbf{P}_{k'} : k-L_2-M \leq k' \leq k+L_1, k'\neq k\}\}\end{array} \quad (41)$$

and $\mathbf{E}_{\triangle}$ the matrix of dimension $N_T\left(L_w+M\right)\times N_T$ as

$$\mathbf{E}_{\triangle} = \left[\begin{array}{ccc}\mathbf{0}_{N_T\times N_T(L_2+M)} & \mathbf{I}_{N_T} & \mathbf{0}_{N_T\times N_TL_1}\end{array}\right]^{\mathsf{T}} \quad (42)$$

the MMSE estimate $\mathbf{z}_k$ takes the form $\mathbf{z}_k = \mathbf{E}_{\triangle}^{\dagger}\underline{\mathbf{z}}_k$ where

$$\underline{\mathbf{z}}_k = \mathbf{F}_k\underline{\mathbf{y}}_k - \mathbf{B}_k\underline{\widetilde{\mathbf{x}}}_k \quad (43)$$

In (43), the forward and backward filters are given by

$$\mathbf{F}_k = \mathcal{H}_w^{\dagger}\mathbf{A}_k^{-1} \quad (44)$$

$$\mathbf{B}_k = \mathcal{H}_w\Theta_{\underline{\mathbf{x}}_k}\mathcal{H}_w^{\dagger} - \text{diag}\left\{\mathbf{G}_{k-L_2-M},\ldots,\mathbf{G}_{k+L_1}\right\} \quad (45)$$

with $\Theta_{\underline{\mathbf{x}}_k}$ =diag$\{\Theta_{\mathbf{x}_{k-L_2-M}},\ldots,\Theta_{\mathbf{x}_{k+L_1}}\}$, $\mathbf{A}_k = \mathcal{H}_w\Theta_{\underline{\mathbf{x}}_k}\mathcal{H}_w^{\dagger}+\sigma^2\mathbf{I}_{N_RL_w}$ and $\mathbf{G}_k = \mathbf{E}_{\triangle}^{\dagger}\mathcal{H}_w^{\dagger}\mathbf{A}_k^{-1}\mathcal{H}_w\mathbf{E}_{\triangle}$. The unconditional approximation can be applied at this level too. The conditional covariance matrix $\Theta_{\underline{\mathbf{x}}_k}$ in $\mathbf{A}_k$ is replaced by its ensemble average

$$\Xi_{\underline{\mathbf{x}}} = \mathbb{E}\left\{\Theta_{\underline{\mathbf{x}}_k}\right\} = \mathbf{I}_{L_w+M}\otimes\Xi_{\mathbf{x}} \quad (46)$$

turning $\mathbf{A}_k$ into $\mathbf{A} = \mathcal{H}_w\Xi_{\underline{\mathbf{x}}}\mathcal{H}_w^{\dagger}+\sigma^2\mathbf{I}_{N_RL_w}$ which does not depend on the time index. The same conclusion holds for $\mathbf{G}_k$ and thus for Θ_{ζ_k}. The sliding-window implementation obviously requires the inversion of matrix $\mathbf{A}$ of dimension L_wN_R. Due to its highly structured nature, efficient algorithms can be used with complexity in $O\left(M^2N_R^3\right)$.

E. Criteria hybridization for ISI and CAI cancellation

In [11], it is demonstrated through Monte-Carlo simulations that if the channel is dispersive enough, the number of antennas sufficiently large, and the load per antenna typically less or equal to 1 bit/c.u, a bank of simple matched filters may eventually replace the N_T Wiener filters during the course of iterations without notable degradation in terms of performance. Our proposed space-time equalizing strategy lends itself well to the design of such very low-complexity configurations, i.e., Max-SNR-based ISI cancellation (in a multidimensional sense) followed by either MMSE-based or MAP-based CAI resolution. It is sufficient to replace $\mathbf{F}_k$ by $\mathbf{F}_k^{GAD}$. The covariance matrix Θ_{ζ_k} becomes

$$\begin{array}{r}\Theta_{\zeta_k} = \sum_{k'\neq k}\mathbf{E}_k^{\dagger}\mathcal{H}_c^{\dagger}\mathcal{H}_c\mathbf{E}_{k'}\Theta_{\mathbf{x}_{k'}}\mathbf{E}_{k'}^{\dagger}\mathcal{H}_c^{\dagger}\mathcal{H}_c\mathbf{E}_k \\ +\sigma^2\left(\sum_{m=0}^{M}\mathbf{H}_m^{\dagger}\mathbf{H}_m\right)\end{array} \quad (47)$$

Moreover, again applying the aforementioned unconditional approximation makes the covariance matrix time independent. For the load per antenna typically greater than 1 bits/c.u, extensive simulations have demonstrated that keeping the MMSE criterion for ISI suppression, at least for the first iteration, is of prime importance to obtain good performance.

IV. Numerical results and discussion

A high-rate STBICM using 16-QAM modulation (Gray labeling) is simulated. The employed code is a rate-3/4 64-state non-recursive convolutional code with $d_{free} = 6$. The transmitted symbol block length is $L = 192$ ($k_o = 2304$ bits included tail) while the cyclic prefix length is equal to the channel memory. Moreover, a simulated coded Matched Filter Bound (coded MFB) which assumes $\underline{\mathbf{x}} = \underline{\widetilde{\mathbf{x}}}$ for both CAI cancellation and N_T-dimensional equalization, is used to evaluate the loss inherent to the sub-optimality of the receiver design. We use pseudo APPs to compute Wiener filters. The CAI resolution (MAP criterion) keeps being fed by EXT pmfs. Transmission occurs on a quasi-static 4×4 MIMO channel with $M = 2$ and EXPonential decreasing taps (EXP3) is considered below : on each link (r,t), the $M+1$ channel coefficients are i.i.d circularly symmetric complex Gaussian following pdfs $\mathcal{CN}(0, \sigma_m^2)$ with $\sigma_m^2 \propto \exp(-2m)$. In Fig. 1, our MMSE-based turbo-receiver performance (MMSE LSD in Fig. 1) is compared to the classical approach (MMSE MMSE in Fig. 1) after 5 iterations. Spectral efficiency is $\eta = 12$ bits/c.u. The MAP criterion for CAI resolution is practically implemented using the List-APP Sphere Decoder (LSD) described in [12]. The spherical list is centered on the ML point instead of the received point. The ML point is computed thanks to an accelerated sphere decoder following the Schnorr-Euchner enumeration strategy. The origin-centered list size was taken equal to $N_p = 1000$ points. The average number of candidates per block and per iteration used here to evaluate the $\mathbf{Q}_{t,k}$'s at the CAI detector output is reported on this table for some SNR values:

E_b/N_0	0	2	4	5	6	7	8
N_p	1419	1526	1542	1545	1549	1556	1553

Note that an optimal MAP detection would require the computation of $2^{16} = 65536$ likelihoods for each binary element. The gain brought by the N_T-dimensional approach reaches 3 dB at BLER 10^{-3} when the distance to the coded MFB is near to 2 dB. In addition, our N_T-dimensional approach has faster convergence. We move now to investigate the behavior of our equalizer when $\mathbf{F}_k^{GAD}$ replaces $\mathbf{F}_k$ from the first iteration (MF-IC LSD in Fig. 1) or from the second iteration (Hybrid in Fig. 1). As expected, we observe that MF-IC LSD fails to converge whereas Hybrid closely approaches the performance of MMSE LSD with much reduced complexity.

V. Conclusion

In this paper, the principles of iterated-decision linear equalization have been extended to interleaved STC over MIMO broadband wireless channel. Contrary to the classical approach inherited from MUD, where ISI and CAI cancellations are jointly performed in MMSE sense for each distinct antenna, our approach decouples the two tasks, allowing an additional degree of freedom in the receiver design. ISI resolution still relies on MMSE criterion and operates on multidimensional modulation symbols, whose individual components can be detected in accordance with another criterion. When the optimum MAP criterion is chosen, substantial performance gains over conventional space-time turbo equalization have been observed for different transmission scenarios, at the price of an increased computational complexity (the latter can be managed, e.g., by resorting to a list-APP sphere decoder). The general concept developed in this paper may be essential to conceive space-time turbo equalizers able to fully exploit multidimensional design optimization attempts at the transmitter (e.g., labeling or linear precoding) and to deal with space correlation between antennas. Due to the lack of space, those topics will be reported in further contributions [6].

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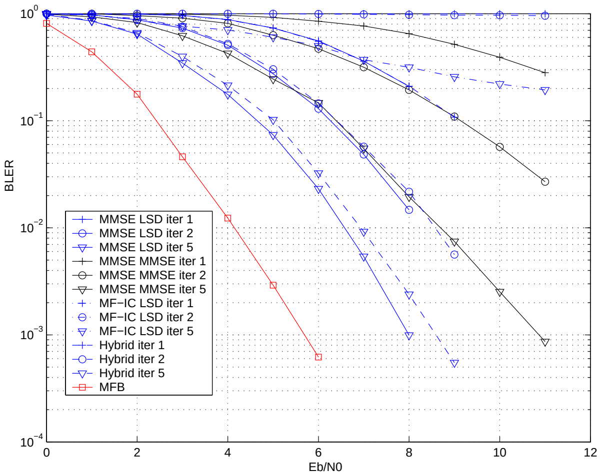

Fig. 1. Block-static System 4×4 EXP3 η=12 bits p.c.u